ELECTRICAL METERS

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96b

ELECTRICAL METERS

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EXTENSION DIVISION OF

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PREFACE

Efficiency is the shibboleth of the modern industrial world. From a physical viewpoint the efficient operation of any plant is mainly a correct application of the laws of conservation and transformation of energy, and hence, the operation of an industrial plant cannot be efficient unless data are available for determining the relation between energy generated or delivered and energy utilized in any manufacturing process.

The data necessary for efficient operation cannot be had unless proper and accurate instruments are used to determine the various quantities that enter into the operation. In any industry where electrical energy is generated or utilized, electrical measuring instruments are necessary for efficient operation.

When the author decided to offer a course treating of Electrical Measuring Instruments, he was surprised to discover that no suitable text was available, and in fact in this country very little had been published treating, in a comprehensive and systematic way, of the various kinds of electrical measuring instruments. The articles in the technical journals and proceedings of technical societies are, of course, numerous and valuable, but they are inaccessible to the average man who may want information concerning the characteristics and principles of operation of some type of measuring instrument.

This text is written primarily to supply the author's needs in correspondence instruction, although it is hoped that others also may find it useful.

Since, in this country, instruments of foreign make are used to such a limited extent, this work is confined almost entirely to instruments made in the United States, and to American practice.

In classifying electrical measuring instruments, the main divisions have been made in accordance with the quantities to be measured, and minor subdivisions according to the principles of operation. Although such a classification necessitates some repetition in describing different instruments whose operation is based on the same principles, nevertheless, for the sake of clearness and simplicity such a classification appears justifiable.

The attempt has been made to explain the fundamental principles in an elementary way, and for this reason many line drawings and vector diagrams are used. The manner in which these fundamental principles are applied in practice is usually exemplified by means of cuts of actual instruments. The illustrations used were selected not because the author considers the instruments better than others not shown, but because they are typical, and used quite extensively.

Great care has been taken to eliminate all errors, yet it is too much to expect that no mistakes will be found. The author will be very grateful to anyone who may discover and report any error.

The author is under great obligations to Dr. M. G. Lloyd, of Chicago; Professor J. P. Jackson of the Pennsylvania State College, and Professor R. C. Disque of The University of Wisconsin for reading the manuscript and for many valuable suggestions. Thanks are also due to Mr. F. C. Thiessen and Mr. G. R. Wells for making the line drawings and vector diagrams; to the several manufacturers of electrical measuring instruments for their kindness in supplying information and electrotypes of their instruments.

C. M. J

The University of Wisconsin,
Madison, Wisconsin.
November 26, 1912.

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ELECTRICAL METERS

CHAPTER I

FUNDAMENTAL ELECTRICAL PRINCIPLES

Before taking up in detail the discussion of electrical measuring instruments and their application, it will be well to review some fundamental electrical principles with special reference to their application.

1. Energy.—The industrial application of electricity is mainly a process of utilizing energy, and the industrial use of electrical measuring instruments is primarily to secure efficient generation, distribution, and conversion of energy. Energy is thus the important entity in all industrial operations. In fact, the whole series of physical phenomena consists in the transfer and transformation of this entity.

Physicists define energy as the ability of a body or a system of bodies to do work; and work is defined as overcoming resistance through space, or in other words, the motion of a body against a force. Every moving body possesses the ability of doing work, because by virtue of its motion it can set other bodies into motion.

2. Forms of Energy.—For purposes of clearness in discussion and calculation, energy is usually considered under two heads which are determined by the manner in which energy manifests itself. As pointed out, a body in motion is capable of causing motion in another body and hence, a body possesses energy by virtue of its motion. Such energy is called Kinetic.

Again, energy may also be possessed by a body in such a position, or condition, that it is capable of motion and ready to do work when the occasion arises. Such energy is called Potential.

These two forms are not distinct in kind, and one form may readily be converted into the other. Perhaps the simplest illustration of the two kinds of energy and the conversion of one form into the other is a vibrating pendulum. At the extreme positions of its swing the pendulum comes momentarily to rest, hence, its energy of motion, or kinetic energy, is zero. All of the energy is in the potential form. At the lowest or middle

point of its swing the energy of the pendulum is wholly kinetic, and at intervening points it is partly kinetic and partly potential.

3. Conservation of Energy.—Throughout all transformations of energy no body or system of bodies can acquire energy except at the expense of energy possessed by some other system. Hence, to do work is to transfer energy from one system to another, and the amount of energy lost by one system is the exact equivalent of that acquired by the other. This means that no electric generator can ever be made to give out more energy than it receives. Some energy is always dissipated in every transformation, and hence, no machine can have an efficiency of 100 per cent.

4. Electricity and Electrical Energy.—We know not what electricity fundamentally is, we know it only through its manifestations or effects. It matters not, so far as practical results are concerned, whether electricity is a form of energy, or only a vehicle of energy. The fact is that energy always is manifest in connection with the electrical current, and that this energy can be transformed into other forms of energy. It may also be transferred from point to point without the necessity of mass motion. It is this ability to transfer energy without mass motion that makes electricity the only successful medium for transferring energy over long distances.

The transformation of electrical energy is electrical work and is accomplished in many ways. The rate of transformation is power just as in the case of an expenditure of mechanical energy.

5. Analogies.—The kinetic energy of a body in motion is proportional to the square of its speed. If m represents the mass of the body and V_1 its speed, its kinetic energy is given by

$$\text{Kinetic energy} = \tfrac{1}{2} m V_1^2$$

When a force acts upon a body in motion its effect is to accelerate the speed of the body, that is, to change its speed from V_1 to V_2. The kinetic energy then is equal to $\frac{1}{2} mV_2^2$. If the action of the force is such as to increase the speed there is an accumulation or storage of energy equal to $\frac{1}{2}m(V_2^2 - V_1^2)$. In an analogous way whenever a current in a circuit is increased, the energy in the magnetic field is increased. The mechanical energy is recovered when the body slows down to its former speed, and the electrical energy is returned to the circuit when the current decreases to its former value.

Another similarity between mechanical and electrical energy is found in their conversion from kinetic into potential form and *vice versa.* Mechanical energy can be changed to the potential form by compressing a spring. Electrical energy becomes potential when a condenser is charged.

This similarity, or analogy is brought out more forcibly by writing the expressions for energy in the following algebraic forms:

$$1. \begin{cases} \text{Mechanical energy of rotation} & = \frac{1}{2} K\omega^2 \\ \text{Energy of magnetic field} & = \frac{1}{2} LI^2 \end{cases}$$

$$2. \begin{cases} \text{Potential energy in compressed spring} & = \frac{1}{2} Px \\ \text{Potential energy in charged condenser} & = \frac{1}{2} QE. \end{cases}$$

The letters in the above expressions have the following significance:

K is the moment of inertia, ω the angular velocity, L the coefficient of induction and I the current strength. In the second set of expressions P is the maximum pressure to which the spring is subjected, x the distance through which the spring has been compressed, Q the quantity of electricity in condenser, and E the difference of electrical pressure between the terminals of the condenser. The terms here used will be explained more fully later.

6. Magnetism.—Magnetic bodies, or magnets, are bodies which attract or repel each other with a force other than gravitation and which tend to set themselves in a definite direction with reference to the earth's surface. The most obvious property of a magnet is its power to attract iron at a distance. A permanent magnet may be made by placing a bar of hardened steel within a solenoid and sending a current of electricity through the solenoid. The immediate space or region surrounding the magnet possesses unique properties. Some of these may be conveniently shown by placing a sheet of paper over a bar magnet and sprinkling iron filings on the paper. On examining the pattern produced on the paper by the filings, it will be discovered that they are arranged in lines radiating from one end of the magnet and converging at the other end, Fig. 1. Another method of exploring the field surrounding a bar magnet is by the aid of a small pocket compass. When a small pocket compass is placed near one end of a magnet, the needle of the compass will point toward the magnet. When the compass is moved along one of the magnetic

lines, it will remain parallel to the line. When the compass is brought near the other end of the magnet, the compass needle will be reversed, showing that the properties of the two ends are different at least in one respect. This experimental fact has lead to a conventional statement that the magnetic lines leave the north-seeking pole of the bar magnet and reënter the south-seeking pole. According to this explanation the magnetic lines are closed curves leaving the north pole, curving through the air, reëntering the south pole, and completing the circuit through the metal. The magnetism about the bar is said to be due to a

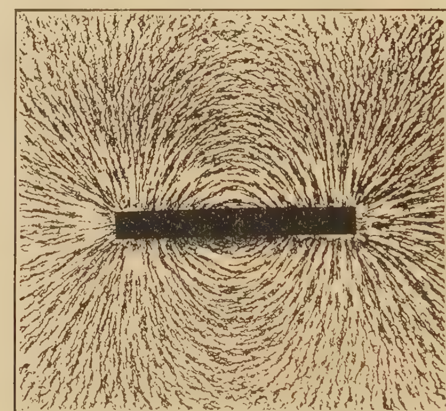


Fig. 1.

magnetomotive force analogous to electromotive force. The magnetomotive force sets up a difference of magnetic pressure between the two ends of the bar, which causes magnetism or magnetic flux from one end of the bar to the other.

7. Properties of Magnetic Fields.—When unlike poles of two magnets are placed near each other the arrangement of the magnetic lines will be as shown in Fig. 2. The lines pass directly from the pole of one to the unlike pole of the other magnet. The attraction between unlike poles may, therefore, be considered as due to a tendency of the lines to shorten.

When two like poles are brought near each other the resulting field is shown by Fig. 3. An inspection of this figure will show that the lines from one pole do not connect with those of the other pole. The repulsion between unlike poles is therefore, due

to a tendency of the lines to repel each other. These figures show that a tension or stress exists in a magnetic field parallel to the lines, and a pressure at right angles to their direction. Further-

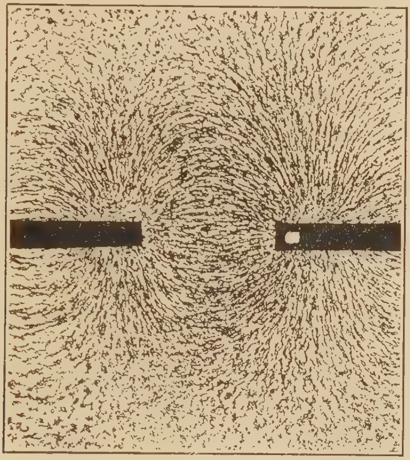


FIG. 2.

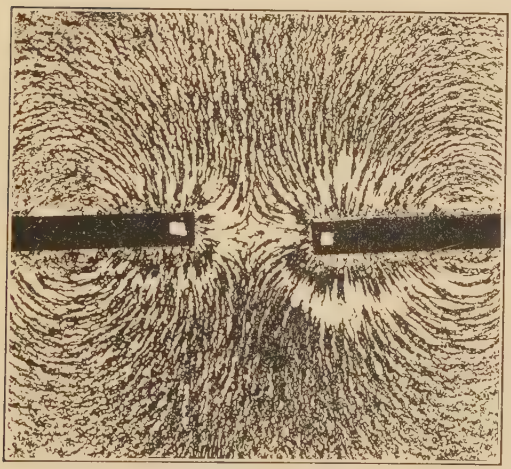


FIG. 3.

more, the force of attraction and repulsion has its seat outside of the iron bar, and within the space surrounding it. This principle should be kept in mind.

8. Strength of Magnetic Field.—The strength of a magnetic field is measured by the force it exerts upon a unit magnet pole. When this force is one dyne the field is said to have unit strength. Since iron filings arrange themselves in lines when subjected to the influence of a magnetic field, it is customary to express the field strength in lines per square centimeter; that is, graphically the density of lines is a measure of the field strength. A field of unit strength is represented by one line per square centimeter in a plane perpendicular to the lines. A field of 10 units would then be represented by 10 lines per square centimeter, etc.

If the cross-sectional area of the field be S square centimeters and if the field strength or density be represented by B the total number of magnetic lines is equal to SB. The total number of lines through a given area is called the magnetic flux. Algebraically

$$\Phi(\text{flux}) = SB$$

9. Relation between Tension and Flux Density.—When a magnetic circuit is made of iron, only a small magnetomotive force is necessary to maintain the magnetic flux. When one or more air gaps intervene, most of the magnetomotive force is utilized in forcing the flux through or across the air gap. As a result, there is manifest a force of attraction between any two parts of a magnetic circuit which are separated by an air gap. Both from experimental and theoretical considerations, it has been determined that this force is proportional to the square of magnetic flux per unit cross-section, or put in algebraic form the tension may be expressed as follows:

$$F = KSB^2,$$

where F is the force, B the flux density, S the area, and K a constant. If F is to be expressed in dynes per square centimeter, the expression becomes

$$F\ (\text{dynes}) = \frac{B^2}{8\pi}$$

In pounds per square inch

$$F\ (\text{pounds}) = \frac{(2.54)^2\ B^2}{445{,}000 \times 8\pi}.$$

10. Magnetic Field Surrounding an Electric Wire.—The behavior of a compass needle near a wire through which a current

is flowing is much the same as when near a magnet. This shows that the space surrounding an electric wire is a magnetic field.

The magnetic lines surround the wire in concentric circles as shown in Fig. 4. The dark spot in the center of the figure repre-

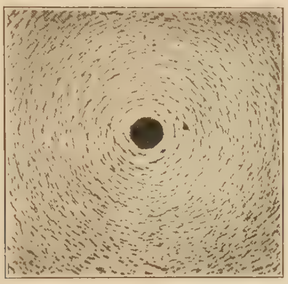


FIG. 4.

sents a cross-section of the wire. The direction of the magnetic lines is determined in accordance with the experiment shown in Fig. 5. If a current passes from south to north along a wire held above a pivoted magnetic needle, the needle will be deflected westward.

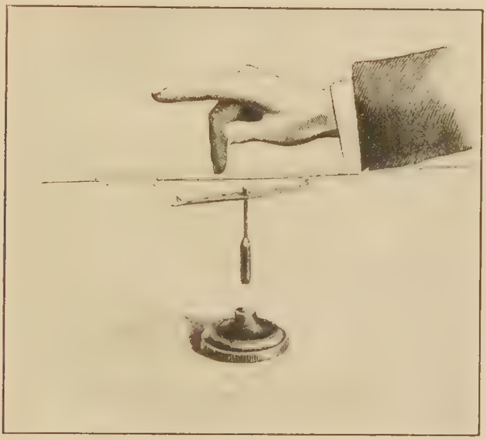


FIG. 5.

The north-seeking end of the magnetic needle is pushed aside by the field surrounding the wire. The direction of the lines below the wire must then be westward. In any given case the direction of the lines may be determined by the following rule:

Grasp the wire with the right hand, the thumb pointing in the direction of the current, the fingers will then point in the direction of the magnetic lines. The magnetic field surrounding a straight wire may be considered as a series of concentric cylinders.

11. Field of a Circular Coil.—When the wire is coiled into a circular loop, the magnetic lines enter one side of the loop, pass through, and spread out at the other side, Fig. 6. The loop of wire thus has the properties of a magnet; the north-seeking pole being where the lines leave, and the south pole where the lines enter the coil. A magnet brought near the loop will be attracted or repelled, depending on whether unlike or like poles are brought near each other.

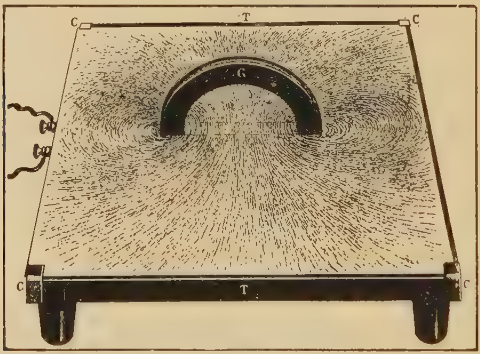


FIG. 6.

12. Solenoids.—It has already been pointed out that, when an electric wire is wound into a coil, all or nearly all the magnetic lines pass in at one end and out at the other. Many types of instruments make use of some form of a solenoid. When the solenoid possesses an iron core, the combination is called an electromagnet. Figs. 7 and 8 show the general distribution of the magnetic lines in both a solenoid and electromagnet. Upon referring to the figures in question, it will be noticed that the lines do not all go the whole length of the solenoid, but some escape between the convolutions. The magnetic field is thus not uniform throughout the length of the solenoid. In fact, the coil has to be of considerable length to get a uniform field of 10 cm. length.

When the solenoid has an iron core, fewer magnetic lines take "short cuts" between the convolutions of the coil, but more lines continue throughout the whole length of the solenoid. This is due to the fact that the iron offers less opposition or reluctance, as it is

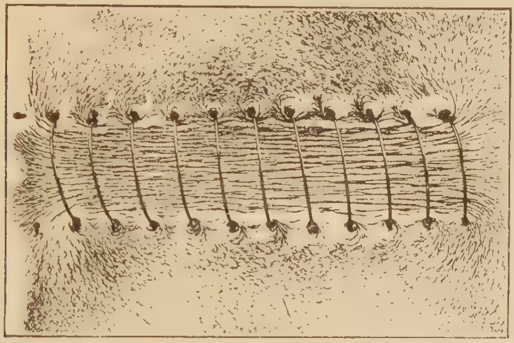


FIG. 7.

called, to the establishment of the magnetic lines than air. It requires less magnetomotive force to establish a given number of magnetic lines in iron than in air, so when a definite magnetic

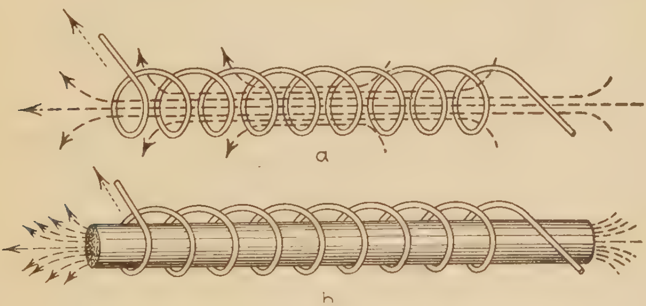


FIG. 8.

field is established within a solenoid, the introduction of an iron core will greatly increase its strength.

The strength of magnetic field within a solenoid is given by the formula

$$H = \frac{4}{10} \pi n I$$

where n is the number of turns per unit length—1 cm.—of solenoid, and I is the current in amperes. This formula applies only to the middle portion of the solenoid where the field is uniform. Toward the ends the value falls off rapidly. When an iron core is placed within the solenoid, the flux density within the iron is

$$B=\frac{4}{10}\pi\mu nI$$

μ is a quantity depending upon the quality of the iron and may have many values under different conditions. It is always greater than one and may be as large as 10,000.

The cores of electromagnets should be made of the softest and purest iron, but for alternating currents the core must be made of laminated iron or iron wire. The laminations prevent the formation of eddy currents which tend to flow at right angles to the direction of the magnetic lines. The elimination of eddy currents prevents excessive heating of the core.

13. Law of the Magnetic Circuit.—The relation between the magnetizing force of a solenoid and the resulting magnetic flux may be conveniently expressed in the form of Ohm's law.

Analogous to resistance in an electrical circuit there is a quantity called reluctance in the magnetic circuit. The readiness with which any given magnetizing force will build up a magnetic field depends upon the permeability, length, and cross-section of the circuit.

The higher the permeability and the greater the cross-sectional area of the circuit the more readily will the magnetic field be established. On the other hand, the longer the circuit the stronger will the magnetizing force have to be to establish a given field. The effect of these physical properties of a magnetic circuit is called reluctance and is equal to

$$\mathcal{R}=\frac{L}{\mu A}$$

Where L is the length of circuit, μ the permeability, and A the cross-section.

The magnetizing force due to a solenoid carrying a current I is

$$\text{m.m.f.}=\frac{4}{10}\pi NI$$

Where N is the total number of turns on solenoid.

Analogously to Ohm's law the flux $\Phi=\frac{\text{m.m.f.}}{\mathcal{R}}=\frac{\frac{4}{10}\pi NI}{\frac{L}{\mu A}}$

When the magnetic circuit is not uniform, the reluctance of each part must be determined separately. The reluctance of the whole circuit is the sum of the reluctances of its parts.

14. Force Exerted upon a Wire in a Magnetic Field.—Since a wire carrying a current is surrounded by a magnetic field, a force of attraction or repulsion will be experienced when the wire is introduced into a field due to a magnet; this force will act at

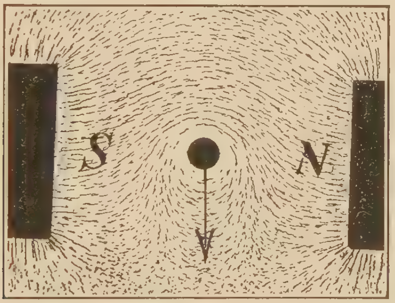


Fig. 9.

right angles to the wire, and to the field of the magnet, Fig. 5.

If I represents the current flowing in the wire, H the strength of magnetic field, and l the length of wire at right angles to field, the force is given by $F=IlH$.

The direction of the force, as indicated by the arrow, may be determined by the following rule:

If the thumb, index, and middle fingers of the left hand be held at right angles to each other, the thumb pointing in the direction of the field, the index finger in the direction of the current, the middle finger will point in the direction of the force acting upon the conductor. If this conductor is free to move it will move in the direction of the force. Thus in Fig. 9 the direction of the field is from N to S, and if the current flows up through the paper the force will be in direction indicated.

15. Force Between Parallel Wires Carrying Currents.—Two parallel wires carrying electric currents will be either attracted or repelled depending upon whether the currents are flowing in the same or opposite directions.

If the currents flow in the same direction the magnetic lines will combine so as to encircle both conductors, Fig. 10. The tension along the lines will be manifest as a force tending to draw the wires together.

When the currents are in opposite directions, the direction of the fields between the conductors is the same, and hence, the pressure at right angles to the lines will tend to force the wires farther apart.

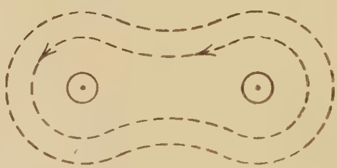


FIG. 10.

The intensity of the force in either case is proportional to the product of the currents in the two wires. This relation is derived as follows:

At a distance x from a wire carrying a current I, the strength of field is

$$H=\frac{2I}{x}$$

The force between a field H and a current per unit length of conductor is

$$F=HI', \ I' \text{ is the current in the second conductor}$$

Since $$H=\frac{2I}{x}$$

$$F=\frac{2I\times I'}{x}, \text{ per unit length of conductor.}$$

When I and I' are in absolute units, x in centimeters, the force is in dynes.

Note.—The student can find the derivations of the two assumed equations, $H=\frac{2I}{x}$ and $F=HI'$ in books on Physics.

When the two currents are equal, $I=I'$, the expression becomes

$$F=KI^2$$

where K is a proportionality factor.

These relations are very important as they have numerous applications in electrical measuring instruments.

16. Electrolytic Conductors.—The passage of an electric current through some liquids is accompanied by different phenomena from its passage through solids. In fact, in regard to their conductivity, liquids may be divided into three groups, viz.:

1. Those which make fairly good insulators, or which are non-conductors for practical purposes such as paraffin, mineral oil, etc.

2. Those which conduct like solids without undergoing any chemical change, such as molten metal, mercury, etc.

3. Those in which the passage of the current is accompanied by chemical decomposition, such as solutions of acids, salts of the metals, and some other chemical compounds.

Liquids of the latter class are called electrolytes. The process of their decomposition by the passage of the electric current is called electrolysis, and the cell in which electrolysis is carried on is called an electrolytic cell.

17. Faraday's Laws.—During the years 1833 and 1834 Faraday investigated the relation between the quantity of electrolyte decomposed by an electric current, and the strength of the current. The result of his investigations he expressed as follows:

1. The mass of the solution decomposed is proportional to the quantity of electricity which passes through it.

2. The mass of any substance liberated by a given quantity of electricity is proportional to the chemical equivalent of the substance.

The first law means that a given current of electricity flowing for a given time will deposit the same mass or weight of a given element from a solution, irrespective of the concentration of the solution that contains the element, or of other conditions.

According to the second law, the mass of substance deposited will depend upon its combining weight, which is called chemical equivalent. Thus, when a solution of copper salt is used as the electrolyte, the mass of copper deposited will depend on whether a cupric or cuprous salt is used. The chemical symbol for cupric chloride is $CuCl_2$, and for cuprous chloride $CuCl$. From this it will be seen that two atoms of copper in the cuprous compound

take the place of one atom in the cupric compound. The combining weight is twice as great, and twice as much copper will be deposited by a given current from a cuprous solution as from the cupric solution. The law also states that if solutions of different compounds be decomposed, the weight of material deposited by a given current is proportional to the combining weight of the materials or elements forming the compounds. Thus, one ampere sent through a solution of silver nitrate for an hour will deposit 4.025 g. of silver. The same current sent through a solution of copper sulphate will deposit only 1.184 g. of copper in 1 hour. These laws are the fundamental principles of the operation of electrochemical measuring instruments. The electrochemical equivalents of some metals are given in the following table:

TABLE I

Metal	Electrochemical equivalent in milligrams per coulomb
Aluminum	0.0936
Copper	0.6588
Copper	0.3290
Gold	0.6818
Iron	0.2894
Iron	0.1929
Lead	1.0731
Nickel	0.3040
Silver	1.1180
Zinc	0.3385

It is noticed that two values are given in the table for copper and iron. This is because each of these has different valencies as explained above. The value 0.3290 for copper usually applies when copper is deposited in an electrolytic cell. The table may be reduced to English units by remembering that 1 g. is equal to 0.0353 of an ounce avoirdupois.

18. Heat Effect.—The physical principle made use of in some instruments is the expansion of the metals by heat. When the temperature of a wire is raised, it expands; and since some of the energy of a current is always converted into heat, the strength

of current may be measured by means of the expansion of a wire suitably arranged. Hot-wire instruments are due to an adaptation of this principle.

19. Practical Electrical Units.—In the application of electricity many terms are constantly met, and a clear comprehension of the meaning of these terms will aid materially in understanding their industrial application. Among the most important terms are the names of the fundamental electrical units: ohm, ampere, volt, coulomb, watt, joule, henry, and farad.

20. Resistance.—Every electrical conductor offers a resistance to the flow of electricity. This resistance depends upon the material of which the conductor is made, the length of the conductor, and its cross-sectional area. The resistance of a conductor is analogous to the resistance a water pipe offers to the flow of water. This resistance will depend upon the roughness of its surface, or upon the material of which it is made. A long pipe will offer more resistance than a short pipe of same diameter, and a pipe of large diameter will offer less resistance than one of same length but of smaller diameter. It must be remembered, however, that the cause of the resistance of a conductor to the flow of electrical current is not the same as the cause of the resistance of a water pipe to the flow of water. They are analogous only.

The resistance of any conductor can then be written in the following form:

$$R=\frac{rl}{A},$$

where R is the total resistance, r, the resistance of a piece of the conductor of unit length and of unit cross-section, A its cross-sectional area, and l its length.

The ohm is the unit of resistance and is defined as the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 g. mass, of a constant cross-sectional area and of a length of 106.3 cm.

The ohm is thus a definite quantity and the resistance of any conductor is expressed in terms of it. In the formula $R=\frac{rl}{A}$, l and A may be expressed in any units, provided r expresses a resistance based on these units. The definition given for the

ohm assumes l to be in centimeters and A in square centimeters. In this country the Brown and Sharpe, or American wire gauge has been generally adopted where a gauge is to be used. In many cases it is better to specify the actual diameter or cross-section of a wire, and for this purpose the "mil system" has been introduced. In this system the mil is the unit of length and is equal to 0.001 in.

Since the areas of any two circles are proportional to the squares of their diameters, if the area of a circle one mil in diameter be taken as the unit area, the area of any other circle may be expressed as the square of its diameter in mils. The unit area is called the circular mil (C. M.) and is, as above expressed, the area of a circle 0.001 in. in diameter. Area in circular mils is equal to diameter squared, and the area expressed in square measure is equal to $0.7854 \times d^2$ (diameter squared). The circular mil is, therefore, equal to 0.7854 of a square mil.

It is seldom necessary to convert the area of round conductors into square measure. The wire tables which are in common use usually give the sizes in the B and S (Brown and Sharpe) gauge, its diameter in mils, its area in circular mils, and various other properties of wire depending on the completeness of the tables.

Wires larger than 0000 B and S, that is, of a greater diameter than 0.46 in. are usually designated by their diameters in mils or their cross-sectional area in circular mils.

The unit of a conductor most commonly used is a conductor one foot long and one mil in diameter called the mil-foot. The resistance of a mil-foot of copper of 98 per cent. conductivity is 9.59 ohms at 0° C. or 32° F. This value may be used in our resistance formula which then becomes

$$R = \frac{9.59l}{A},$$

l being expressed in feet and A in circular mils.

21. Change of Resistance with Temperature.—The resistance of most conductors changes with the temperature. The resistance of pure metallic conductors increases with increase in temperature. For pure metals the increase per ohm per degree is practically the same for all. This increase per ohm per degree change in temperature is called temperature coefficient of resistance, and for pure metals is nearly 0.004 per degree Centigrade.

The resistance of a conductor at any temperature $t°$ C. is given by the following:

$$R_t = R_o(1 + at).$$

R_o is the resistance of conductor in ohms at 0° C., a the temperature coefficient, and t the temperature. The resistance of most alloys also increases with increase in temperature, but to a much smaller extent than pure metals. Thus an alloy of 84 parts by weight of copper, 12 parts by weight of nickel, and 4 parts by weight of manganese, called manganin, has a temperature coefficient of resistance which is negligible for practical purposes. Although the temperature coefficient of maganin is very slight, it is positive between 0° and about 50° C. When the temperature is increased above 50° C. the resistance of manganin slightly decreases.

Carbon and all acid and salt solutions have negative temperature coefficients of resistance. That is, the resistance of these decreases as the temperature increases.

22. Electric Current.—The term "electric current" has already been used several times without explanation, and perhaps an explanation will not aid much in giving a clear understanding of the quantity.

Since the transfer of energy by water through pipes is in many ways analogous to the transfer of energy by electrical means, the terminology in one case is used to some extent in the other.

When water flows through pipes the energy transferred by it in a given time depends upon the current and head, or pressure. The current is the number of gallons or cubic feet of water per second, or some other unit of time. The current is then the rate of flow of water.

Electrical energy may be transferred along a conductor, and while the energy is being transferred the conductor is surrounded by a magnetic field. The transfer of energy is said to be by means of a current of electricity. Thus, the rate of flow of electricity is also called a current. The two cases are evidently analogous.

An electric current is a name given to a continuous transference of electric energy from its source to other parts of the circuit or to the space surrounding the circuit.

In measuring a water current it is possible to measure the quantity of water discharged and thus, the rate of flow. It is

not practical to measure an electric current in this way. The electric current is measured by means of its effect, and any effect which is proportional to some power of the current strength may be used for determining unit current, and hence, for measuring the current. The practical unit current has been defined in accordance with Faraday's first law as follows:

The *Ampere* is the unvarying electric current which, when passed through a standard solution of nitrate of silver in water, deposits silver at the rate of 0.00111800 g. per second. An ampere will thus deposit 4.025 g. of silver per hour.

22A. Electromagnetic Unit of Current.—Another definition of unit current rests upon the fundamental principle that about every electric current there is a magnetic field. The intensity of this magnetic field at any point varies directly as the current strength and inversely as the distance of the point from the conductor. See Article 15.

If a magnet be introduced into such a field, a force will be exerted upon it. The unit current is defined in terms of this force. According to these principles, unit current is defined as that current which when flowing through a conductor 1 cm. long, bent into an arc with 1 cm. radius, will exert a force of one dyne on unit magnet pole placed at the center of the circle of which the arc is a part. The ampere is one-tenth of this absolute unit.

23. Electromotive Force.—The real cause of an electric current is called electromotive force. Without going into details, we may say that electromotive force can be generated in three ways:

(1) Chemically, as in voltaic cell.

(2) Thermally, as when the junction of two metals is heated.

(3) Mechanically, as in the case of the static induction machine or when a wire is moved across a magnetic field.

Of these three methods the last is the all important one in industrial practice. It consists in the application of the principle that a wire moved in a magnetic field in such a direction as to cut across the magnetic lines has an electromotive force induced in it. That is, the reaction between the magnetic field and the mechanical force causing the motion is manifest as an electromotive force between the terminals of the wire. The value of this electromotive force will depend upon the strength of the magnetic field, the length of wire, and the speed with which it is moving across the field. This principle is used in the construction of all dynamo-electric machines, which form the main

means for the conversion of mechanical into electrical energy and *vice versa.*

Volt.—Since the resistance of a conductor is comparable to the resistance offered by a pipe to the flow of water, and the electrical current is comparable to the current of water, we may compare the electromotive force or electrical pressure to the water pressure causing a flow of water. Although this comparison is not exact, it still serves to give a better understanding of the relation of the electrical quantities involved. Water pressure can be measured in terms of pounds per square inch, but usually it is expressed as a head of so many feet. In the same way, the difference of electrical pressure between the terminals of a battery may be considered as a difference of electrical level. The current will then flow from a point of higher to a point of lower electrical level, when the circuit is closed. This difference of electrical pressure or electromotive force, is expressed in volts, and the volt is defined as that difference of pressure which will cause a current of one ampere to flow through a resistance of one ohm.

24. Quantity.—The quantity of water flowing through any given pipe in a given time may be expressed as the strength of current multiplied by the time. That is, if a unit current gives a cubic foot of water per second, a two unit current would give 2 cu. ft. per second, or 4 cu. ft. in 2 seconds.

Similarly, a unit current of electricity flowing for 1 second gives a definite quantity of electricity. This quantity is called the *coulomb* and is defined as the quantity of electricity conveyed by a current of one ampere in 1 second of time. The total quantity conveyed by a current of I amperes in t seconds is then given by

$$Q = It, \text{ assuming } I \text{ to be constant.}$$

25. Energy.—Referring again to our analogy we may consider unit work to be done when a cubic foot of water is delivered under a head of 1 ft. The amount of work done by a head of h ft. delivering q cu. ft. of water will then be hq.

In our electrical analogy, the head was analogous to electrical pressure, and the number of cubic feet of water is analogous to the number of coulombs. A current delivering q coulombs of electricity under a pressure of E volts will then do Eq units of work. The unit of electrical work is the *joule* and is defined as

the work expended in a circuit when one coulomb is transferred under a pressure of one volt. Since the number of coulombs delivered by a current of I amperes in the time t is It, the amount of work expended by a current of I amperes in the time t and under a pressure of E volts is

$$\text{Work} = EIt \text{ joules.}$$

Watt-hour.—One watt-hour equals 3600 joules.

26. Power.—The watt is the unit of power and is equal to one joule per second. The watt is also equal to $\frac{1}{746}$ of a horse-power. The number of joules of work expended by a current of I amperes in t seconds, as above expressed, is EIt; the number of joules per second will then be $EIt \div t$ or EI. That is, the power of a current of I amperes flowing under a pressure of E volts is EI watts.

27. Inductance.—It was briefly pointed out in Article 23 that when a conductor moves across a magnetic field an electromotive force is induced in the conductor. Evidently it is immaterial whether the field is stationary and the conductor moves, or the conductor is stationary and the magnetic field moves; the necessary condition is relative motion between conductor and field.

This relative motion may be secured in several ways; the one in which we are at present interested consists in changing the magnetic flux around a conductor by means of the current in the conductor.

Consider the case represented by Fig. 11 where a battery B supplies current through a variable resistance R to an electromagnet M. Let us suppose the circuit open and no initial or residual magnetism in the core. Upon closing the circuit a current will begin to flow through the electromagnet coil. This current will magnetize the core and thus cause a number of magnetic lines to thread through the coil. If the resistance R, is varied, the current will change and consequently, the number of magnetic lines threading through the coil is either increased or decreased. In other words, any change in the current is accompanied by a change in the magnetic flux passing through the coil.

According to the principle of electromagnetic induction, whenever the number of magnetic lines linked with a circuit is changed an e.m.f. is induced in the circuit. Consequently, every time the current in the coil changes, an e.m.f. is induced in it. The

e.m.f. induced by the building up or decay of the magnetic field within the coil, due to the variation of current in the coil, is called self induction. The induced e.m.f. is in such a direction as to oppose the applied electromotive force. In other words, while the current is changing the electromotive force of self induction opposes any change. If the magnetic lines threading through the coil are due to a current in an adjacent coil, the phenomenon is the same, but it is called mutual induction.

From Fig. 6 it is evident that each magnetic line through the center of the coil is linked with each turn; or what amounts to

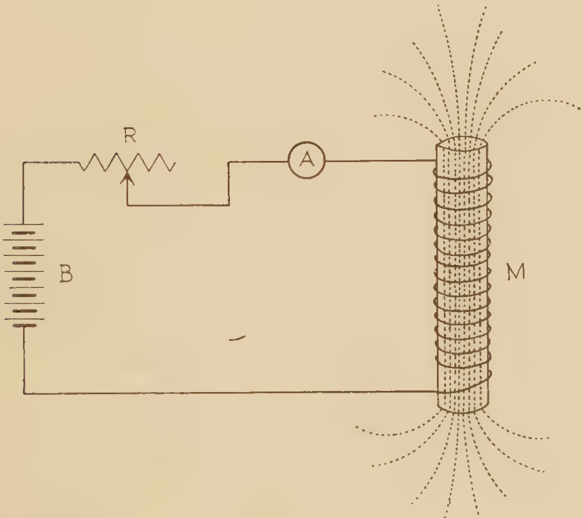


FIG. 11.

the same thing, the lines due to each turn are linked with every other turn as well. The e.m.f. of self induction then depends not only upon the current but also upon the arrangement of conductors in the coil. If Φ is the total flux due to one turn through the coil, and if n is the number of turns, the total flux is $n\Phi$. This total flux is proportional to the current and hence we may write

$$n\Phi = LI$$

The constant L is called the coefficient of self induction, or simply the self inductance of the coil.

The electromotive force of self induction depends upon the rate of change of magnetic field. This may be expressed by

$$E = -n\frac{d\Phi}{dt} = -L\frac{dI}{dt}$$

Where $\frac{dI}{dt}$ represents the rate of change of current.

According to this expression the inductance L is defined as the ratio of the induced counter e.m.f. to the time rate of change of current.

Henry.—Self and mutual inductions are physical quantities of the same nature, and hence the same unit must be used for both. This unit is called the henry and is defined as the inductance in a circuit in which the induced e.m.f. is one volt, when the current changes at the rate of one ampere per second.

28. Capacity.—If two metal plates be separated by a good insulator and the two plates be connected to a source of electrical pressure, a momentary current will flow into the plates. The intensity of the current will depend upon the ability of the plates to hold a charge of electricity. This ability of a conductor or a system of conductors, to store electricity is called electrical capacity. The capacity of a system of conductors is determined by their arrangement, number, and material separating them. The quantity of electricity that a condenser will hold is determined by the capacity of the condenser and by the electrical pressure applied. Algebraically this is expressed by

$$Q = EC$$

Where Q is the quantity of electricity, E the pressure, and C the capacity.

Farad.—The unit of capacity is called the farad and is that capacity which is charged to a difference of pressure of one volt by one coulomb.

The effect of inductance and capacity in alternating-current circuits will be given later.

29. Ohm's Law.—The relation between current and electromotive force in a circuit was first enunciated by Dr. G. S. Ohm in 1827, and is known as Ohm's law. It may be stated as follows:

The current strength in any circuit is directly proportional

to the sum of all the electromotive forces in the circuit. This relation expressed algebraically is

$$E=KI$$

or

$$\frac{E}{I}=K, \text{ a constant.}$$

This holds for both direct and alternating-current circuits so long as the physical conditions surrounding the circuit remain unchanged. For direct-current circuits K is equal to what is called the resistance of the circuit and under these conditions

$$E=RI$$

or

$$\frac{E}{I}=R.$$

Thus the ratio of the electromotive force to current is constant so long as physical conditions remain constant. If, for instance, the temperature changes, this ratio will change. This is explained by saying that the resistance changes.

In alternating-current circuits the total e.m.f. must include the e.m.f.'s of mutual induction, self induction, and capacity. When these are considered, Ohm's law, as stated still holds. What quantities enter into the expression

$$\frac{E}{I}=K$$

when alternating currents are considered is explained in Chapter IV.

30. Pressure Drop in D. C. Circuits.—If a current I flows through a circuit, the e.m.f. necessary to force it through a resistance R is by Ohm's law

$$E_R=IR$$

Thus the product of current by resistance is equal to the difference in electrical pressure between two points. This quantity IR is called pressure or voltage drop.

EXAMPLES

1. What current is taken by an incandescent lamp of 198 ohms resistance when an e.m.f. of 110 volts is applied to its terminals?

Solution.—$I=\frac{E}{R}=\frac{110}{198}=.55$ ampere

2. A trolley wire of No. 0 B. & S. gauge has a resistance of .519 ohms per mile. What is the drop in voltage due to line resistance between the station and car 2 miles out on the line, when 20 amperes are flowing?

Solution.—Drop $= IR = .519 \times 2 \times 20$
$= 20.76$ volts.

31. Energy Loss.—The flow of electric current through a conductor is analogous to the flow of water through a pipe. The pressure forcing the water through the pipe is analogous to the electrical pressure which maintains the electrical current. Work is defined as force times distance through which it acts. Evidently, if the water pipe is closed and no water flows, considerable pressure can be exerted and still no work be done. If, however, the water is flowing under a given pressure a certain amount of work will be done which is proportional to the quantity of water times the pressure. This is seen to be true, if the pipe be vertical, in that case, the height of the pipe is proportional to the pressure and the height in feet times the weight of water delivered at the top equals foot-pounds of work.

Similarly, the work done in transferring a certain quantity of electricity through a conductor is proportional to the pressure times the quantity of electricity.

If we define our unit of work as the product of unit quantity times unit difference of pressure, the total amount of work will then be represented by the total difference of potential times the current times the time, or

$W = EIt$, since It is the total quantity of electricity delivered.

Power is the rate of doing work, or is the work done per second, hence,

$$\text{Power, or } P = \frac{W}{t} = \frac{EIt}{t} = EI$$

If the difference in pressure between any two points, as A and B of a conductor, is E and the current is I, the energy spent per second in that portion of the circuit is IE. But, as has been shown,

$$E = IR,$$

$$\text{Hence, } P = IE = I \times I \times R = I^2R.$$

This is known as power loss or energy loss per second due to resistance.

EXAMPLES

1. A current of 1.5 amperes flows through a circuit of two ohms resistance. How many watts are dissipated as heat?

Solution.—Power, $P = I^2R = 1.5^2 \times 2$

$= 4.5$ watts.

2. How many watts are expended in a 16 candle-power 110-volt lamp which takes 0.5 of an ampere? How many watts per candle?

Solution.—$P = 110 \times .5 = 55$ total watts consumed

$\frac{55}{16} = 3.44$ (nearly) watts per candle.

The effects of both the voltage drop and loss of energy due to resistance, are objectionable. The voltage drop produces a difference of pressure between the generator and receiving circuit. The I^2R losses cause heating of the conductors, thus increasing the fire hazard. They also cause a deterioration of the insulation and lessen the efficiency of the distributing systems.

Summary

The *Ohm* is the unit of resistance. It is the resistance offered by a column of mercury of uniform cross-section, 106.3 cm. in length, weighing 14.4521 g. at 0° C.

The *Ampere* is the practical unit of current, and is that current which when passing through a standard solution of silver nitrate deposits silver at the rate of 1.118 mg. per second.

The *Volt* is the practical unit of electrical pressure and is the electromotive force which will produce a current of one ampere through a resistance of one ohm. It may also be defined as the $\frac{10000}{10183}$ of the e.m.f. of the Weston standard cell at 20° C.

The *Coulomb* is the unit of quantity of electricity and is the quantity of electricity conveyed by one ampere in 1 second.

The *Joule* is the unit of work, and is represented by the work expended in one second by a current of one ampere under an electrical pressure of one volt.

Joules $= k \times$ volts $\times$ amperes $\times$ time.

The *Watt* is the unit of power and is equal to one joule per second. The watt is also equal to $\frac{1}{746}$ of a horse-power.

Watts $= k \times$ volts $\times$ amperes. In direct current circuits $k = 1$.

The *Kilowatt* is the practical unit of power and is equal to 1000 watts.

The *Kilowatt-hour* is the practical unit of work and represents 1000 watts supplied for a period of one hour. It is equal to 3,600,000 joules.

The *Henry* is the unit of inductance and is defined as the inductance of a circuit in which the induced e.m.f. is one volt when the inducing current changes at the rate of one ampere per second.

The *Farad* is the unit of capacity and is that capacity which is charged to a difference of pressure of one volt by one coulomb. The farad is too large for practical use, hence, the micro-farad $\left(\frac{1}{1000000} \text{ farad}\right)$ is commonly used.

CHAPTER II

CLASSIFICATION OF INSTRUMENTS

32. Classes of Meters.—The discriminating characteristics of a classification of electrical measuring instruments may be the quantity to be measured, or the principle upon which the instruments operate. The plan here followed bases the main divisions of measuring instruments upon the quantities to be measured, while minor subdivisions are according to the principles of operation. The quantities measured by electrical instruments are: current, electromotive force, quantity, power, energy, frequency, power-factor, and phase. Frequency, power-factor, and phase will be explained later.

The instruments for measuring these quantities are ammeters, voltmeters, coulometers or ampere-hour meters, wattmeters, watt-hour meters, frequency meters, power-factor meters, and synchroscopes.

33. Groups of Instruments.—As already stated, the minor subdivisions of instruments will be based on the principles of their operations. Accordingly, we have electromagnetic, electrodynamic, electrostatic, and thermal groups.

34. Electromagnetic Instruments.—Any instrument that makes use of the interaction between the magnetic field surrounding an electric conductor and the magnetic field around an iron core, belongs to the electromagnetic group. The application of the principle is made in various ways and accordingly we have the following types of electromagnetic instruments:

(1) The movable-core type.
(2) The movable-coil permanent magnet type.
(3) The induction type.

35. Electrodynamic Instruments.—The force between two conductors carrying electric currents is called electrodynamic attraction or repulsion. Hence, electrodynamic instruments are those whose actuating forces are due to currents flowing through coils without iron cores.

36. Electrostatic Instruments.—Electrostatic instruments are actuated by forces of attraction and repulsion between electric charges. Their action is independent of magnetic forces.

37. Thermal Instruments.—The actuating forces of the thermal group are due to the expansion of a wire by the heat generated in it when an electric current is flowing. These are usually called hot-wire instruments.

38. Controlling Forces.—Where any physical quantity is measured in terms of the force it exerts, provision must be made for the application of some counterforce whose intensity will increase in proportion to the actuating force. Since the common methods of measuring electrical quantities are in terms of their force effects, the moving system of every meter must be counterbalanced in some way. This counterbalancing force must be so adjusted that the deflection or speed of the moving part, as the case may be, is always proportional to the actuating force. If this were not done, the movable system would deflect to the extreme position, or would race, upon the application of a force sufficient to overcome the friction of the movable parts. The controlling forces employed for this purpose are:

(1) The resisting force of a spring
(2) The torsion of some filament
(3) The attraction of gravity
(4) The attraction of permanent magnets
(5) The attraction of induced and inducing currents
(6) The mechanical friction of a rotating fan.

The first four of these controlling forces are utilized in ammeters, voltmeters, and wattmeters of both the indicating and recording forms, while the fifth and sixth are applied in watt-hour meters of various types. When a spring is used it may be one of two forms, *i.e.*, it may be helical in form, or in the form of a spiral. The helical form is used to control both in torsion and by axial extension. The fundamental law of the relation of the actuating force and distortion of spring within the limits of elasticity is—the actuating force is proportional to the distortion produced. The strength of either form of spring will be increased by decreasing the number of turns, or by increasing the sectional area of the spring. Springs are usually made of some elastic non-magnetic material. In most cases the material used is phosphor-bronze.

The use of the torsion of a metal or silk fiber suspension is limited almost wholly to laboratory instruments. The same relation between the actuating force and the resulting deflection holds as in the case of the springs.

The third form of control is very satisfactory, when it can be applied. The application of this method of control permits the construction of comparatively cheap and relatively accurate instruments. In this method the law, mentioned above, connecting the actuating force with the distortion does not hold. This will be made clear by referring to Figs. 12 and 13. Fig. 12 shows a typical arrangement for gravity control. *P*, the pointer, is balanced by the weights *B* and *C*. When the pointer moves to the right, *C* rises and *B* falls so that the moment of the

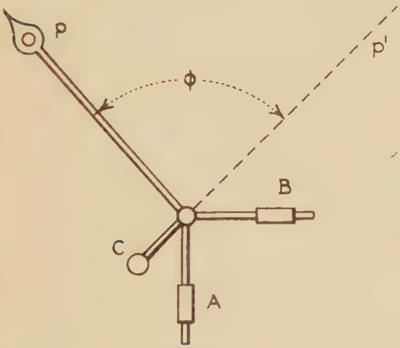


FIG. 12.

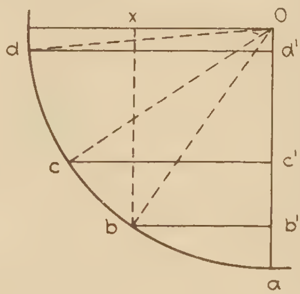


FIG. 13.

two about the point of suspension is zero. The controlling force is furnished by the weight *A*, which moves to the left and up as the pointer moves to the right. The moment of the force tending to bring the weight back to its vertical position or the pointer to its zero position, will vary with the position of *A*. The moment of *A* around the point of suspension *O* is equal to

$$w \times bb' \text{ at } b$$
$$w \times cc' \text{ at } c$$
$$w \times dd' \text{ at } d, \text{ etc.}$$

If we call $od = ob$, etc. $= r$ and if the angle through which the pointer has been deflected be represented by θ, then $bb' = r \sin \theta$; $cc' = r \sin \theta$, etc., and thus the moment tending to bring pointer back to zero is in general,

$$\text{moment} = wr \sin \theta$$

Since w and r are constants, the resisting moment varies as the sine of the angle of deflection. The graduations of the scale on such an instrument are not uniform, being close together at the beginning and end of the scale, and relatively far apart in the middle.

The fourth method of control is used only to a slight extent. The objection to this method is that in the presence of a magnetic field the indications are likely to be in error without the knowledge of the user. This error in the reading of the instrument is due to the influence of the outside field upon the permanent magnet of the instrument.

39. Magnetic Shielding.—It is a well-known principle that the reluctance of iron is much less than that of air. Thus, when a piece of iron is introduced into a magnetic field, the magnetic lines will be concentrated in the iron. The lines will deviate from a straight line and pass through the iron instead of through the air.

The movable or working systems of some meters are quite readily affected by a magnetic field and, to eliminate this influence, they are enclosed in an iron case. When this is done, the external field does not penetrate the instrument, and thus the readings are unaffected unless the external field is very strong.

40. Friction of Supports.—Several ways have been tried to support the moving system with varying degrees of success. The usual way of supporting the moving coil is to mount it upon two hardened steel pivots with sharpened points resting in sapphire, or diamond jewel bearings. Although the movable system is made as light as possible, the pressure per unit area is considerable on account of the small area of contact between pivot and bearing. Owing to this small contact, the wear and friction on the pivot increases with time, especially in portable instruments, unless some provision is made for relieving this pressure, when the instrument is not in use.

One method of securing a resilient support for jewels is shown in Fig. 14. In this figure DD is a two piece shaft fitted at top and bottom with cup-shaped jewels. In the middle or at junction of the two halves of shaft is a collar C which carries a small diameter screw over which the halves of shaft DD are threaded. The distance between the jewels can thus be adjusted by screwing or unscrewing the collar C. Surrounding each half of shaft is a spiral spring F one extremity of which bears against the collar, and

the other against a cup shaped guide bushing at end of shaft. This structure permits of longitudinal motion of the shaft against the tension of the spiral springs, and, consequently, in case of a sudden jar the coil G with its attached pointer will transmit the pressure of the pivot E, until the lower surface of G comes into contact with the upper surface of the stationary core H, thus preventing damage to jewels.

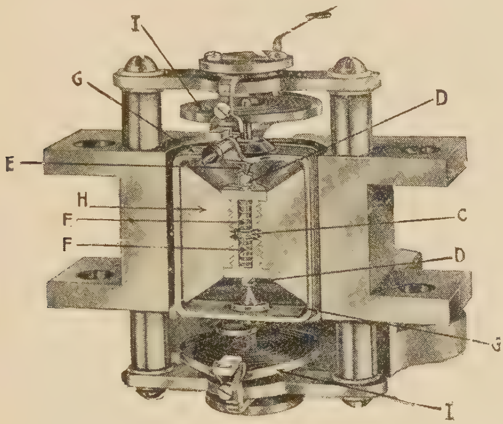


Fig. 14.

CHAPTER III

CURRENT AND PRESSURE MEASURING INSTRUMENTS

41. Ammeters and Voltmeters.—These two classes of instruments will be discussed together since, with the exception of the electrostatic voltmeter, they are alike in most respects. The names ammeter and voltmeter are clearly derived from ampere and volt, the units of current and pressure respectively.

42. Uses of Ammeters and Voltmeters.—The main difference between ammeters and voltmeters lies in their use. Ammeters are connected in series with the circuit in which the current is to be measured. That is, they are connected in such a way that the total or proportional part of the total current passes through the instrument. Voltmeters, on the other hand, are connected in parallel. That is, the terminals of the voltmeter are connected to two points, between which the difference of pressure is to be measured. Fig. 15 shows a standard connection for a voltmeter and an ammeter. In so far as construction is concerned, ammeters and voltmeters are similar. With the exception of the electrostatic voltmeters, all voltmeters are ammeters graduated in volts. The moving system is actuated by the current passing through the instrument, and in each instrument the deflection depends upon the strength of this current.

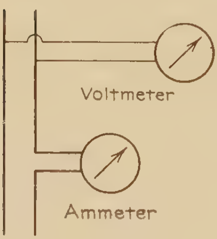


FIG. 15.

The pressure drop due to resistance between any two points of a conductor is IR, as already shown. Hence, if R is constant, the pressure drop varies with the current.

In ammeters with shunts this pressure drop along a resistance is utilized for sending a current through the movable system. The resistance of the movable system being constant, the current through it is directly proportional to the pressure drop and thus, to the main current. Since the current through the movable system is proportional to pressure drop, it is evident that the instrument may indicate the voltage or current. Thus, if E is the difference

of pressure between the voltmeter terminals, Fig. 15, and if R is the resistance of voltmeter coils, the current through the voltmeter is $I = \frac{E}{R}$. Since R is practically constant, I varies as E. The current which causes a deflection of the movable system, thus varies as the pressure between voltmeter terminals. In place of graduating the instrument in terms of the current passing through it, it is graduated in terms of the difference of pressure between terminals. The action in every respect is the same as that of an ammeter. The current through the ammeter is limited by the circuit to which the ammeter is connected. The current through the voltmeter is limited by the resistance of the instrument itself.

In order that the energy lost in each instrument may be as small as possible there is a great difference in the resistances of the two kinds of instruments. Since the load current or a certain per cent of it, passes through the ammeter, its resistance must be very low in order that the I^2R, or power loss, may be small. Since the voltmeter is connected as a shunt to the main circuit, its resistance must be very large in order that the current may be small. Thus, voltmeters have coils of relatively high resistance—several thousand ohms—while ammeters have low resistance coils—only a few thousandths of an ohm.

43. Range of Instruments.—The current intensity that an ammeter can indicate is mainly determined by the resistance of the movable coil. In practice two methods are used for increasing the range, viz., shunts for direct-current and transformers for alternating-current ammeters.

44. Ammeter Shunts.—There are so many types of ammeters that it is practically impossible to make a general statement of principles. The following will be found to apply mainly to ammeters whose movable system is actuated by current passing through it. The movable coil carries only a small part of the current so that, when the instrument is made to be used as an ammeter, a shunt for the main current must be provided. This shunt is nothing more than a low resistance which bears a constant ratio to the resistance of the movable coil. For small currents the shunt is mounted within or as a part of the instrument. For measuring large currents, however, the shunt is mounted separately, and the instrument is connected to the terminals of the shunt as shown in Fig. 16. The instrument used in this way is in reality a millivoltmeter which measures the

voltage drop across the shunt. The current strength is obtained in accordance with Ohm's law. This may be made clear by considering the following example: Assume that the shunt has a resistance of .001 ohm and that the instrument reads 40 millivolts. According to Ohm's law the drop across the shunt is

$$IR = E$$

$$\text{or } I \times .001 = .040$$

$$\text{hence } I = \frac{.040}{.001} = 40 \text{ amperes.}$$

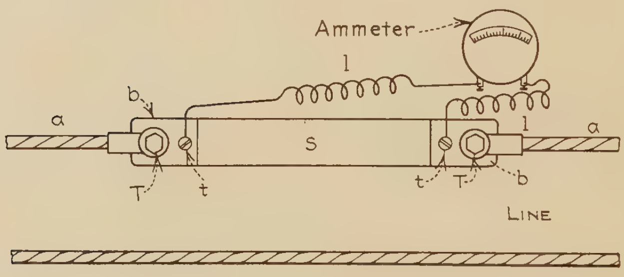


FIG. 16.

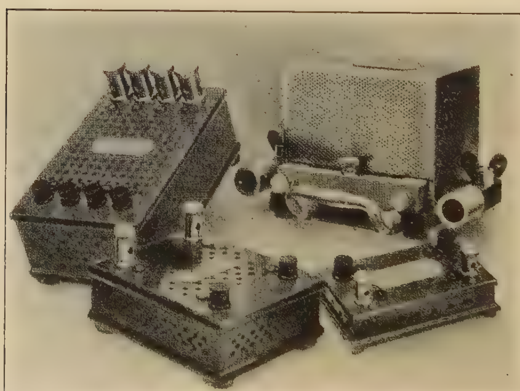


FIG. 17.

If the shunt and instrument are to be used together constantly the shunt and millivoltmeter are calibrated together, and the scale is graduated in amperes.

Since the shunts carry the main current and become heated, they must be made of some material whose temperature coefficient is very small. An alloy of copper, manganese, and nickel

called managnin, fulfills this requirement more fully than any other and for that reason is almost universally used for ammeter shunts. Fig. 17 shows some standard Weston shunts.

45. Range of Voltmeters.—Two methods are also in general use for increasing the range of voltmeters. One is by means of resistance coils connected in series with the instrument and the other by what are called potential transformers.

46. Voltmeter Multipliers.—It would be very inconvenient to make a movable coil of sufficient resistance to measure high differences of potential. To do this economically would necessitate large coils. The same thing can be accomplished in another

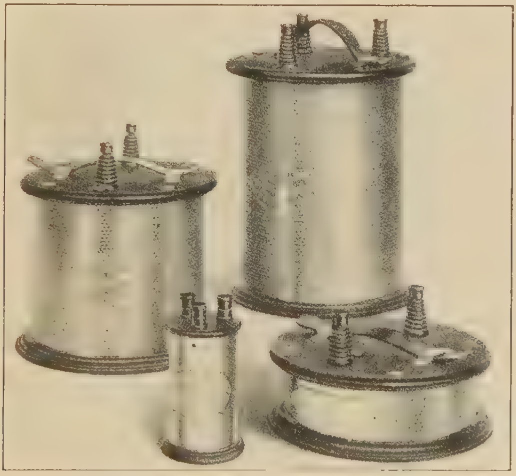


FIG. 18.

way; that is, by mounting in series with the movable coil of the instrument a resistance sufficiently large to reduce the current to the desired value. By providing different resistances, or multipliers as these are commonly called, the range of instrument can be varied considerably. In some cases these multipliers are mounted inside of the instrument case, and the circuit is connected to the proper multiplier by means of a push button, separate binding post, plug switches, or some other simple device.

The current in the multiplier is relatively small, so its resistance must be comparatively high. Manganin, constantan, or some other alloy of high specific resistance and low temperature

coefficient is used. Multipliers for Weston voltmeters are shown in Fig. 18.

47. The Movable Core Type.—To this type of instruments belong all those in which a piece of soft iron is acted upon by an electromagnetic field. The electromagnetic field is formed by an electric current flowing in a stationary solenoid, while the soft iron is pivoted and attached to a pointer which moves over a graduated dial.

One of the oldest and simplest instruments of this type is shown in Fig. 19. *C* is the stationary solenoid, *P* is the soft iron core pivoted at *O* so that it can move freely up and down. To the arbor *O* are also attached the pointer *N* and the counter weight *Q*. By referring to the discussion on controlling forces it will be seen that gravity is the controlling force in the plunger type of instrument, as that shown in Fig. 19 is usually called.

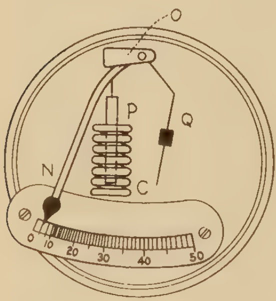


Fig. 19.

In ammeters of this type the main current flows through the solenoid *C*, and the core *P*, is drawn into the solenoid against the force of gravity on *Q*. The coil *C*, as shown, usually consists of a few turns of heavy wire. In voltmeters, however, the solenoid is made of many turns of fine wire. It is customary to place an additional resistance in series with the voltmeter coil of this type in order to reduce the current sufficiently when used to measure high voltage. By changing the resistance of the multiplier, the range of the instrument can be varied through wide limits. Fig. 20 shows the main features of a Westinghouse plunger type ammeter.

Another form of this type of instrument is the Thomson inclined coil ammeter. The essential features of this instrument are shown in Fig. 21, and the complete instrument is shown in Fig. 22. As shown in diagram of Fig. 21, the stationary coil makes approximately an angle of 45 degrees with the shaft upon which is mounted a soft iron vane, also making approximately an angle of 45 degrees.

The operation of the instrument is as follows: The current flowing through the stationary coil sets up a magnetic field as shown in Fig. 21. When the pointer is at zero of the scale the

plane of the vane is nearly at right angles to the magnetic lines. When the magnetic field is set up, the vane moves so as to become parallel to the magnetic field. Opposing this motion is the spiral spring at the upper end of the shaft. Since the pointer is

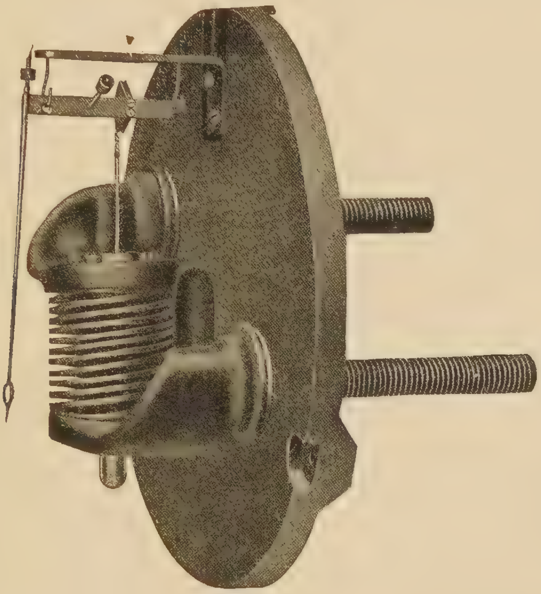


FIG. 20.

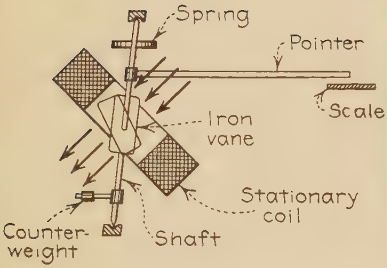
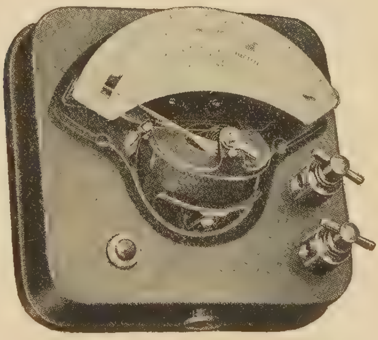


FIG. 21.



FIG, 22.

also rigidly attached to the shaft it moves over the scale, coming to rest when the torque, due to controlling spring is equal to the torque, due to the action of the magnetic field on the vane. The counter-weight balances the moving parts.

The inclined coil movable vane instrument is much more compact than the plunger type; the movable parts are lighter, reducing friction and making the instrument much more sensitive. The gravity control of the plunger type is replaced by the spiral spring control. As has been already shown, the counter-force in the gravity control method is proportional to the sine of the angle of deflection, while the counter-force of the spring is directly

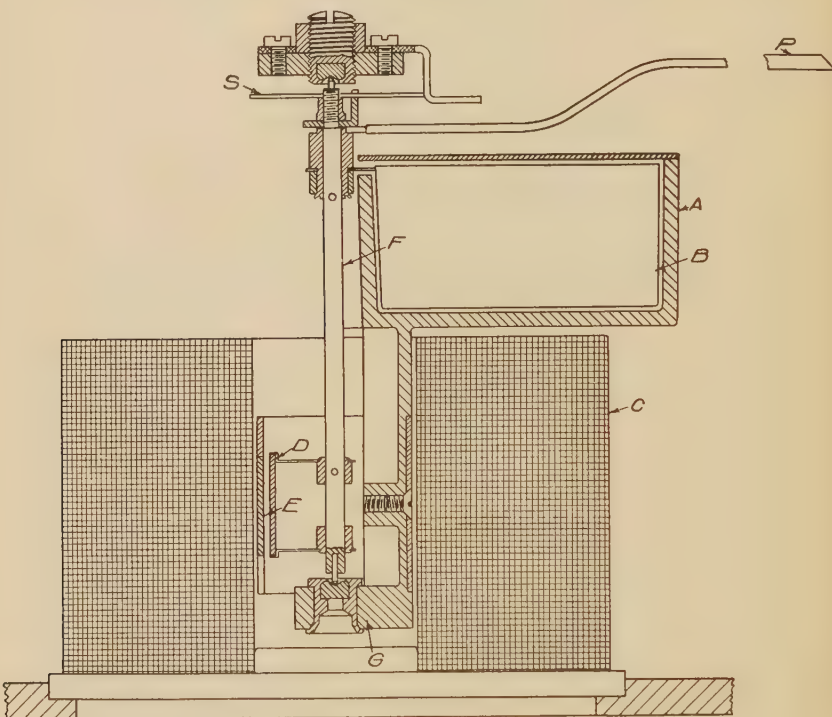


Fig. 23.

proportional to the deflection. This permits the use of a more uniform scale, although the graduations cannot be made exactly uniform on account of the fact that the deflecting force acting on the vane is not directly proportional to the current.

Another method of applying the principle of reaction between magnetic fields is utilized by the Weston Electrical Instrument Company in their so-called "Soft-Iron" instruments. The essential parts of an instrument of this type are shown in Fig. 23.

Referring to the diagram, E is a fixed piece of iron of nearly triangular shape, bent into cylindrical form. Concentric with this is a movable piece of similarly shaped iron D which is secured to the staff F, which carries the pointer P. The movable element is mounted on a support G and inserted in the field coil C, which for the voltmeter, is connected in series with a non-inductive resistance. Under the magnetizing action of the field coil the two iron pieces tend to separate and thus produce a torque which is opposed and controlled by the spring S. The torque for any deflection is approximately proportional to the square of the current in the field coil. The two iron pieces D and E are shaped so as to give a nearly uniform scale.

48. Damping.—As is clear from the diagram, damping is produced by a vane B, which moves in an air damper box A. The movable core, or soft iron instruments, may be used on both direct and alternating circuits. This is due to the fact that the iron core of the plunger type, and the vane of the inclined coil type are attracted by a magnetic field no matter what its direction. When the instrument is to be used on alternating current, the core must be of laminated iron or of a bundle of soft iron wire; this is done to prevent the flow of eddy currents and consequent alteration of the force.

49. Approximate Equation for Pull on Iron Core.—The magnetic field (H) within a solenoid is equal to $H = 1.257\ nI$, where n is the number of turns per centimeter length, and I is current in amperes. When an iron core is introduced into the solenoid, the number of magnetic lines is greatly increased.

Although the permeability of the iron core is not constant but varies with the intensity of magnetization, nevertheless it may be assumed that the flux density is approximately proportional to the current. This may be written

$$B = KI$$

The pull exerted by the solenoid upon the iron core is proportional to the product of the strength of magnetic flux and current, or

$$\text{Pull} = K'BI$$

But as has already been shown, B is proportional to I, so by replacing B by I we get pull $= K_1'I^2$. That is, the pull on the plunger is proportional to the square of the current flowing through the solenoid. When the instrument is used to measure

alternating current, the value of B is at each instant proportional to the current. Since the current varies B will vary, but at each instant the pull will be proportional to the square of the current at that instant. We may thus write instantaneous pull $=ki^2$, where i is the instantaneous current. The direction of the pull does not change, for as the current reverses the magnetic field, due to the current reverses, or the field and current change their signs together. The inertia of the movable system does not permit it to follow the instantaneous fluctuations of the pull; the plunger will assume a position determined by the average pull. By higher mathematics it can be shown that the average pull is proportional to the mean square of the instantaneous values of the current. The square root of the mean square value is called the effective value, and is the value given by all alternating-current instruments. See page 53.

The movable core type of instrument when calibrated with direct current should then give exact effective values of alternating currents. In practice this is not quite true. The magnetic field does not increase uniformly with the current, and also the effect of eddy currents and hysteresis is appreciable. The difference, however, is not great; the readings differing only slightly.

50. Movable Coil Permanent Magnet Type.—In this type of meter the magnetic field remains constant and is due to a permanent magnet of the horse shoe form. Between the poles of the magnet is mounted a rectangular coil in such a way that it can revolve through a considerable arc within a specially designed magnetic field. When in use the current flowing through the coil tends to set up a magnetic field at an angle to the field of the permanent magnet. The reaction between the two fields causes a deflection of the pointer which is rigidly attached to the shaft of the moving coil.

The general principles of this type of instrument are well shown in Fig. 24. The permanent magnet is shown in the diagram by NS, N being the north and S the south pole. The rectangular coil C carries the current causing the deflection. Usually this is only a small per cent of the total current to be measured. The controlling force is furnished by two spiral springs, one at the upper, the other at the lower end of the shaft. The springs are coiled in opposite directions so that when the pointer is deflected, one of the springs is coiled up, while the other is uncoiled. This scheme compensates for any inequalities that may exist in the

springs. In most cases the springs also serve to make connection between the coil C and the external circuit.

In this type of instrument the direction of the main field remains constant. The deflection of the pointer will be in one direction when the current flows from t to t' and in the opposite direction when the current is reversed. This being the fact, it is evident that instruments of this type cannot be used to measure alternating current. They are, however, very efficient and convenient direct-current instruments. The instruments of the Weston Electrical Instrument Company, made in accordance with the foregoing principles, have practically been the standard direct-current instruments for years.

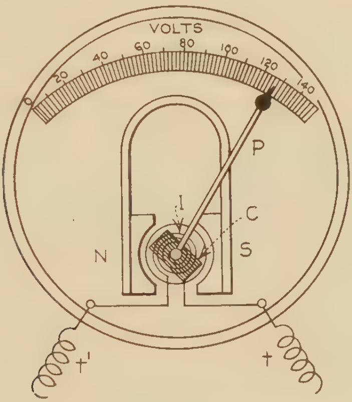


FIG. 24.

In Fig. 25 are shown the movable elements of eight different makes of ammeters while in Fig. 26 the same movable elements and magnets are assembled. The instruments represented in these figures are all of American make.

The cross-section of the movable coil is usually rectangular and it usually surrounds a soft iron cylindrical core. The function of the core is two-fold; to concentrate the field and to secure a distribution of flux in the air gap such that uniform graduations are possible.

51. Torque Exerted by a Magnetic Field upon a Rectangular Coil.—Let Fig. 27 represent both a side and top view of a rectangular coil in a permanent magnetic field of uniform distribution and strength H. Assume the plane of the coil to lie

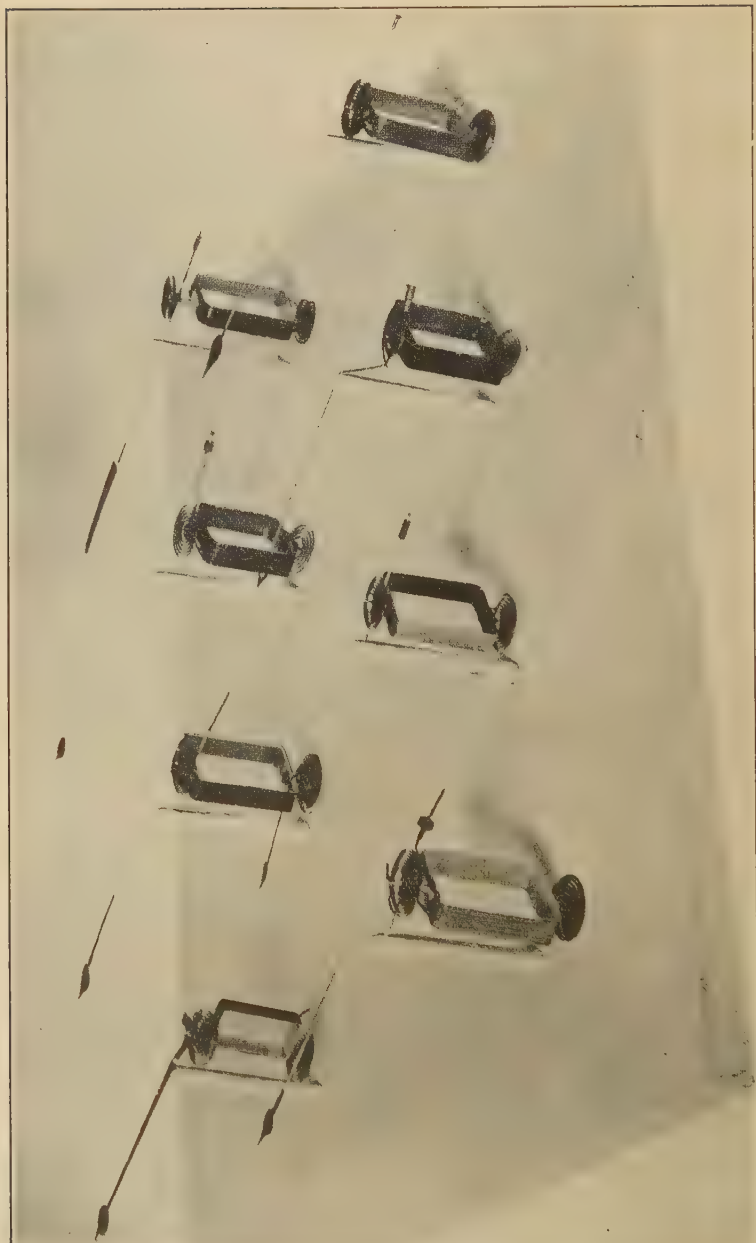


Fig. 25.

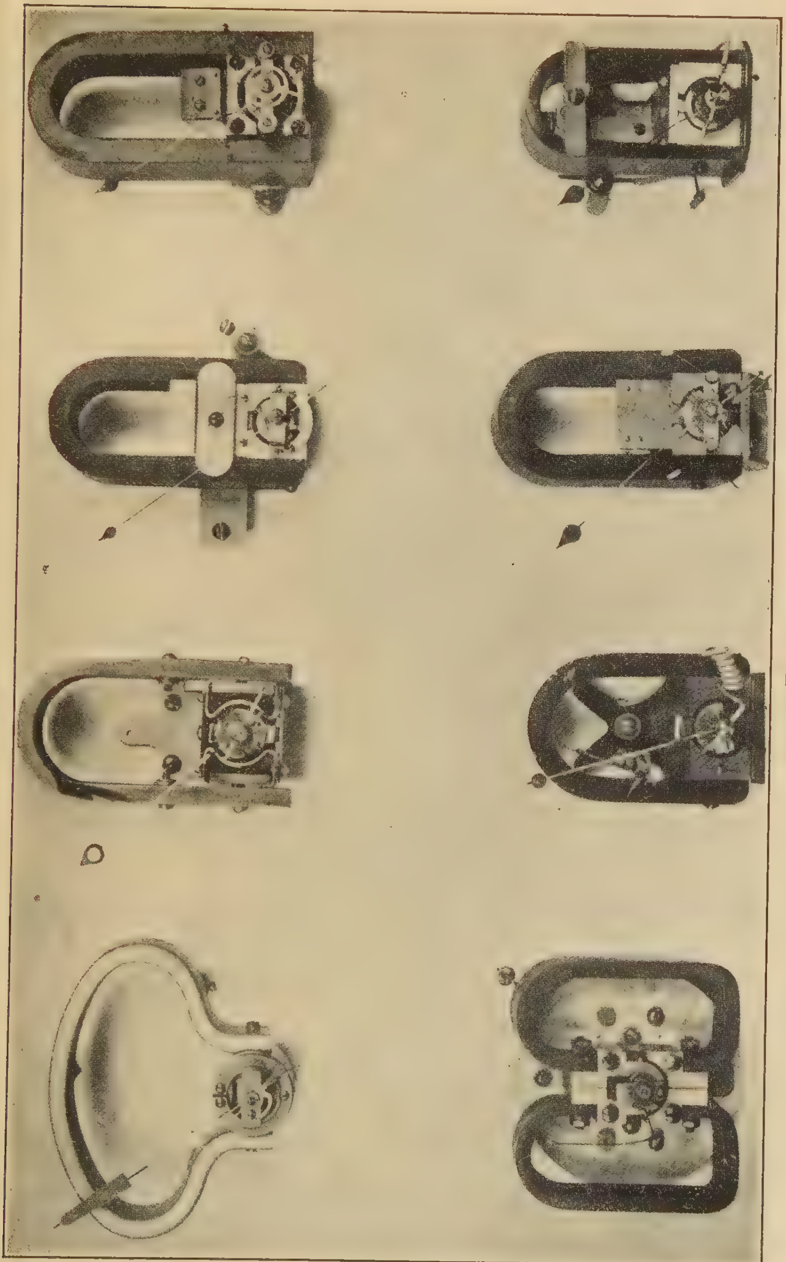


Fig. 26.

parallel to the field. When a current flows through the coil a turning moment or torque will be manifest due to the interaction of current and magnetic field. This torque will tend to turn the coil in such a way that its plane will be at right angles to the permanent magnetic field. The force per unit length of conductor carrying a current I in a field of strength H, is HI. If a be the breadth and b the height of coil the force on one side per turn is bHI, and for N turns it is $NbHI$. For both sides of the coil it will be twice this, or $2NbHI$. If the coil is pivoted at the middle point of a the torque is

$$\begin{aligned} T &= 2NbHI \times a/2 \\ &= NabHI \end{aligned}$$

T is the torque in dyne centimeters tending to turn the coil when a and b are in centimeters and I in absolute units.

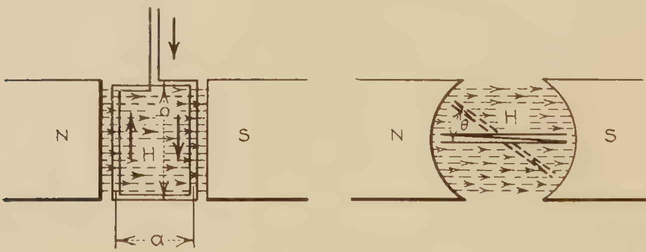


FIG. 27.

When the coil is turned through an angle θ about the axis of rotation the component, or one may say, the intensity of the field producing torque will be different. This component is equal to $H \cos \theta$. Hence, if the field is uniform as assumed, the torque becomes

$$T = abNIH \cos \theta$$

In well-designed instruments the pole pieces of the permanent magnet are shaped so that $H \cos \theta$ is practically constant within the limits of motion of the coil. The quantities a, b, and N are constant for any given coil, and if $H \cos \theta$ is constant, we may replace $abNH \cos \theta$ by a constant K, the expression for torque then becomes

$$T = KI$$

This shows that in well-designed instruments of the movable coil, permanent magnet type, the torque is proportional to the first power of current. Since the opposing torque of a spring is proportional to the angle of twist, the coiled spring method of control is ideal, and is universally used upon this type of instruments. These principles make possible practically uniform graduations.

CHAPTER IV

FUNDAMENTAL PRINCIPLES OF ALTERNATING CURRENTS

52. Introduction.—Before discussing induction type instruments, some general principles of alternating currents will first be reviewed.

In Chapter I were discussed some of the fundamental principles of power and work. It was there shown that the power in a direct current circuit is given by IE watts, where I is the current in amperes and E the electromotive force in volts impressed upon the circuit. Similarly, the energy consumed or utilized by the circuit in t seconds is EIt joules.

From this, and from the other considerations discussed in Chapter I, it is evident that the measurement of power in direct-current systems is comparatively a simple operation. All one needs to know is the current in amperes and voltage, when the power is readily obtained as the product of the two. Thus an ammeter and voltmeter suffice for power determination in direct-current circuits.

When power is to be measured in a circuit in which alternating current is flowing, due to an alternating electromotive force, other considerations enter. These considerations we shall now briefly discuss.

53. Alternating Current.—An alternating current or electromotive force is one which begins at zero, increases to a maximum in one direction, then decreases to zero and again increases to a maximum in the other direction, finally decreasing back to zero again in one cycle. These cycles continue one after another so long as the current is flowing. Thus, in an alternating-current system the terminals of the circuit are alternately positive and negative and the current flows first in one direction and then in an opposite direction.

54. Generation of an Alternating Pressure.—The simplest method of generating an alternating pressure is that indicated in Fig. 28. Here the rotating armature consists of two conductors A and B, connected so as to form a loop. The loop is supported on the shaft D, and centered between the two magnet poles

N and *S*. The ends of the loop are connected to two insulated metal slip rings, C_1 and C_2. The circuit is completed through the external circuit *R* by means of metal contacts or brushes bearing on the rings. Assuming that the magnetic lines pass straight across the armature from the *N*-pole to the *S*-pole, when the loop is rotated the sides *A* and *B* of the loop have electric pressures induced in them. Since the conductors are moving across the field in opposite directions, the pressures in the conductors will be oppositely directed with reference to the plane of the paper. But the two conductors are so connected

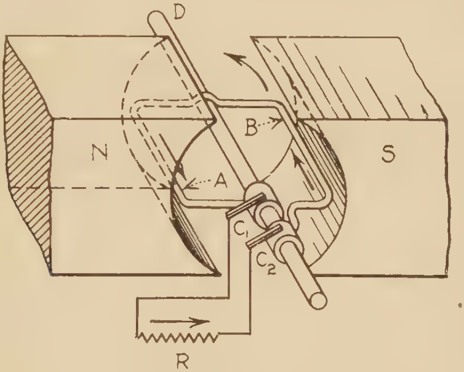


FIG. 28.

that the two pressures act together around the loop; therefore, the pressure between the brushes will be twice that developed in either conductor.

Starting with the plane of the loop in a vertical position, the conductors are moving parallel to the magnetic lines and consequently, no lines are being cut and no pressure is induced in the loop. As the loop rotates from its vertical position, the angle at which the lines are cut approaches a right angle until the loop has turned through 90° when the conductors *A* and *B* will be passing under the centers of the poles *N* and *S* respectively. At this instant they are cutting the lines at right angles and consequently, the maximum pressure will be induced in each conductor. As the coil or loop rotates beyond this point, the number of lines cut by the conductor in unit time decreases. When the loop has advanced another 90° it will again be in a vertical position, but *A* will be at the bottom and *B* at the top. At this instant the pressure is again zero. During this half rotation the current in

external circuit will pass through exactly similar changes in strength.

During the next half rotation, conductor A will pass up in front of the S-pole, and B down in front of the N-pole; consequently, the pressure will go through the same changes that it went through the first half revolution, but the pressure will be directed in the opposite direction. The current through external circuit will flow in the opposite direction also, and its changes in value will follow those of the pressure.

The pressure and current, therefore, pass through a complete cycle of changes; starting from zero they increase to a maximum, then decrease to zero; reverse in direction, increase to a negative maximum, then again decrease to zero, after which the same cycle of changes is repeated.

55. Law of Fluctuation of Alternating Pressure and Current.—In Fig. 29 let C and C_1 represent the cross-sections of the two conductors of a loop, as in Fig. 28, rotating about an axis perpendicular to the plane of the paper at O. Since the pressure induced in one conductor is equal to that induced in the other, it will be sufficient to consider the variation of pressure in one conductor only. Assume the conductor to be revolving counter-clockwise as indicated by the arrow, and let ϕ be the angle made by the plane of the coil with the line OA; OA is taken as the reference line. If the coil rotates

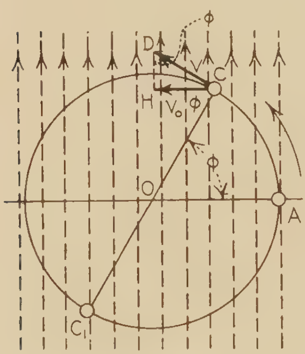


Fig. 29.

with uniform speed, C will move around a circle with uniform speed, V, and ϕ will be a uniformly increasing angle. Assume that the coil has turned through the angle ϕ and reached the position shown in the figure.

By many experiments it has been determined that whenever a wire is moved across a magnetic field an electromotive force is developed. The intensity of the electromotive force will depend upon three quantities: the strength of the magnetic field, the length of the wire within the field, and the speed with which it is moving. No electromotive force is induced when the wire moves along the direction of the field; hence, it is evident that

the intensity of the pressure will also be influenced by the direction of motion of the wire. Expressing these experimental facts in a mathematical equation we get $e = HlV_0$ in absolute units. If we wish to get e in volts, the above expression must be divided by 10^8, or 100,000,000, since 10^8 absolute units equal 1 volt. The expression becomes

$$e \text{ (volts)} = \frac{HlV_o}{10^8}.$$

where

H=strength of field in lines per square centimeters.

l=total cutting length in centimeters of all conductors in series.

V_o=the velocity at right angles to the magnetic lines in centimeters per second.

Let CD, Fig. 29, perpendicular to OC represent graphically to scale the direction and magnitude of the constant velocity V, with which C is moving around the circle.

Let DH be a line drawn through D parallel to the magnetic lines, and CH a line drawn through C perpendicular to the magnetic lines. It is evident that at this instant the same number of magnetic lines would be cut if the conductor moved horizontally with a velocity V_o, equal to CH as when it moves around the circle with the velocity V. V_o is the velocity at right angles to the lines and it is the velocity which must be used in the formula for e.

From geometry, the angle made by the lines DC and DH is equal to the angle ϕ. The ratio of V_o to V, $\left(\frac{V_o}{V}\right)$, is called the sine of the angle ϕ; consequently, $V_o = V \sin \phi$. Substituting this value for V_o in the formula for e we get

$$e = \frac{HlV \sin \phi}{10^8}.$$

In this expression e is the instantaneous pressure.

When $\phi = 0$, $\sin \phi = 0$ and the induced pressure is zero.

When $\phi = 90°$, $\sin \phi = 1$ and the induced pressure is a maximum. The expression $\frac{HlV}{10^8}$ represents the maximum value of the e.m.f. We can then write $E_m = \frac{HlV}{10^8}$, and $e = E_m \sin \phi$; E_m means maximum electromotive force.

The instantaneous value of the pressure is, therefore, proportional to the sine of the angle which the plane of the coil makes with a plane at right angles to magnetic field.

The successive values of the e.m.f. which are induced as the coil passes under the N and S-poles, can be graphically represented, as in Fig. 30.

Let the base line AB represent 360° and let it be divided into equal parts, each part representing 30°. At points representing 0°, 30°, 60°, 90°, etc., draw vertical lines whose lengths represent the maximum value of the alternating quantity times the sin 0°, sin 30°, etc. Between 180° and 360° the sine is negative and the values are drawn below the base line. This is in agreement with

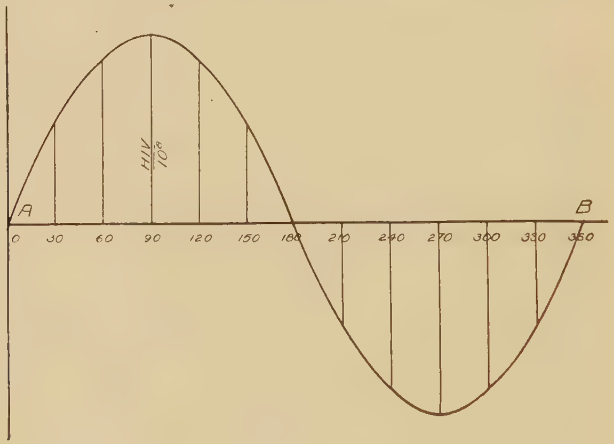


Fig. 30.

the fact already mentioned that the pressure changes in direction as the conductors pass from under one pole under another pole.

A curve drawn through the extremities of the vertical lines is called a sine curve. If the maximum ordinate is E_m, then the sine curve will be the sine curve of the pressure. If the maximum ordinate represents the maximum value of the current, the sine curve will be a sine wave of current.

Another method of graphically constructing a sine curve is shown in Fig. 31. The radius of the circle, to the left, is made equal to the maximum value of the sine curve. Let OM be an extension of the line of reference AB and draw radial lines ON, OP, OQ, etc., making angles of 30°, 60°, etc., with the base line OM. From the extremities of these radii draw horizontal

lines intersecting the vertical lines erected at points on AB representing the corresponding angles. The points of intersection will be points on the sine curve. Between 0° and 180° the ordinates are above the axis and from 180° to 360° they are below. A curve drawn through all the points of intersection will be a sine curve. At the point on AB representing 30°, the ordinate equals RN. But from trigonometry RN equals $ON \times \sin 30°$. If ON were drawn to represent E_m then RN equals $E_m \sin 30°$. In general the curve shows graphically the value of y in

$$y = A \sin x$$
$$\text{or } e = E_m \sin \phi.$$

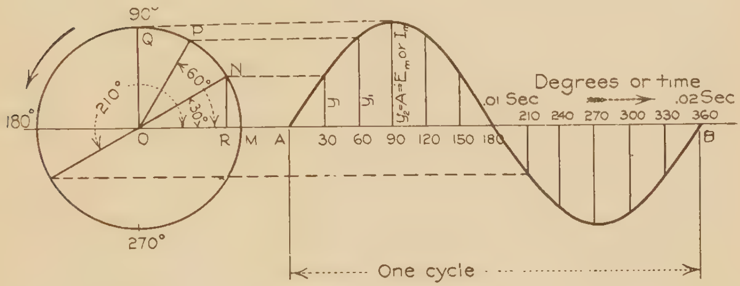


FIG. 31.

If ω represents the angle described by ON in unit time, one second, in t seconds it will describe an angle equal to ωt, and if ωt is the time required to describe angle ϕ, then $\phi = \omega t$. The expression for induced pressure then becomes

$$e = E_m \sin \omega t.$$

There is some advantage in considering AB, Fig. 31, as an axis of time, and plotting successive values of t horizontally rather than the angle ϕ or ωt. For instance, if ON makes 50 complete revolutions per second, it will make one revolution in 1/50 or .02 second. The time required to describe an angle of 30° is 1/12 of .02 second. Thus the intervals $A-30°$, etc., can just as readily be used to represent intervals of time. The significance of this will be seen when we consider phase difference.

Although the positive and negative loops of an alternating current or pressure curve are nearly always alike, in general the fluctuations do not follow a simple law. Since any complex wave

whose alternate loops are alike may be represented by the sum of a series of sine and cosine curves having different amplitudes and frequencies, an alternating current wave may be represented thus:

$i=I_1 \sin x+I_3 \sin 3x+I_5 \sin 5x+$ etc. $+I_1 \cos x+I_3 \cos 3x+I_5 \cos 5x+$etc.

An analysis of such a representation of current or pressure is beyond the limits of this text. The following discussion is based on the assumption that the current or pressure curve may be represented by the first term on the right-hand side of the equation; thus $i=I_m \sin x=I_m \sin \omega t$.

56. Cycle, Frequency, Period, Alternation.—Referring to Fig. 31, we may assume that the electromotive force induced within one coil of an armature, while under a north pole, is represented by the ordinates above the horizontal axis. Similarly, the ordinates under the horizontal axis represent the electromotive force induced in coil when under the south pole. These two sets of values are called a cycle. The number of cycles per second is called the frequency. The time required for the electromotive force to change through one cycle is called a period.

Since the pressure and current pass through a complete cycle of values when the coil or conductor passes under a north and south pole, the number of cycles per revolution of an armature is equal to the number of pairs of poles. The frequency will then be equal to the number of cycles in one revolution multiplied by the number of revolutions per second or

$$f=\frac{p}{2}\times n$$

where p is the number of poles and n the number of revolutions per second. If the revolutions per minute are given, as is usually the case, this number must first be divided by 60 to get the revolutions per second.

There are always two alternations for each cycle, hence the number of alternations in any unit of time will be two times the number of cycles in the same unit of time.

57. Instantaneous Value.—Representing the fluctuations of an e.m.f., or current by a sine curve, the instantaneous value is represented by the distance from the horizontal axis to the curve at that particular instant. Thus in Fig. 31 the vertical lines y, y_1, y_2, etc., represent the instantaneous values at the ends of

the intervals of $\frac{1}{600}$, $\frac{1}{300}$, $\frac{1}{200}$, etc., seconds after the point N has passed through M.

58. Maximum Value.—The instantaneous value at the point marked 90°, or at the end of $\frac{1}{200}$ second, is greater than that at any other point between 0° and 180°, and is consequently a maximum value. The numerical value at the point marked 270° is equal to that at 90°; its direction, however, is downward and is, therefore, a negative maximum value. In alternating-current problems the term maximum has reference to only the numerical value and not to its direction.

59. Average Value.—The average value of an alternating e.m.f. or current is the average of all the instantaneous values for half a cycle, or the average of all the instantaneous values for a complete cycle, irrespective of sign. The average or mean value of a series of quantities is in general,

$$\text{average } (a) = \frac{a_1 + a_2 + a_3 + \ . \ . \ . \ a_n}{n}$$

where a, a_2, a_3 . . . a_n represent the successive values and n is their number.

Assuming that the instantaneous values of an alternating e.m.f. vary according to a sine law, or that the alternating quantity is harmonic, the average value will be equal to the sum of the instantaneous values divided by their number. In other words, it will be equal to the area between the curve and base line divided by the base line. It can be shown by calculus that this is equal to $2/\pi \times$ maximum value $= .636 \times$ maximum value. The average value is used in some calculations.

60. Effective Value or Root-mean Square Value.—The effective value is very important in alternating-current problems.

When a direct current is sent through a resistance, the energy converted into heat per second is

$$\text{Heat} = I^2R \text{ joules.}$$

An alternating current varies in intensity from instant to instant; its heating value is, however, at each instant equal to i^2R, where i is the current at the instant considered. The heat developed per cycle will then be the average of i^2R for the cycle. Since the resistance R remains constant, the heat developed per cycle must be equal to R times the average of i^2. Thus an alternating current, whose average square is I^2, will develop the same amount

of heat per second in a resistance R as a direct current whose value is I.

If a direct current I be sent through an ammeter of the electro-dynamometer type, a torque proportional to I^2 will be developed. If the same ammeter is used to measure an alternating current, the torque at each instant will be proportional to i^2. This torque is always exerted in the same direction irrespective of the direction of the current. The resulting torque will be proportional to the average of i^2, and if the deflection with direct current is equal to that with alternating current, we may write

$$KI^2 = K \text{ average } i^2$$
$$\text{and } I = \sqrt{\text{average } i^2}$$

That is, when the deflection is the same, the value of the direct current must be equal to the square root of the average of the squares of the instantaneous values of the alternating current. This square root of the mean square value is called the effective value, or "root-mean square value," of an alternating current or pressure. It can easily be shown that for a harmonic current the effective value is $\frac{1}{2}\sqrt{2}$ times the maximum value; that is

$$I = .707\, I_m$$
$$\text{and } E = .707\, E_m.$$

61. Effect of Inductance.—We learned in Article 10 that the wire along which an electric current is flowing is surrounded by a magnetic field. In a direct-current system this magnetic field remains constant, both in intensity and direction, so long as the current remains constant. The building up of the magnetic field requires energy which must be furnished by the initial current. Since no energy can be stored in any system unless that system reacts upon the source of energy or working substance, it follows that the magnetic field reacts upon the current to which it is due. This reaction prevents the sudden rise of current within a circuit to the maximum value, as indicated by Ohm's law.

Again, when the circuit is opened and the current ceases, the energy that has been stored in the magnetic field is returned to the circuit and attempts to keep the current flowing. This energy manifests itself as a spark at the terminals of the circuit. Since the current cannot rise to a maximum value immediately

upon closing the circuit, neither can it immediately fall to zero when the circuit is opened. This reaction of magnetic field upon the current is known as induction.

The effect of induction may be considered as analogous to the action of a fly-wheel on a steam engine. When first the steam is turned on, some of the energy of the steam is converted into energy of motion, or kinetic energy of the fly-wheel. The fly-wheel reacts upon the engine until steady speed is reached. When steady speed is reached, no more energy is given to the fly-wheel, but all of it goes toward running the machinery. When the steam is shut off, the engine does not at once come to a dead stop; the fly-wheel keeps it in motion for some time until its kinetic energy has all been given back to the engine and machinery.

62. Effect of Capacity.—In addition to inductance every circuit possesses some capacity, that is, the ability to become charged, or to store up electricity. One conductor alone has very little, but more than one, arranged in suitable ways, may possess considerable capacity. When so arranged, the device is called a condenser.

An electrical condenser may be considered analogous to an air tank. Suppose we have an air tank that under one atmospheric pressure holds a certain definite quantity of air, say 5 lb. We can define the capacity of the vessel in terms of the number of pounds of air it holds, and call it a 5-lb. tank.

If the pressure is doubled, the tank will hold 10 lb. of air. Since we have defined the capacity of the tank in terms of unit (one atmosphere) pressure, we cannot call it a 10-lb. tank. A 10-lb. tank under same conditions will hold 20 lb. of air.

Furthermore, suppose the tank to be exhausted, evidently no back pressure will be exerted when air is first admitted to the tank. As soon as some air is admitted to the tank, back pressure begins to manifest itself, and when the back pressure equals the applied pressure, no more air enters the tank. We thus see that the amount of air entering per unit time depends upon the back pressure and this back pressure will depend upon the capacity of the tank. For instance, if we put 5 lb. of air in a 10-lb. tank, the back pressure will be one-half as great as when 5 lb. of air are put into a 5-lb. tank. We can then say that unit capacity of a tank is such that when 1 lb. of air is forced into it the pressure will be equal to one atmosphere. Evidently, a certain amount of

work will be done in forcing the air into the tank, and we could define unit capacity in terms of the work expended.

The capacity of electrical conductors is analogous to the capacity of the air tank discussed above. The capacity of a condenser or system of conductors is usually defined in terms of the quantity of electricity required to raise the difference of pressure between the terminals by one volt. In accordance with this definition the quantity of electricity that a condenser will contain is equal to the product of the capacity and pressure, or $Q=EC$. The flow of an alternating current within any circuit depends not only upon the resistance of the circuit, but also upon any inductance and capacity that may be contained in or connected with the circuit.

The continual surging back and forth of the current in an alternating circuit gives rise to very important inductance and capacity effects in certain parts of the circuit, and the resulting peculiarities that distinguish the alternating from the direct-current circuit. The two factors mentioned above may be far more important than resistance, and in some cases may entirely control the flow of the current.

63. Phase Difference.—It has been pointed out that the current wave is in form much the same as the electromotive force wave in an alternating-current circuit. The constants of the circuit—that is, resistance, inductance, and capacity—will, however, influence the time at which the current will reach a maximum.

This fact will probably be understood from the analogy of the fly-wheel as already given. If the pressure applied to the fly-wheel is constant it may be considered as analogous to the electromotive force applied to a circuit possessing inductance. The speed of the fly-wheel may be considered as analogous to the current flowing. The pressure applied to the fly-wheel is maximum at the start while its speed does not reach a maximum until later. We can say that the speed lags behind the pressure. Similarly, the current in an inductive circuit does not reach a maximum until some time after the electromotive force. We thus say that the current lags behind the pressure. The time between the positive or negative maximum values of electromotive force and current is the phase difference. This phase difference depends upon the relative values of resistance and inductance of the circuit.

When the circuit contains capacity, the analogy of the air

tank serves to give an idea of the relations of these quantities. When the air is first admitted into the empty tank the current will be a maximum, and the counter-pressure a minimum. The current of air thus leads the pressure. Similarly, when a source of electromotive force is connected to a circuit possessing capacity, the current is a maximum ahead, in time, of the pressure; and we say the current leads. Again, the phase difference is repre-

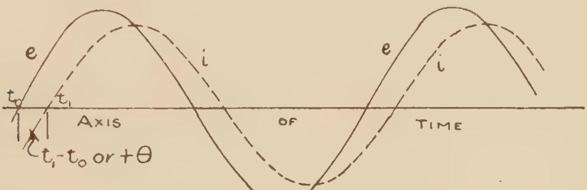


FIG. 32.

sented as the difference in time between the maximum values of current and electromotive force.

The relative positions of the electromotive force and current curves are shown in Figs. 32 and 33. Fig. 32 shows the conditions in a circuit having resistance and inductance; and Fig. 33, the conditions in a circuit having resistance and capacity. θ is the phase difference.

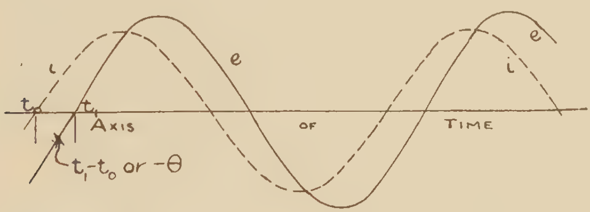


FIG. 33.

64. Power in Alternating-current Circuits.—Since in direct-current circuits the power is equal to the product of current and pressure, the instantaneous power in an alternating-current circuit will be equal to the product of the instantaneous values of current and pressure. These, however, vary from time to time, so the average power per cycle will be the average of the instantaneous values of the product of current and pressure.

When the current and pressure are in phase, that is, pass

through their maximum and zero values at the same time, we have conditions as shown in Fig. 34. The power curve is obtained by multiplying together the instantaneous values of current and pressure, and, as the figure shows, the power reaches a maximum at the same time as current and pressure. The power curve is, however, always positive, for the product of two positive quantities is positive, and likewise the product of two negative quantities is positive.

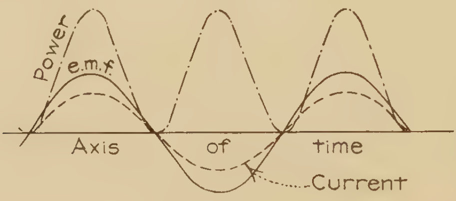


Fig. 34.

In Fig. 35 we have conditions that are somewhat different. Here the current lags behind the electromotive force. The power curve has both positive and negative loops, that is, loops both above and below the horizontal axis. The average power supplied to the circuit will thus be the difference of the average of the positive and negative loops. It is thus evident that the average product of current and pressure in an alternating-current

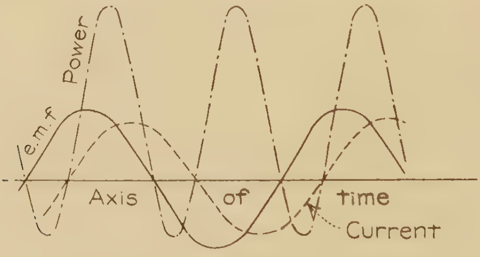


FIG. 35.

circuit does not give the average power unless the current and pressure are in phase as shown in Fig. 34. The power will depend not only upon the current and pressure, but also upon the phase difference between them. Ordinarily, the product of an ammeter reading and voltmeter reading will not give the true power in an alternating-current circuit. In general, the power is less than this product.

As has already been shown, the phase difference will depend upon the inductance and capacity of the circuit, and hence the power will depend upon the inductance and capacity.

65. Phase Angle.—We have defined phase difference as the interval of time that elapses between the maximum values of pressure and current. Physically this is exactly what the phase difference is. For purposes of computation, however, the phase difference is best expressed as an angle and, accordingly the constant by which we multiply the product of the effective current and effective electromotive force to get the actual power in a circuit is usually written as the cosine of the phase angle, thus

$$\text{Power} = IE \cos \theta$$

where I and E are the effective values of current and pressure, or, in other words, are the values that alternating-current ammeters and voltmeters indicate. Cosθ is the cosine of the phase angle and is called the power-factor. Another definition of power-factor will be given later. When the current and pressure reach the maximum value at the same time the difference in phase is zero and $\cos \theta = 1$. When this is the case, the power is a maximum.

This fact, that the alternating current and pressure causing the current may be out of phase, is fundamental in measuring alternating-current quantities, and should be mastered. As already stated, power computed from ammeter and voltmeter indications may be very much in error in some cases. Wattmeters, however, take account of the power-factor and their indications give correct values.

CHAPTER V

ALTERNATING-CURRENT CIRCUITS

66. Single-phase Circuits.—A single-phase circuit consists of two line wires, and is fed by a single-phase generator. The armature of the single-phase generator contains a single winding, the two ends of which are connected to two collector rings. In the revolving-field type of generator the single-phase generator is provided with two terminals to which the external circuit may be connected.

The electromotive force of a single-phase generator fluctuates between positive and negative values and is well represented in Figs. 32, 33, and 34. Similarly, the current in a single-phase circuit fluctuates between positive and negative values, as indicated in figures mentioned above.

The single-phase circuit is thus similar to a two-wire, direct-current circuit. The flow of power in the circuit is, however, considerably different. It can be shown that the power in a single-phase circuit fluctuates with a frequency double that of the electromotive force or current. A curve showing the fluctuations of power in a single-phase circuit is shown in Fig. 35.

67. Polyphase Circuits.—A polyphase circuit may consist of either two, three, or more phases; the three-phase circuit being the most common.

The winding of the armature of a quarter-phase, commonly called two-phase, generator consists of two distinct sets of coils. This winding is so arranged that, when the coils of one set are under the field poles, the coils of the other set are midway between the field poles. Thus, when the electromotive force in one set of coils is a maximum, that in the other set is zero. The electromotive force curves of a quarter-phase generator are shown in Fig. 36. Calling the angular distance between two consecutive poles of an alternator equal to 180 degrees, the difference in phase between the electromotive forces in the two sets of windings is 90 degrees, as indicated in figure.

A quarter-phase circuit usually contains four wires, each connected to the terminals of one set of coils on the armature. For

simplicity, it may be looked upon as consisting of two single-phase circuits. In some cases two of the return wires are joined and the circuit consists of only three wires. This system is not very common. The quarter-phase generator is also being displaced

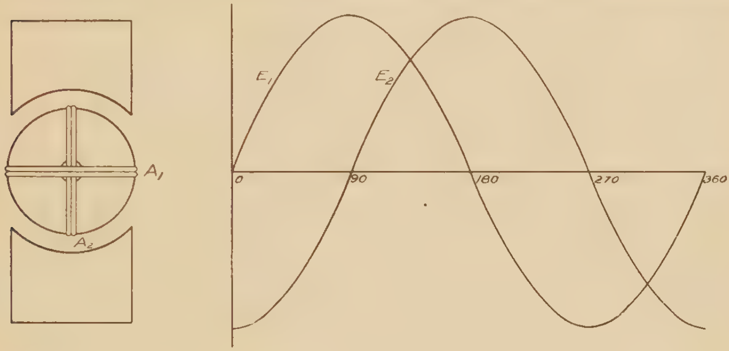


FIG. 36.

by the three-phase machine, which is more efficient, everything considered.

68. Three-phase Circuits.—The voltage relations in a three-phase system are represented in Fig. 37. The windings on the armature consist of three semi-distinct sets of coils so arranged that each occupies approximately one-third the distance between

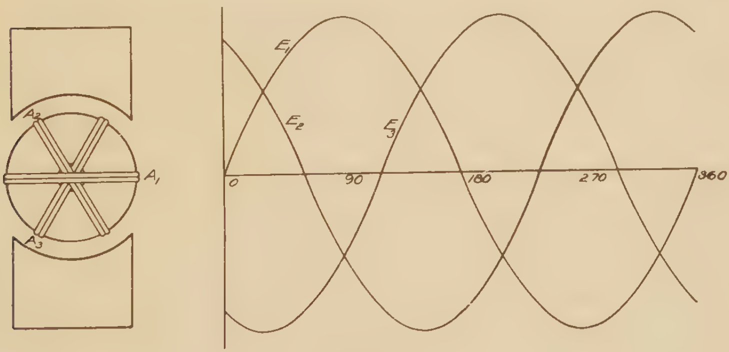


FIG. 37.

two field poles of the same polarity. The phase difference between the separate electromotive forces is then 120 degrees, or 1/3 of 360 degrees, since the distance between two like poles is 360 electrical degrees.

The three-phase system may also be looked upon as three single-phase systems whose voltages are out of phase by 120 degrees. In practice, however, this would necessitate six line wires, which would make the system very complicated and expensive. The efficiency of the system lies in the fact that three ends of the armature coils may be joined together and the other three ends

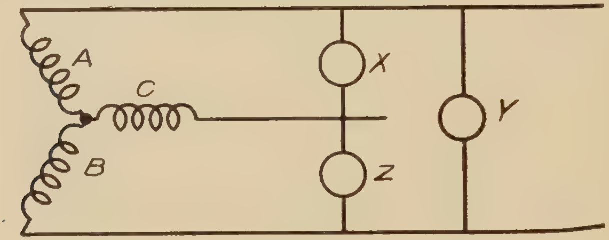


Fig. 38.

to the line wires, as shown in Figs. 38 and 39. The lamps, or other receiving circuits, are then connected between the line wires as shown by x, y, and z. The manner of connecting the armature coils, shown in Fig. 38, is known as the Y connection, and that shown in Fig. 39 as the delta connection. A, B, and C in each of these figures represent the separate phase windings.

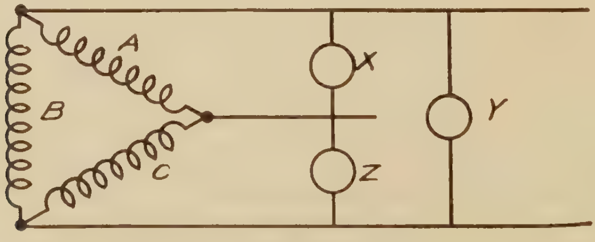


Fig. 39.

69. Current and Voltage Relations in Three-phase Circuits.—Representing the maximum value of the pressure generated in each phase by a line whose length is proportional to the numerical value of the maximum pressure, Fig. 40, three equal lines, making angles of 120 degrees with each other, will represent, both in magnitude and phase, the maximum three-phase pressures generated in a Y-connected armature. The instantaneous values of the

separate pressures will then be equal to the projections of E_{1m}, E_{2m}, and E_{3m} upon the vertical line YY'. At the instant represented by the figure these instantaneous values are for E_{1m}, Oe_1; for E_{2m}, Oe_2; and for E_{3m}, Oe_3. Representing Oe_1, Oe_2 and Oe_3 by e_1, e_2 and e_3 respectively the following relations hold:

$$e_1 = E_{1m} \sin \theta$$
$$e_2 = E_{2m} \sin (\theta + 120°)$$
$$e_3 = E_{3m} \sin (\theta + 240°).$$

By rotating E_{1m}, E_{2m}, and E_{3m} counter-clockwise, e_1, e_2, and e_3 will fluctuate as the sine of an angle, but differing in phase by 120 degrees. Hence, they may be properly represented by the sine curves of Fig. 37.

The three lines E_{1m}, E_{2m}, and E_{3m}, Fig. 40, are called vectors. Vectors cannot be added algebraically, that is, the sum of E_{1m}

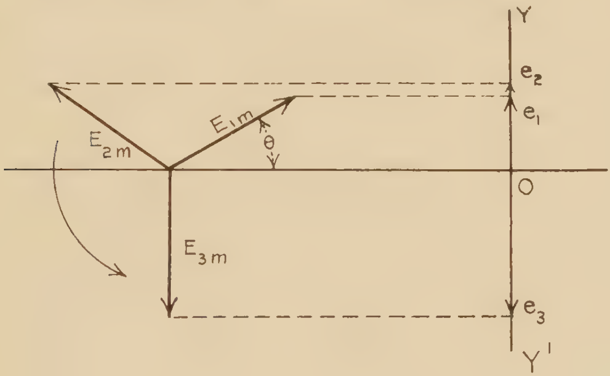


Fig. 40.

and E_{2m} is not the algebraic sum of their numerical values. Vectors are combined geometrically. To get the sum of two vectors, form a parallelogram with the given vectors as sides; the diagonal will then represent the sum, both in magnitude and direction.

The vector difference is obtained in much the same way. The direction of vector to be subtracted is reversed and the two vectors are then added.

According to this method of addition and subtraction, the pressure across y, Fig. 38, is equal to the vector difference between the pressures generated in windings A and B respectively. Representing the pressure developed in winding A by vector E_A,

and that in winding B by E_B, Fig. 41, the vector difference is equal to E, which is the pressure across y. Numerically

$$E = E_A \cos 30° + E_B \cos 30°$$
$$= \tfrac{1}{2}\sqrt{3}E_A + \tfrac{1}{2}\sqrt{3}E_B$$
$$= \sqrt{3}E_A = \sqrt{3}E_B$$

when $E_A = E_B$, as is usual in practice.

In the foregoing demonstration E, E_A, and E_B represent either maximum or effective values. Hence, we may, in general, say that the pressure between the mains of a Y-connected three-phase circuit is equal to $\sqrt{3}$ times the pressure developed in each armature winding.

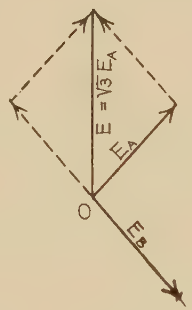


Fig. 41.

The current in each main must be the same as the armature current, as is evident from the connections.

In the delta-connected system the conditions are reversed. The connections plainly show that the pressure between mains is the same as that developed in one armature winding. The current in either main, however, is the vector difference between currents in two windings, and hence, according to what has just been said, current in one main equals $\sqrt{3}$ times current in one armature winding. The power in any polyphase system is not pulsating as in a single-phase system, but is constant or steady.

CHAPTER VI

INDUCTION PRINCIPLE

70. Introduction.—The fundamental principle of induction meters, as well as motors, was discovered by Arago in 1825. He found that if a copper disk is pivoted on the axis of a magnetic needle, its plane being horizontal, and rotated, the needle will be deflected. This principle is illustrated in Fig. 42. Above the copper disk *C*, but not touching, is a glass plate *G*. In the middle of the glass plate and directly above the center of the arbor is pivoted a magnetic needle. When the copper disk is rotated, the needle is deflected in the direction of rotation. If the position of disk and needle be changed, and the needle rotated, the disk will rotate in the same direction as the needle.

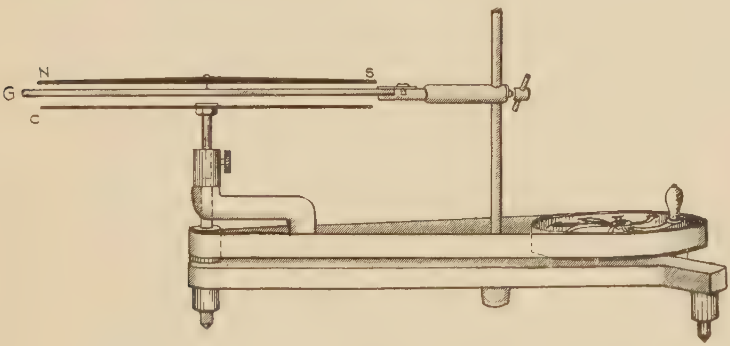


Fig. 42.

Faraday explained this phenomenon on the supposition that relative motion between needle and disk resulted in producing electric currents in the disk, and that the reaction between the magnetic field produced by these currents and the field due to the magnet caused the rotation of needle in the first case, and of the disk in the second case. This explanation has been verified many times since. Whenever a conductor cuts across a magnetic field an electromotive force is induced. If the conductor forms a closed circuit, a current will flow through the circuit and a reaction is set up between the conductor and magnetic field.

In the experiments of Arago and Faraday the magnetic field was caused to rotate by rotating the magnet to which the field was due. It is evident such a device is not suited for measuring instruments. What is needed is relative motion between the field and conductors, without the corresponding motion of magnets.

71. Rotating and Revolving Magnetic Fields.—In practice there are two types of moving magnetic fields which may be designated by the two terms, rotating and revolving. In everyday language the two words, rotating and revolving, are used interchangeably, but there is a distinction in their meanings which should be kept in mind.

A body is said to rotate when it has a circular motion about its own center or axis; to revolve is said of a body that moves in a curved path, as a circle or ellipse, about a center outside of itself. According to this distinction a rotating magnetic field is one that turns about an axis passing through the field; and a revolving magnetic field is one that moves around an axis outside of itself.

72. Production of Rotating Field.—To produce a rotating magnetic field of constant intensity, and rotating at a constant speed, necessitates polyphase currents. In Fig. 43 is shown

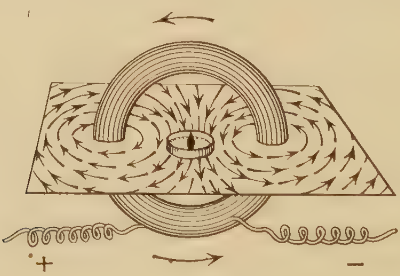


FIG. 43.

a circular coil carrying a current which sets up a magnetic field at the center of the coil. When the current is alternating, the field is also alternating, increasing and decreasing with the current and reversing as the current reverses. Representing the current by

$$i = I_m \cos \omega t$$

we may represent the instantaneous value of field strength by

$$h = H_m \cos \omega t$$

A horizontal section of two such coils with their planes at right angles is shown in Fig. 44. When coil AA' is connected to one phase of a quarter-phase circuit, and BB' to the other phase, the currents will produce alternating fields in the two coils. The direction of each field will always be at right angles to the plane of the coil producing it, but the intensities of the two fields will differ by one-quarter of a period.

The instantaneous values of the field intensities can be represented by

$$h_1 = H_{1m} \cos \omega t$$

and

$$h_2 = H_{2m} \sin \omega t$$

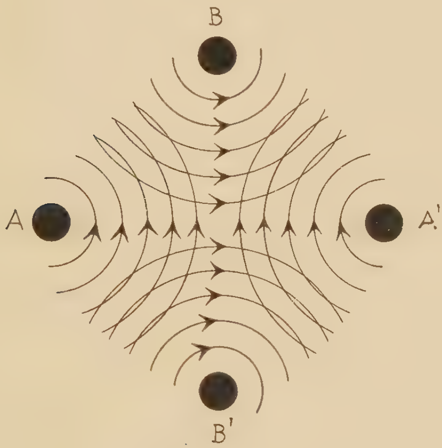


Fig. 44.

The resulting field will be the vector sum of h_1 and h_2. Since h_1 and h_2 are at right angles to each other the instantaneous value of the resultant is

$$h = \sqrt{h_1^2 + h_2^2}$$
$$= \sqrt{H_{1m}^2 \cos^2 \omega t + H_{2m}^2 \sin \omega t.}$$

When $H_{1m} = H_{2m}$, that is, when coils are exactly alike,

$$h = \sqrt{H_{1m}^2 (\cos^2 \omega t + \sin^2 \omega t)}$$

or

$$h = H_{1m}, \text{ a constant.}$$

The rectangular components are h_1 and h_2 and when $H_{1m} = H_{2m} =$

H_m, that is, when the maximum values of the component fields are equal, we have

$$h_1 = H_m \cos \omega t$$

and $$h_2 = H_m \sin \omega t$$

or $$h_1^2 + h_2^2 = H_m^2 \text{ the equation of a circle.}$$

Therefore, the resultant field rotates at a constant angular speed which is determined by the frequency.

In practice the two component fields seldom have equal maximum values, consequently, the following is more important.

73. Rotating Field Produced by Unequal Component Fields.—Two harmonically alternating fluxes at right angles to each other in space may be replaced by two rotating fluxes of different magnitudes rotating in opposite directions but having the same angular speeds. Thus in Fig. 45, let $OA = 2H_{1m}$ and $OB = 2H_{2m}$,

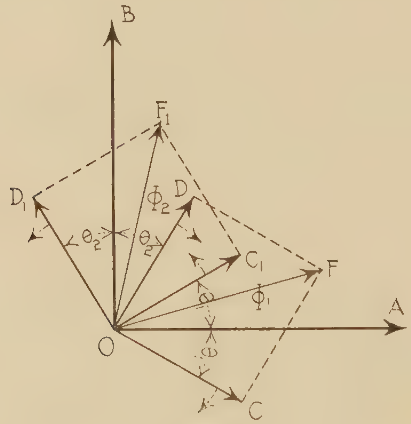


FIG. 45.

represent two harmonic fluxes, produced by two circular coils having a common center but having planes at right angles to each other. OA and OB are fixed in direction but vary in magnitude according to the sine law. The flux OA may be considered as being the projection of two equal vectors, each equal to $\frac{1}{2}$ OA, rotating in opposite directions at a uniform speed so as to make one complete revolution while the values of OA pass through one cycle.

Let OC and OC_1 represent the two component vectors at the instant they make an angle θ_1 with OA. Evidently, at this

instant the intensity of the field along OA is given by $2\,OC \cos \theta_1 = 2\,OC \cos \omega t$

where $$\omega = 2\pi \times \text{frequency}.$$

Similarly OB may be considered as the resultant of two vectors OD and OD_1, each equal to $\frac{1}{2}\,OB$.

Combining the two components that rotate in the same direction we get the two components OF and OF_1, which differ in magnitude, rotate in opposite directions, but have same angular speeds.

The numerical value of OF and OF_1 in terms of OC and OD, and thus in terms of OA and OB, can be obtained analytically as follows:

$$OF^2 = OC^2 + OD^2 - 2OC \times OD \cos ODF$$

and $$OF_1^2 = OC_1^2 + OD_1^2 - 2OC_1 \times OD_1 \cos OD_1F_1$$

but $$OC = OC_1 = H_{1m} \text{ and } OD = OD_1 = H_{2m}$$

Then $$OF^2 = H_{1m}^2 + H_{2m}^2 - 2H_{1m}H_{2m} \cos ODF$$

and $$OF_1^2 = H_{1m}^2\ H_{2m}^2 - 2H_{1m}H_{2m} \cos OD_1F_1.$$

The angle ODF is the supplement of $\angle COD$.

But $\angle COD = \angle AOD + \angle COA = \frac{\pi}{2} - \theta_2 + \theta_1$, and hence,

angle $ODF = \pi - \left(\frac{\pi}{2} - \theta_2 + \theta_1\right) = \frac{\pi}{2} + (\theta_2 - \theta_1)$

Similarly angle OD_1F_1 can be shown to equal $\frac{\pi}{2} - (\theta_2 - \theta_1)$.

Now $\frac{\pi}{2}$ is the physical angle between the fluxes OA and OB, but $\theta_2 - \theta_1$ is the time phase difference expressed as an angle between the alternating fluxes of which OA and OB represent the maximum values. Representing this phase difference by θ_o we finally get:

$$OF^2 = H_{1m}^2 + H_{2m}^2 + 2H_{1m}H_{2m} \sin \theta_o$$
$$OF_1^2 = H_{1m}^2 + H_{2m}^2 - 2H_{1m}H_{2m} \sin \theta_o.$$

That is, in place of one resultant field rotating with a uniform speed in one direction there are two fields with different amplitudes rotating in opposite directions. This principle has important application in induction type instruments.

74. Production of a Revolving Magnetic Field.—Two quarter-phase currents may also be used for producing a revolving field. The manner in which the stator of a quarter-phase motor is connected is indicated in the diagram of Fig. 46. The short

heavy lines represent the conductors in the slots of the stator core, and the light lines represent the end connections. The curved lines outside may be considered as representing the front end connections, and, under this assumption, the curved lines within will represent the back connections. An examination of the diagram will show that the current in conductor 1 passes across the iron core from front to back while in conductor 11 it passes from back to front. The two groups of conductors, A and A', together with the connecting wires may thus be considered

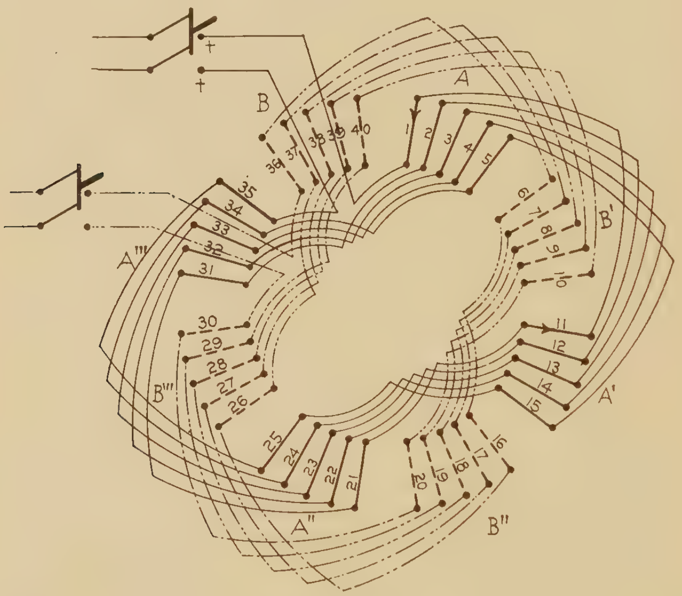


Fig. 46.

as forming a coil which surrounds a portion of the iron core. Similarly B and B', which surround another portion of the core, will form another coil. Assuming a direct current to be flowing through phase A while phase B is open, it is evident that a north pole will develop between A and A', and a south pole between A' and A'', etc. In all there will be four poles. Such an arrangement of conductors is called a four-pole winding.

A simplified diagram of an end view of a four-pole two-phase induction motor is shown in Fig. 47a. In this diagram the conductors are represented by circles. For clearness the end

connections are omitted. When current in winding A–A', etc., is maximum that in winding B–B', etc., is zero. The position of the magnetic lines at this instant is indicated by dotted lines with arrow heads. The current in group of conductors A is toward the observer, and in group A' away from the observer. Under these conditions a south pole is formed at S under the stator core.

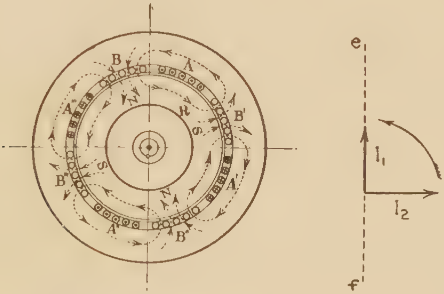


FIG. 47a.

One-eighth of a period later the current in winding A–A', etc., will have decreased to .707 of its maximum value and that in winding B–B', etc., will have increased to .707 of its maximum value. The currents will at this instant be equal and as they will be flowing in the same direction, the position of the magnetic field

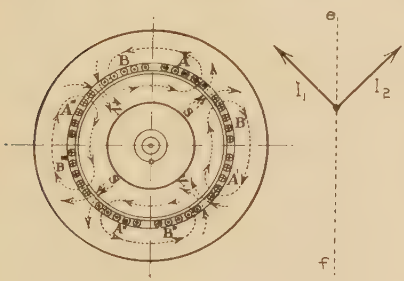


FIG. 47b.

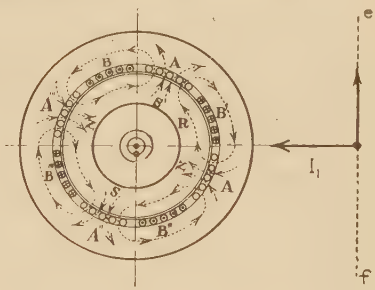


FIG. 47c.

will be as indicated in Fig. 47b. The magnetic fluxes due to the two currents will combine forming a resultant pole halfway between A and B'. The same thing will hold true with reference to the other groups of conductors. Thus during one-eighth of a period of the current, the magnetic poles have shifted one-sixteenth of the circumference of the stator.

After another one-eighth of a period the conditions will be as represented in Fig. 47c. At this instant the current in winding A–A', etc., is zero and that in B–B', etc., is a maximum. The magnetic poles have again shifted one-sixteenth of the stator circumference. The magnetic field which is confined to the outer periphery of the rotor and the inner periphery of the stator revolves independently of the iron core. A revolving magnetic field can be obtained from any polyphase circuit by means of appropriate windings.

75. Speed of Revolving Field.—The speed with which the polarity will be transferred around the stator core will depend upon the frequency of the alternating current. The number of revolutions per minute will depend upon the number of pairs of poles per phase, and upon the frequency. If the frequency of the supply pressure be f, and if there be p pairs of poles per phase, then the field will make one complete revolution in $\frac{p}{f}$ seconds. It will, therefore, make $n = 60\frac{f}{p}$ complete revolutions per minute.

CHAPTER VII

INDUCTION TYPE AMMETERS AND VOLTMETERS

76. Application of Induction Principles to Meters.—If a suitably mounted hollow conducting cylinder, or disk, be placed inside a rotating field, currents will be induced in it, due to the relative motion of the two, as in the experiment of Arago. The currents will react with the magnetic field in such a way as to cause rotation of the cylinder or disk. The reaction will be in such a direction as to oppose the motion of the magnetic field in accordance with the general law of induction. The cylinder or disk will thus rotate in the same direction as the magnetic field.

Induction meters operate in accordance with these principles. For single-phase instruments, the principle of rotating, or revolving, field is obtained in a modified way.

77. Induction Ammeters and Voltmeters.—One method of producing a revolving, or in this case what may be called shifting, magnetic field for single-phase instruments is shown in Fig. 48. The current circuit of the meter is represented by the winding *ABC*. The coil *B* surrounds the laminated iron core *I*. In the end of this iron core is a slot through which and around one-half of core is wound a heavy band of copper as shown at *E*. The alternating current flowing through coil *B* induces a flux in the iron core. When the flux is increasing, a part of it will pass through the core of the short-circuited copper band, inducing a current in it. This current is in such a direction that it opposes the building up of the magnetic flux within the space surrounded by the band. While the current is increasing, the magnetic density of the part of iron core that is not surrounded by the copper band will be greater. On decreasing current, the conditions are, however, reversed. The flux density of the unwound part of the core will decrease to zero before that in the other part. The flux thus shifts from the unwound portion to the wound part of the core.

As this shifting flux penetrates the disk *D*, currents are induced in it which react with the magnetic flux, causing the disk to

rotate. The controlling force is supplied by a coiled spring as in other indicating instruments.

It is plainly evident that the reaction between shifting field and induced currents in the disk is a function of both the intensity of the shifting field and frequency of current. If no provision were made for correcting the influence of frequency, changes in frequency would affect its indications. To compensate for frequency effects, the main coil B is made of low resistance and high inductance. The terminals of the coil B are connected to a non-inductive shunt, S. The circuit is, therefore, divided into two branches, one containing resistance but no inductance and the other inductance but of negligible resistance. At normal fre-

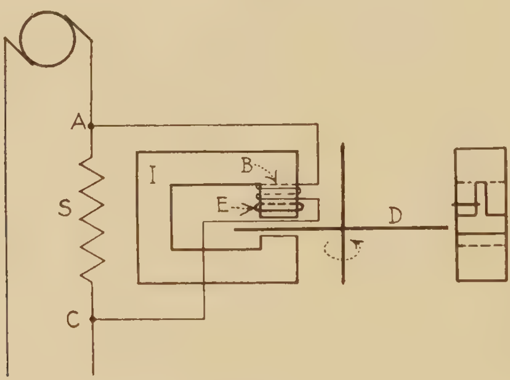


FIG. 48.

quency the current will divide in a certain ratio between the two branches. An increase in frequency will cause more of the current to flow through the shunt, thereby decreasing the current in the meter coil. This reduced current, however, will be more effective in producing torque on account of its higher frequency. By properly adjusting the shunt, the two effects are made to neutralize each other so that the registration is not affected by considerable variation in frequency.

The principles of the voltmeter are identical with those of the ammeter, with the difference that there is connected in series with coil B a high resistance, non-inductive coil. This particular form of induction meter is no longer on the market, although it is still used.

78. Series Transformer Principle.—A most ingenious method of producing a rotating magnetic field was invented by Mr. Frank Conrad, and is used by the Westinghouse Electric and Manufacturing Company in their induction instruments.

In so far as principles of construction are concerned, a series transformer contains two independent windings the same as a shunt transformer. The difference between the two kinds of transformers lies mainly in their use. The primary winding of a series transformer is connected in series with the line, while the other has its primary shunted across the line. Fig. 49 is a diagram of a series transformer winding as applied to an induction

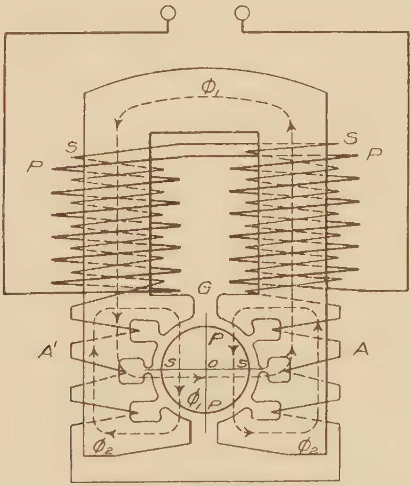


FIG. 49.

meter. It is evident from the diagram that there are two distinct windings, P, the primary winding carries the line current, and S, the secondary, is short circuited. The circle represents the cylindrical meter movement. The dotted lines marked Φ represent the magnetic fluxes set up by currents in the separate coils.

The current to be metered develops magnetism in the core; this magnetism in turn induces a current in the secondary winding. This induced or secondary current, according to the principles of induction is nearly 180 degrees out of phase with the primary current. If it were not for the losses, and magnetizing current it would be exactly 180 degrees out of phase. If

the magnetizing current were negligible, the ampere turns of the primary current would exactly equal the ampere turns of the secondary, or algebraically

$$N_1I_1 = N_2I_2.$$

That is, the magnetizing effect of primary current would just balance the demagnetizing effect of secondary current.

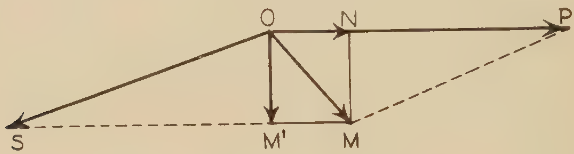


Fig. 50.

A vector diagram for primary and secondary ampere turns is shown in Fig. 50. *OP* represents the product of primary current and primary turns, and *OS* represents the product of secondary current and turns. The magnetizing force, which sets up the flux in the core common to the two coils, is the resultant of the primary and secondary ampere turns, and will, therefore, be represented by *OM*. *OM* may be considered as the resultant of two components, *ON* in phase and *OM'* in quadrature with *OP*. *ON* will represent the ampere turns necessary to supply energy for the losses, and *OM'* will represent the true magnetizing ampere turns. This magnetizing component is primarily responsible for the fact that the phase difference between the primary and secondary currents is not exactly 180 degrees. A vector diagram of the electric and magnetic quantities is shown in Fig. 51.

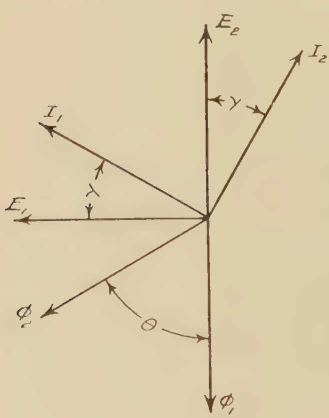


Fig. 51.

The combined primary and secondary ampere turns produce a flux ϕ_1 which in time is nearly one-quarter of a period ahead of the flux ϕ_2 due to the auxiliary secondary winding. In space the two fluxes are at right angles to each other as shown in Fig.

49. These are plainly the conditions necessary for producing a rotating magnetic field.

Within the rotating magnetic field is placed the movable element, which consists of a light aluminum cylinder mounted on

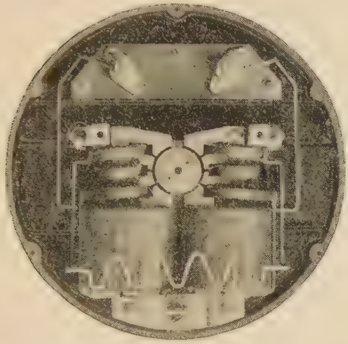


Fig. 52a.

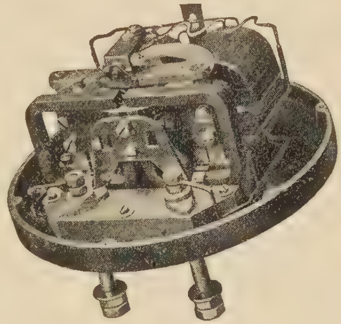


Fig. 52b.

a shaft. The shaft is supported between jewels, as in other types of meters. The rotating field induces eddy currents in the cup-shaped movable element and thus creates a torque which is

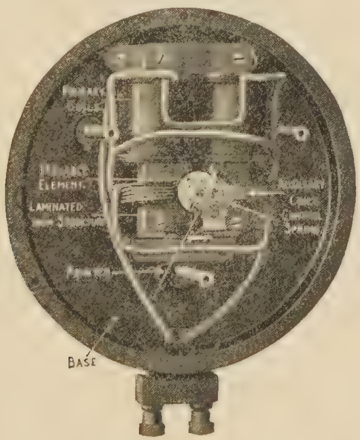


Fig. 52c.

balanced by a coiled spring exactly as in other types of meters. The structural features of a portable ammeter are shown in Figs. 52a and 52b, and of a switch board voltmeter in Fig. 52c. The

principles of construction of ammeters and voltmeters are exactly alike, with the exception that in the voltmeter the primary coil is wound with fine instead of coarse wire, and an external non-inductive series resistor of zero temperature coefficient is used.

79. Relation Between Current and Torque.—There are so many varying quantities involved that no exact expression for the torque or deflecting force with reference to the current can be given. A general relation can be determined as follows:

The flux in the iron core is proportional to the current to be measured and the induced currents in the rotating disk are proportional to the flux. The deflecting or rotating force is proportional to the product of flux and eddy currents, hence, the deflecting force is proportional to the square of the flux, which in turn is roughly proportional to the square of the current in the main coil. The motion is usually opposed by spiral springs and thus the angle of deflection is roughly proportional to the square of the current.

According to the principles of Article 73, if $H_1 = \phi_1$, $H_2 = \phi_2$, $OF_1 = \Phi_1$ and $OF = \Phi_2$ the two fields rotating in opposite directions are given by

$$\Phi_1^2 = \phi_1^2 + \phi_2^2 - 2\phi_1\phi_2 \sin\theta$$

$$\text{and } \Phi_2^2 = \phi_1^2 + \phi_2^2 + 2\phi_1\phi_2 \sin\theta$$

θ is the time phase difference between ϕ_1 and ϕ_2 as represented in Fig. 51.

Since the cylinder turns only until the driving torque equals the counter torque of spring, the speed of drum is zero. The actuating torque is due to the cutting of the cylinder by the two fluxes Φ_1 and Φ_2 in opposite directions. The deflection will be in the direction of the greater torque.

The eddy currents in the drum due to Φ_1 and Φ_2 are proportional to Φ_1 and Φ_2 and to the speed of these which is $2\pi f$. If the eddy currents lag γ degrees behind the induced voltage the torque due to Φ_1 is

$$T_1 = fK_1 \Phi_1^2 \cos\gamma$$

and that due to Φ_2 is likewise given by

$$T_2 = fK_1\Phi_2^2 \cos\gamma$$

The driving torque is equal to

$$T_1 - T_2 = fK_1 \cos \gamma \left(\Phi_1^2 - \Phi_2^2 \right)$$
$$= 4fK_1 \phi_1 \phi_2 \sin \theta \cos \gamma$$

but $\phi_1 = \frac{K_2 I_1}{f}$, and $\phi_2 = K_3 I_1$, where I_1 is the primary current.

$$\text{Then } T = T_1 - T_2 = \frac{4fK_1 K_2 K_3 I_1^2}{f} \sin \theta \cos \gamma$$
$$= 4K_1 K_2 K_3 I_1^2 \sin \theta \cos \gamma$$

K_1 depends mainly upon the impedance of the drum and varies inversely with it. In place of K_1 we may write $K_1 = \frac{1}{Z}$,

Where Z is the impedance of cylinder; then

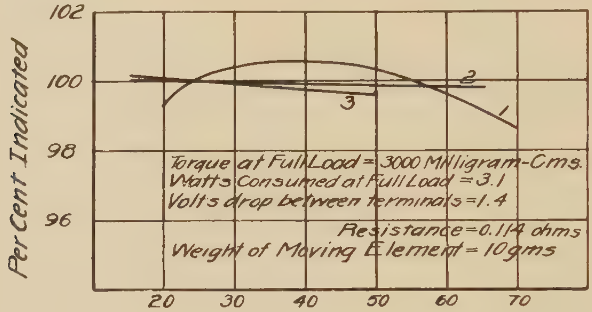
$$T = \frac{K_0 I_1^2}{Z} \sin \theta \cos \gamma.$$

80. Influence of Frequency.—In the expression given above for torque, Z, θ, and γ are quantities varying with frequency, and in order that the meter may be independent of frequency Z must vary as $\sin \theta \cos \gamma$. This can only be approximated, although the maximum error due to changes of frequency from 25 to 60 cycles is not over 1/2 per cent.

81. Influence of Temperature.—Variation in temperature will mainly affect the resistance of the movable cylinder. In order to correct for this variation, the secondary coil is arranged to have a temperature coefficient of resistance such as to exactly cancel the effect of the variations of resistance in the cylinder. To do this another series transformed principle is used. If in a series transformer the primary current be kept constant, the secondary current will remain nearly constant while the secondary resistance is varied over a considerable range. Thus, any increase in the secondary resistance causes a proportional increase in the flux of the core. This is made possible by working the core at a low flux density.

The Westinghouse ammeter described has the secondary circuit wound partly with copper and partly with a wire of low temperature coefficient, the resulting temperature coefficient of the secondary circuit being such as to increase the flux in the

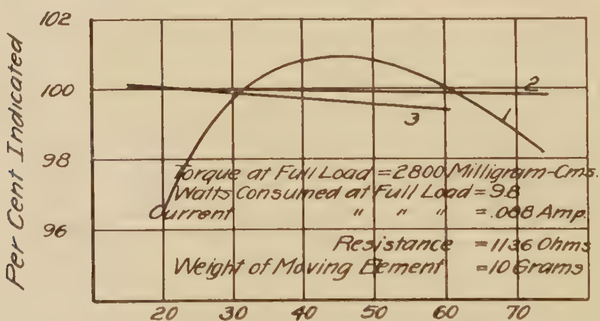
iron when the temperature rises, thus compensating for rise of temperature of aluminum cylinder. Figs. 53 and 53a show the performance curves of ammeters and voltmeters respectively.



Cycles, for curve No. 1 which shows the effect of changes in frequency. Minutes in circuit for curve No. 2 which shows the effect of self-heating at 2/3 load. Air temperature in °C. for curve No. 3 which shows the effect of variations in room temperature.

Fig. 53.

82. Scale.—A direct reading scale of the induction type of instruments will have the same disadvantages as those whose deflection is proportional to the square of the quantity to be



Cycles, for Curve No. 1 which shows the effect of changes in frequency. Minutes in circuit for curve No. 2 which shows the effect of self-heating at 2/3 load. Air temperature in °C. for curve No. 3 which shows the effect of variations in room temperature.

Fig. 53a.

measured. In practice, this disadvantage is overcome in two ways. One method makes use of a cam-shaped disk, which is mounted so that less and less of it lies between the poles of the

iron core as it rotates. A proper shaping of the disk permits the construction of a practically uniform scale for about 300 degrees of arc. A second method, made use of by Siemens and Halske, consists in using an auxiliary weight so mounted that it reinforces the tendency of the moving element to rotate at the zero end of the scale, while at the upper end of the scale, it is vertically below the spindle. The instruments manufactured by the above-mentioned firm have nearly a uniform scale extending over an arc of 90 degrees.

CHAPTER VIII

ELECTRODYNAMIC AMMETERS AND VOLTMETERS

83. Introduction.—The principle upon which the operation of these instruments depends is, as its name implies, the mutual attraction and repulsion between adjacent circuits carrying electric currents. The principle that currents flowing in the same direction in parallel wires attract, and, when flowing in opposite directions repel, is the fundamental principle of the instruments of this class. The repulsion and attraction is due to the interaction of magnetic fields produced by the currents, but as instruments possess no iron core, the interaction is called electrodynamic instead of electromagnetic.

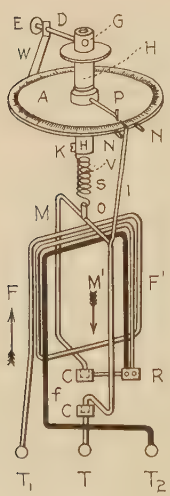


Fig. 54.

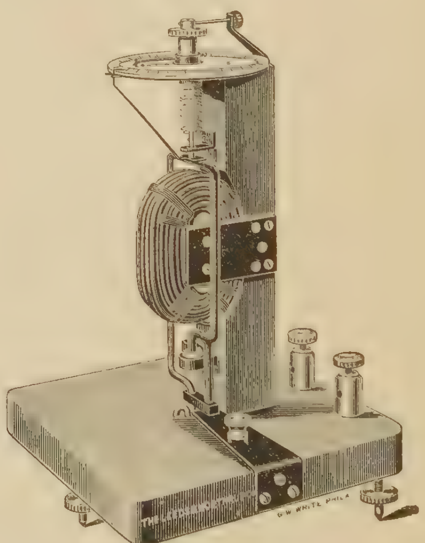


Fig. 55.

84. Electrodynamometer Type.—The essential features of an instrument of this type, which for a long time was extensively used for measuring alternating currents, is shown in Fig. 54. The completed instrument is shown in Fig. 55. As shown in Fig. 54, the instrument contains two coils, one fixed FF' and

one movable MM'. The movable coil is placed outside of the fixed coil and is supported by a fiber which offers very slight resistance to motion of coil. The lower ends of coil dip into mercury cups which are connected to binding posts.

The controlling force is furnished by a spiral spring, one end of which is attached to coil MM' and the other to the torsion head H. The index pin P is also rigidly connected to the torsion head, which passes through the fixed graduated plate A, and may be turned by the milled head at the top of the instrument.

85. Operation of Electrodynamometer Ammeter.—The circuit within which the current is to be measured is connected in series with T and T_1, or T and T_2, depending upon the magnitude of the current. Assuming that direct current is to be measured and that the positive terminal is connected to T_1, it will be noticed that the current flows up the side of the stationary coil marked F and down the side marked F'. After passing through the junction block R and the connecting link RC the current enters the movable coil at C and flows up through the side M and down through the side marked M', finally leaving the instrument through binding post T.

Keeping in mind the principle of parallel circuits stated in Chapter I, we see that the current flows in the same direction in sides F, M', and F', M. Since the side coil FF' is fixed and cannot move, the coil MM' will be deflected in such a direction that the side M approaches F, and M' approaches F'. The motion of the coil is stopped when the pointer I strikes the pin N. By turning the torsion head G, the pointer I can be brought back to its original or zero position. The force tending to deflect the coil is then measured by the angle through which the torsion head has been turned, for within the limits of elasticity of the helical spring the force causing a distortion or twist is strictly proportional to the angle through which it has been twisted. In order to measure the current, however, it is necessary to get an expression for the force in terms of the current in the coils. Both theory and experiment show that the force is proportional to the product of the current in M and F. In the case considered the same current flows through both coils, hence, the force of attraction must be proportional to the square of the current. Now, since the force is proportional to the angle of deflection, and likewise to the square of the current, it is evident that the square of the current must be proportional to the angle of deflec-

tion. Letting I equal current strength, K^2 a proportionality factor, and θ the angle of deflection, we may write the foregoing relation as follows:

$$I^2 = K^2\theta.$$

$$\text{Whence } I = K\sqrt{\theta}.$$

This is the fundamental equation for instruments of this type. It will be noticed that the current is proportional to the square root of the angle of deflection, and, hence, the scale of such an instrument, if it were to be direct reading, could not be uniform. In practice, the scale is graduated in degrees, and the current is determined from calibration curves or computed in accordance with the above formula after the constant K has been determined.

It has been assumed that the current to be measured was direct, or continuous. One of the advantages of the electrodynamometer ammeter is that it can also be used on alternating-current circuits. That th edeflection is in the same direction on alternating current as on direct current, will be evident on noticing that when the current reverses in the stationary coil it likewise reverses in the movable coil. The current will always be in the same direction in F as in M, and in F' as in M', and consequently, the deflection on alternating current is the same as on direct current. Also, both by theory and experiment, it has been shown that the deflecting force on alternating current is proportional to the square of the effective current. Hence the instrument gives true effective values of alternating currents.

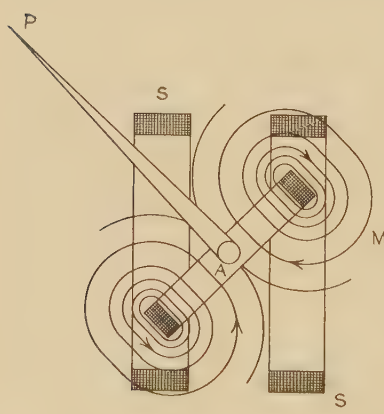


Fig. 56.

86. Voltmeters.—The electrodynamic principle can be used to measure difference of pressure as well as current. The arrangement of the coils for the dynamometer type voltmeter is shown in Fig. 56. SS are the fixed coils, which are connected in series, and A is the movable coil to which the pointer P is rigidly

attached. They are similar in mechanical construction to the permanent magnet, moving coil type of instrument. The electrical features differ in that the magnetic field in this type of instrument is due to the current flowing in the coils, which is proportional to the voltage to be measured. In the permanent magnet type of instrument the magnetic field is due to the permanent magnets, and the current in the moving coil only is proportional to the difference of pressure.

87. Effect of Inductance Upon Reading of Electrodynamometer Voltmeter.—In well designed voltmeters of this type the inductance is reduced to a minimum, and the inductance errors are practically negligible; nevertheless, the influence of this quantity may sometimes be appreciable. The relation between a direct electromotive force and an alternating electromotive force causing the same deflection may be determined as follows:

Let E = D. C. e.m.f. causing a given deflection
E' = A. C. harmonic e.m.f. causing same deflection
L = Inductance of coils
R = Resistance of coils

then the deflecting force on direct current is proportional to $\frac{E^2}{R^2}$, that is, to the square of current in coils.

Since the deflection is assumed to be the same when A. C. e.m.f. is measured, the effective current must be equal to the direct current. The effective current

$$I = \frac{E'}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\text{and} \quad \frac{E}{R} = \frac{E'}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\text{whence,} \quad E' = \frac{E\sqrt{R^2 + \omega^2 L^2}}{R}$$

$$= E\sqrt{1 + \frac{\omega^2 L^2}{R^2}}$$

When R is large in comparison with $L\omega$, the second term under the radical sign has little effect; when this is not the case the difference between E and E' will be appreciable.

88. Construction.—The dynamometer type of voltmeter contains no iron, and, as the three coils are all connected in series, it

is well suited for alternating-current measurements. A phantom view of the essential features of a Weston dynamometer voltmeter is shown in Fig. 57. The Roller Smith Company apply

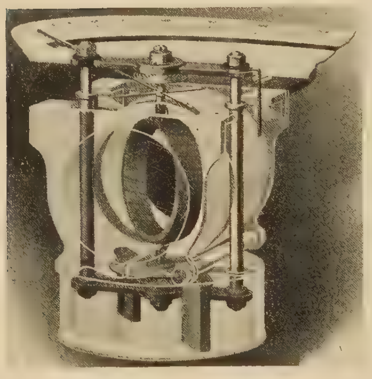


Fig. 57.

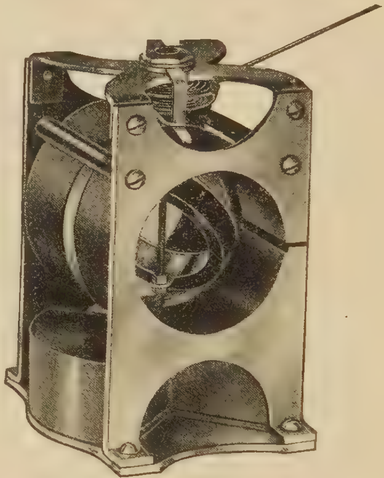


Fig. 58.

the same principle in their dynamometer type of voltmeters. In this case, however, the coil is mounted with its plane inclined at an angle to the arbor or shaft which supports it, Fig. 58. Such a construction makes it possible to fasten the coil to the shaft by small projecting lugs which are integral with and project from the sides of the coil frame. By clamping the lugs between appropriate nuts, the coil is rigidly fastened to the arbor. The inclined coil voltmeter of the General Electric Company is also similar in construction. In these instruments both the fixed and movable coils are inclined with reference to the shaft carrying the movable coil, Fig. 59.

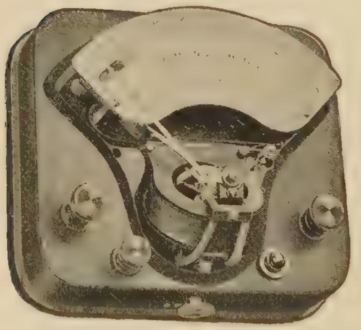


Fig. 59.

The principles of operation are the same in all cases. In every case the force causing a deflection varies as the product of currents in fixed and movable coils. When these currents

are the same, the deflecting force depends upon the square of the actuating current. On account of the change in relative position of coils when in use, exact preportionality does not exist between the deflecting force and the square of current. Since in the instruments of the dynamometer type the controlling force is due to a spiral or helical spring whose counterforce varies directly with the deflection, such instruments do not have uniform scales when direct reading. The graduations are crowded together at the beginning and end of scale, especially at the beginning. To overcome this objection the Keystone Electrical Instrument Company has designed a controlling system which is

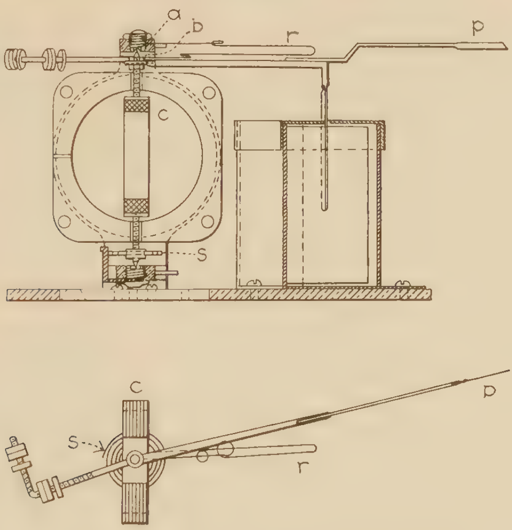


FIG. 60.

worthy of mention. The controlling system consists of two springs, one spiral and one in the form of a very elongated *U*. These are shown in Fig. 60. One end of this auxiliary spring is attached to the supporting frame while the other end is fastened to the movable coil. When the pointer is at zero, the tendency of the spring is to cause a deflection against the reaction of spiral spring. When the pointer has been deflected a predetermined distance, the direction of pressure of the long looped spring reverses. Thus for the first part of the deflection the pressure of the looped spring is against the spiral spring, and for the last part it acts with the spiral spring.

This effect will perhaps be more clearly understood if stated algebraically. The reacting force of a spring is proportional to the distance through which it has been deflected. Hence, if δ represents the angle through which spiral spring has been deflected the reaction may be expressed by

$$F_s = k\delta.$$

Suppose that when the deflection is δ_o the pressure of the looped spring is zero, then when the deflection is δ, the pressure due to the looped spring will be proportional to $\delta - \delta_o$

or

$$f = k(\delta - \delta_o).$$

When the pointer is at zero the force due to the spiral spring must be just equal to the force due to looped spring. Call this initial force F_o, then at any deflection δ, the controlling force

$$F = F_s + f + F_o$$

$$= K\delta + k(\delta - \delta_o) + F_o.$$

This expression shows that when $\delta = 0$

$$F = F_o - k\delta_o.$$

When $\delta = \delta_o$, $F = K\delta + F_o$, which is greater by F_o than the normal reacting force would be were the spiral spring alone used.

When $\delta > d_o$, the two springs act together. Thus, during the first part of the deflection the controlling force is due to the differential action of the springs, and during the latter part it is due to the cumulative effect.

89. Ampere Balance.—A standard instrument for measuring current is known as Kelvin's balance, Lord Kelvin being the inventor and designer of the instrument. The operation of the instrument is based on the electrodynamic attraction between stationary and movable coils, in much the same way as the electrodynamometer discussed above. In Fig. 61 are shown the essential features of the instrument. As shown in the figure, the instrument consists of four fixed and two movable coils. The fixed coils are designated by A, A', A'', A''', and the movable coils by B, B'.

The coils are all connected in series by connections as shown in the diagram. The winding of the lower coil A, is reversed with

reference to the winding of the upper coil. The current thus flows in one direction in one coil and in the opposite direction in the other coil of the couple. In the same way the current flows in opposite directions in the stationary coils. Assuming that the current flows counter-clockwise in coil A, it will flow in a clockwise direction in coil A'. Thus, if coil A attracts coil B, coil A' will repel coil B'. On the other side the winding of B' being reversed, it will be repelled by A'' and attracted by A'''. As a result of this attraction and repulsion, the coils BB' will swing around the suspension C, which also serves for conducting the current into the movable coils.

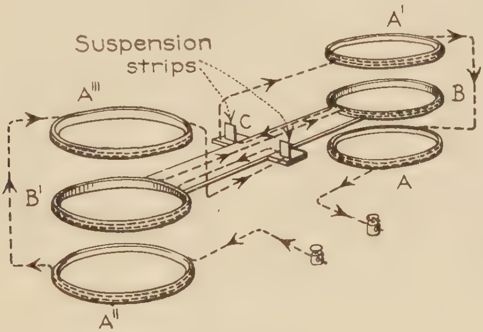


FIG. 61.

The controlling force in this case is gravity acting on the rider which slides along a graduated beam fastened to BB'. When no current is flowing through the instrument, a weight is placed in a pan which balances the movable coils and the rider at its extreme left position, which corresponds to the zero position of the scale. When a current is flowing through the coils, the rider is moved to the right along the beam until the coils are again balanced. The value of the current is then indicated by the position of the rider. The balance arm is supported by two trunions, each hung by an elastic ligament of fine wire, through which the current passes into and out of the movable coil at the end of balance arm. The instrument as manufactured is shown in Fig. 62.

With ampere balances four pairs of weights (sliding and counterpoise) are supplied with each instrument. The carriage and its counterpoise constitute the first pair. These weights are adjusted in the ratios 1, 4, 16, 64, so that each pair gives a whole

number of amperes, or half amperes, or quarter amperes, or some decimal subdivisions or multiples of these magnitudes on the upper or inspectional scale.

For the adjustment of the zero, a small metal flag is provided. The flag is operated by a fork, having a handle outside the case. To adjust the zero reading, the sliding weight is placed with its pointer at the zero end of the scale, and the flag is turned to one side or the other, until it is found that with no current passing the balance rests in its zero position.

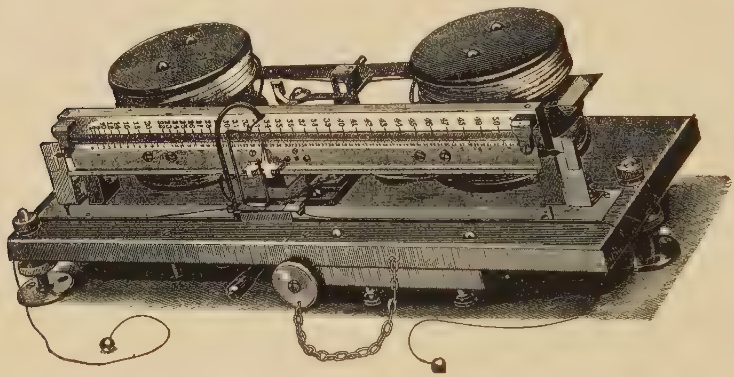


Fig. 62.

When a current is passed through the instrument, the balance arm is displaced, and to measure the current the rider is slipped along the trough until the balance arm is again brought to its zero position. The strength of current is then indicated on the upper fixed scale by the pointer of the sliding weight. For greater accuracy the reading of the lower scale must be taken. Each number on the upper, or as it is called inspectional, scale is twice the square root of the corresponding number on the fine scale of equal divisions. Thus if the reading on the lower scale is 292, that on the fixed scale will be $34.18 = 2 \times \sqrt{292}$. A table of double square roots is furnished with the instrument. The reading, multiplied by the constant of weight used, give the current.

EXAMPLE

A centiampere balance was used to calibrate a milliammeter. The milliammeter reading was 668 milliamperes, and balance reading 326.8 on lower scale. How much is the ammeter in error if the constant for weight used is 2?

Solution.—

$$2\sqrt{326.8}=36.14$$

$$I=2\times 36.14 \text{ centiamperes}=722.8 \text{ milliamperes.}$$

$$\text{Error}=722.8-668=54.8 \text{ milliamperes.}$$

90. Uses of Kelvin Balance as a Voltmeter.—In order that the Kelvin balance may be used as a voltmeter it is only necessary to increase its resistance. When so used, the resistance of the operating coils is about 50 ohms, and special coils are provided to be connected in series. The resistance of these coils ranges from 400 to 2000 ohms, and the maximum voltage that can thus be measured is 500 volts. It is evident that the Kelvin balance operates upon the electrodynamometer principle, yet it is usually classed separately because the controlling force is gravity; the planes of the coils are horizontal instead of vertical; and the movable coils do not rotate around a central axis, but revolve about an axis midway between them.

The relation between the current strength and the force of attraction is the same as that of the electrodynamometer already discussed, *i.e.*,

$$I^2=K^2F$$

but when the coil is balanced the force is proportional to the distance of the rider from the extreme left of the scale, hence

$$I^2=K^2l$$

or $I=K\sqrt{l}$, which is of the same form as the equation for the electrodynamometer.

From this equation we see that the scale which reads in amperes cannot be uniform. On the other hand, in order to obtain the current from a reading on the uniform scale we must extract the square root of the reading and multiply this square root by the constant of the instrument.

The commercial instruments are made in seven sizes ranging in capacity from 0.01 ampere to 2500 amperes. Detailed instruction for using the balance is always furnished with the instrument.

91. Westinghouse Dynamometer Ammeter and Voltmeter.—Fig. 63 shows how the Westinghouse Electric and Manufacturing Co. has adapted the Kelvin balance principle to one type of instrument. The figure shows the general arrangement of the measuring elements, the letters referring to the following parts:

A, A', A^2, A^3 fixed coils
C', C^2 movable coils
B non-inductive resistance
D controlling spring
E torsional head
F pointer attached to movable element
G pointer attached to torsion head.

The four fixed coils and two movable coils are all connected in series, and in series with part or all of the resistance B, depending upon whether small or large electrical quantities are to be measured. Precisely as in the Kelvin balance the meter depends for

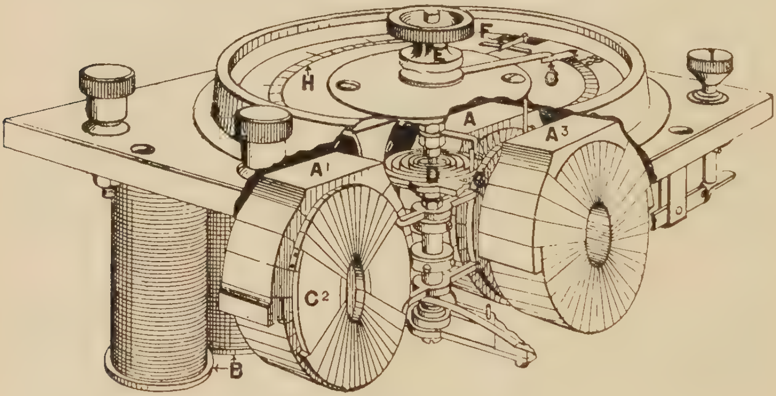


FIG. 63.

its operation upon the electrodynamic action between the fixed and movable coils. The controlling force is, however, a spiral spring instead of gravity. The influence of gravity is eliminated by mounting the coils with their planes vertical instead of horizontal. There is considerable similarity between the Westinghouse dynamometer type of instrument and the electrodynamometer. In both cases the planes of the coils are vertical and the indications of the instruments are obtained in the same way. When a current is sent through either instrument, the movable coil is deflected and, by means of the torsion head, the movable element is brought back to its zero position. The deflection is indicated by the angle between the two pointers. The same law, viz.,

$$I=K\sqrt{\theta}\ \text{holds.}$$

92. Influence of Earth's Magnetic Field.—Since the earth is surrounded by a magnetic field which has a definite direction and value, there is bound to be a reaction between this field and a coil carrying a current. In some of the types of instruments so far discussed, the influence of the earth's magnetic field must be considered, when measuring direct currents, if accurate results are expected.

In instruments of the electromagnetic type, the strength of the operating field is usually so great in comparison with the strength of the earth's field that this influence is practically negligible.

In some of the dynamometer type of instruments this influence may be appreciable. This is true with reference to those instruments having only one stationary coil. Instruments employing the Kelvin balance principle are astatic, that is, they are not influenced by the earth's field. This is due to the fact that the movable element of the Kelvin balance type of instrument contains two coils wound in opposite directions. The effect of the earth's field upon one coil is thus neutralized by its influence upon the other coil.

To prevent undue influence of the earth's magnetic field upon the Siemen's electrodynamometer, the plane of the movable coil should be placed at right angles to the magnetic meridian. The direction of the magnetic meridian is indicated by a compass needle when not influenced by iron or adjacent magnets.

93. Damping.—For efficient service the pointer of a meter should move rapidly to its proper position on the scale, and there come to rest without vibrating to and fro. To secure this, some system of damping must be employed. In practice there are two methods used, one electromagnetic, and the other mechanical.

In the moving coil permanent magnet type of instrument damping is of the first kind. The movable coil is wound upon a frame of conducting material which forms a short circuited path. As the coil moves in the magnetic field, currents are induced in the frame and retard its motion. Others again have a disk of aluminum attached to the shaft. This moves between opposite poles of permanent magnets as shown in Fig. 52b.

The mechanical method of damping is used in the commercial instruments of the dynamometer type. One of the most serious objections to the Siemen's form of current meter is the lack of damping device, which necessitates considerable time and skill

in making readings. The other forms of dynamometer instruments almost invariably make use of some form of air damping device. In the Weston dynamometer type instruments, which may be considered typical, damping is secured by two light symmetrical vanes enclosed in chambers made as nearly air tight as possible.

With the exception of the Siemen's dynamometer and Kelvin balance forms, instruments of the types discussed are also made for switchboard use.

94. Advantages.—Among the main advantages of the dynamometer type of instrument are its sensitiveness, accuracy, and adaptability to both direct and alternating-current measurements. It may be calibrated on direct current and used on alternating current.

95. Disadvantages.—The Siemen's dynamometer and Kelvin balance are not direct reading; they are not "dead beat," and hence require considerable time and skill in making readings. The necessity for accurate leveling is also a disadvantage.

CHAPTER IX

MISCELLANEOUS AMMETERS AND VOLTMETERS

96. Electrostatic Voltmeter.—The measuring instruments so far discussed require an electric current for their operation. Electrostatic instruments utilize the forces of attraction or repulsion between two electric charges. The gold leaf electroscope is the simplest form of instrument of this type.

When two plates are insulated and placed near each other, a force of attraction will exist between them if oppositely charged. If one of these plates is movable, but its motion is counteracted by some controlling force, the deviation of the movable plate from its normal position will be a measure of the force of attraction. Since the capacities of the two plates are practically

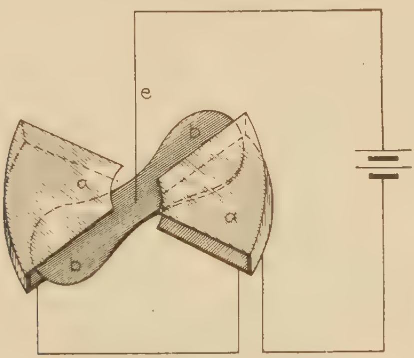


FIG. 64.

constant, the deviation, or force required to prevent deviation, will be a measure of the difference of electrical pressure between the plates. Instruments which make use of this principle of attraction of charges are ordinarily called electrometers. When, however, they are provided with scales graduated in volts they are called electrostatic voltmeters. The adaptation of this principle to commercial instruments is due to Lord Kelvin.

The essential elements of an electrostatic voltmeter are shown in Fig. 64. The stationary element consists of two parts or

quadrants *aa*. The movable element *bb* is a very light figure 8-shaped aluminum vane. The vane is suspended by a fine wire whose elasticity supplies the controlling force. The instrument is practically an air condenser with movable plates. When the connections are made as indicated, the quadrants and vane are oppositely charged and attract each other. If the vane remained stationary, the force of attraction would be proportional to the square of the potential difference between quadrants and vanes. Algebraically

$$F = KE^2$$

This relation is not mathematically exact for the reason that a change in the relative position of vane and quadrants slightly changes the capacity. To measure low voltages the capacity of the instrument must be relatively large. To secure this large

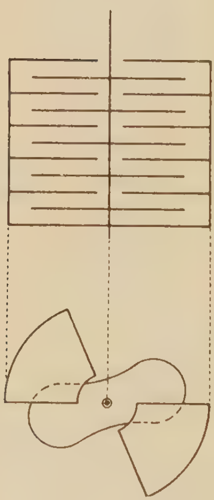


Fig. 65.

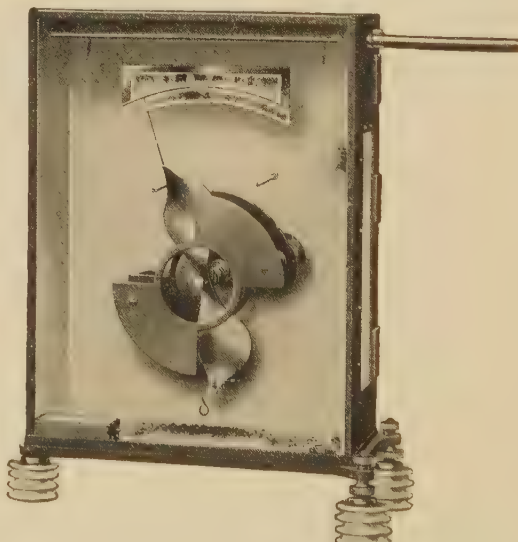


Fig. 66.

capacity, Lord Kelvin mounted several quadrants above each other and between them suspended the same number of vanes. The principle of construction will be readily understood from Fig. 65. Such an instrument may be used for measuring voltages down to 50 volts. The suspending wire supplies the controlling force. For voltages ranging from 400 to 100,000 volts, only one

set of quadrants and one vane are used, Fig. 66. The vane is mounted to swing in a vertical plane and the controlling force is due to the action of gravity upon weights which determine its sensibility. In neither case is the scale uniform. A multicellular form voltmeter for low voltages is shown in Fig. 66a.

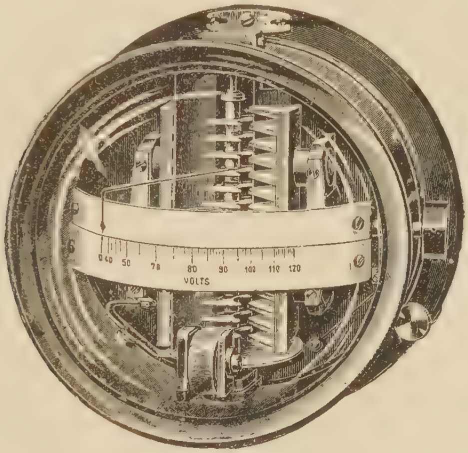


Fig. 66a.

97. Westinghouse Electrostatic Voltmeter.—The principle of mutual attraction between two electrostatic charges of opposite kind has been adapted by Mr. S. M. Kintner of the Westinghouse Electric and Manufacturing Co., in a novel manner. The essential features of this voltmeter are shown in Fig. 67. The measuring elements consist of a series of fixed and movable condensers. The movable element to which a pointer is attached is suitably pivoted and provided with spring control.

As shown in the diagrammatic sketch, the movable part M_1, M_2, consists of two hollow cylinders fixed to a pivoted arm. The curved plates B_1 and B_2 are metallically connected to the inner condenser plates of condensers C_1 and C_2. The operating elements of the meter are immersed in a high grade of insulating oil contained in a metal lined wooden case provided with an insulating cover.

98. Operation.—When the terminals T_1 and T_2 are connected to the circuit whose voltage it is desired to measure, the condensers C_1 and C_2 become oppositely charged. These charges

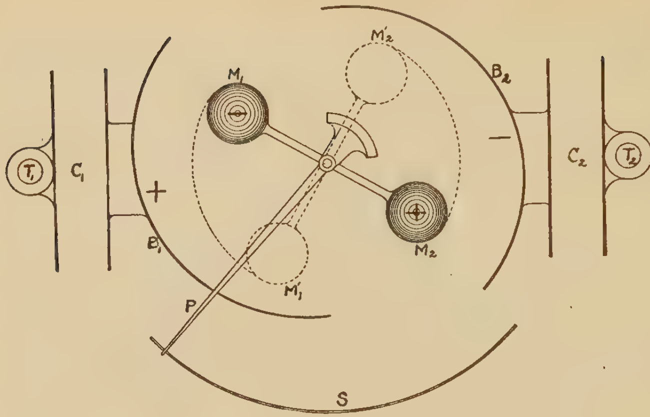


Fig. 67.

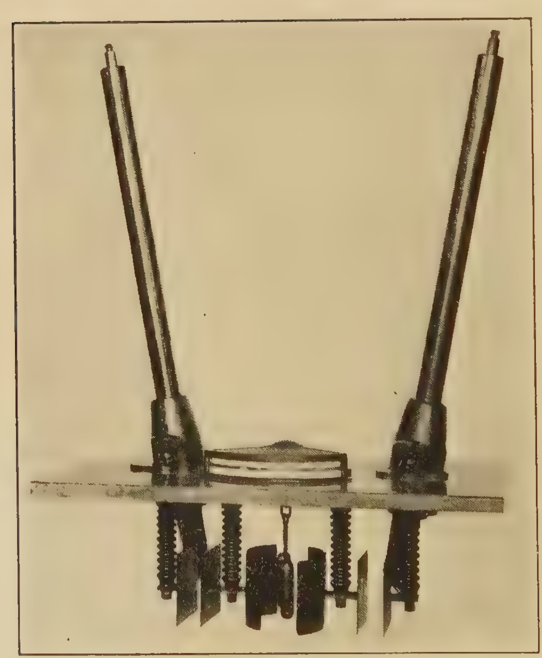


Fig. 68.

induce other charges of opposite polarity on cylinders M_1 and M_2. The consequent attraction between the charges on B_1, B_2 and M_1, M_2 causes a deflection of the movable elements. This motion is made possible by the shape and relative position of plates B_1 and B_2 with reference to the axis of the cylinders. As the cylinders revolve they approach the curved plates B_1 and B_2. In this, as in the Kelvin electrostatic voltmeter, the torque causing a deflection is proportional to the square of the applied pressure. The form and relative position of operating parts is shown in Fig. 68.

99. Insulation.—To secure proper insulation for measuring very high voltages is not only very important but extremely difficult. In the electrometer, or Kelvin form of instrument, the insulating properties of air are mainly relied upon. The high insulating properties of oil, together with its relatively high inductivity, makes its use advantageous in many respects. The most important advantages are:

1. Possibility of more compact construction, as the oil permits the placing of the operating elements nearer together.

2. The force of attraction between stationary and movable elements is greatly increased, both on account of the smaller distance between them and on account of the high inductivity of the oil.

3. The buoyant effect of oil greatly diminishes the pressure and friction of bearings.

100. Damping.—Both the Kelvin multicellular and Westinghouse electrostatic voltmeter use liquid damping. The axis of the Kelvin instrument projects through the bottom of the casing and is provided with a suitable vane. The cup in which the vane swings is narrow and deep and only about one third full of liquid.

In the Westinghouse instrument, the resistance of the insulating oil upon the movable element produces efficient damping.

An electrostatic voltmeter of the electrometer type, manufactured by Hartmann and Braun, employs electromagnetic damping. The moving vane moves between the poles of an electromagnet and as it moves the eddy currents induced effectively damp the movement of the pointer.

101. Advantages.—Among the most important advantages of the electrostatic instruments may be mentioned the following:

They do not consume any electrical current; may be used on either alternating, or direct-current circuits; are entirely unaffected by temperature, external magnetic fields, power-factor,

or frequency. In addition to these they may be used on very high potential circuits.

The Westinghouse voltmeter is adjustable so that it may be used for measuring widely different voltages.

102. Hot-wire Instruments.—When a current of electricity passes through a wire whose resistance is R, the energy transformed into heat is I^2R joules per second, when I, the current in the circuit, is given in amperes. When a stationary condition in the temperature has been reached, the energy converted into heat must be equal to that radiated, and this quantity is proportional to the change in temperature. Hence, it follows that the square of the current, which is proportional to the heat developed, is proportional to the expansion of the wire through which the current flows. It must be remembered, however, that the resistance R, is not independent of the temperature. The wire of most instruments of this type is made of platinum silver whose temperature coefficient is about 0.00024. If R_o is the resistance of wire at room temperature t_o, its resistance at temperature $t_1{}^oC$ is given by

$$R_t = R_o[1 + 0.00024\ (t_1 - t_o)]$$

Thus, a 100 ohm coil will undergo a change of 0.024 ohms per degree Centigrade. This change in resistance modifies, to some extent, the proportionality between square of current and expansion. From a practical point of view this is of no great importance for the scale can be determined by calibration.

103. Hot-wire Voltmeter.—The principle mentioned above was first made use of in an instrument designed by Major Cardew. The wire in the Cardew voltmeter was made of platinum silver and was of such a length that it could be connected across a 110-volt circuit without any series resistance. The wire ran twice from end to end of a long brass and iron tube, passing over insulated rollers at each end. One extremity was fixed while the other, after passing over a pulley, was attached to a spring which kept it taut. The pointer was attached to the pulley which was rotated by the expansion and contraction of the wire. The tube consisted of brass and iron so proportional that its coefficient of expansion was the same as that of the wire. Hence, so long as the temperature of both was the same, the tension of the wire was constant and the readings were independent of external temperature variations.

The arrangement of the wire was such that the instrument was bulky and very inconvenient to handle. It is no longer in use.

In the more modern instruments of the hot-wire type, the working length of wire is from 6 to 8 in. In voltmeters this wire is quite fine, and series resistances are provided. Fig. 69 shows the essential features of a Hartmann and Braun voltmeter. The terminals of the circuit, whose difference of potential is to be measured, are connected to A and B. The resulting current heats the platinum-silver wire causing it to expand. As the tension of AB is lessened, the point C is pulled downward by the tension

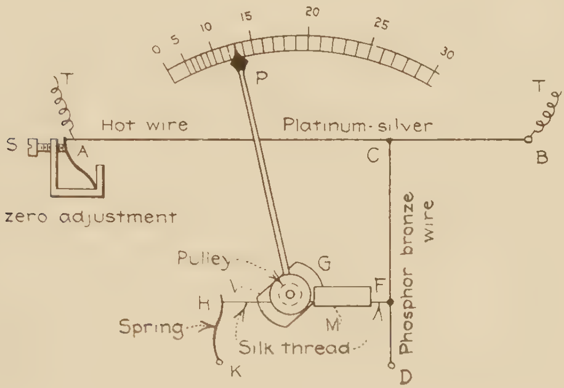


FIG. 69.

of the spring HK. The silk thread HF being wrapped around the pulley G rotates the pointer as the end H of the spring HK moves to the left. By means of the mechanism CD and FH, the pointer P is made to move many times the sag of AB. The tension of the current wire AB is adjusted by means of the screw S.

When an instrument of this type has been in use for some time, the pointer seldom returns to its zero position, but by means of the screw S the zero adjustment of the pointer can be readily made.

The instrument is made "dead-beat" by means of the vane V rotating between the poles of the permanent magnet M. The vane V is mounted upon the shaft with the pulley G and pointer P. As these rotate, the vane cuts across the magnetic field of the

permanent magnet, inducing eddy currents which effectually damp the vibrations of the needle.

The mechanism, as shown in the diagram, is mounted upon a suitable base plate not shown in the figure. The base plate is divided near the point C, one portion consisting of iron and the other of brass. The relative lengths of the iron and brass parts are so designed that the expansion of the base plate is the same as that of the wire itself. Even such an arrangement does not give correct instantaneous readings. This is due to the fact that the wire reaches its maximum temperature almost instantly, whereas the base plate requires considerable time to do so.

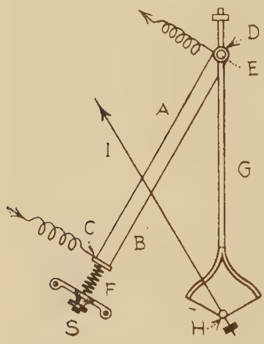


Fig. 70.

Another arrangement of the working wire is found in the Roller and Smith hot-wire meter, the essential features of which are shown in the diagram of Fig. 70.

The current carrying wire in this type of instrument is looped around a pulley, and the two ends are fastened to the same plate C, as shown in the figure. One end of the wire is electrically connected to the plate, while the other end is insulated from it. The wire is kept in tension by the spring which may be adjusted by the screw S.

In voltmeters a non-inductive resistance of very low temperature coefficient is connected in series with the branch A.

The pulley D is rigidly attached to a shaft E to which is also attached the arm G. This arm is divided at one end and counter-balanced at the other. To the two branches of one end of the arm is attached a silk thread which is passed around a suitably pivoted small pulley H, to which is also attached the pointer I.

The current to be measured passes only through the branch A of the working wire A–B, thus heating A only. The resulting expansion diminishes the tension of A, and equilibrium is restored by the spring F pulling B around the pulley D until the strain is equalized. The motion of D carries G with it and the silk fiber rotates the pulley H causing the pointer to move to the right over a properly graduated scale.

In this type of instrument no special provision need be made for changes in the temperature of surrounding air. When the temperature changes, both branches of the working wire are affected alike, expanding alike. This expansion is taken up by the spring without the rotation of pulley *D*.

Since the expansion of the wire *A-B* is independent of the direction of the current, it is evident that hot-wire meters are suitable for alternating as well as direct current.

104. Ammeter.—The principle of the hot-wire ammeter is identical with that of the voltmeter. The construction is practically the same, the working wire being of larger diameter. In order to prevent sluggishness of action, a fairly fine wire is essential, and this introduces a difficulty in the case of ammeters. Messrs. Hartmann and Braun usually so arrange matters that the current is passed through the wire with several parallels,by means of thin silver strips making contact at various points along its length. Even by this means, however, it is found impossible to pass more than a few amperes through the wire, and hence a shunt has to be employed. This entails a considerable loss of energy, since a fall of potential of 0.2 to 0.5 volts is required across the shunt. The large current taken, however, renders the instrument very susceptible to contact errors.

105. Damping.—While the moving element is light and there is no great necessity for a damping device, nevertheless, one is added. This is practically the same as that applied to the Hartmann and Braun hot-wire instruments. The damper is an aluminum disk swinging between the poles of a stationary permanent magnet.

In a bulletin of the Bureau of Standards on "Testing Electrical Measuring Instruments," we find the advantages and disadvantages of the hot-wire instrument stated as follows:

"The hot-wire instrument is not used in this country to any great extent in practical work; its defects are relatively large consumption of energy, uncertainty of zero, errors due to change of surrounding temperature, and to heating when left in the circuit. As the working wire must be run at a fairly elevated temperature, to give proper sensibility, it is easily damaged by sudden overloads, which would do little or no damage to other forms, except the possible bending of a pointer."

The good points of the hot-wire instrument which cause it to be still used for certain classes of work, are its independence of

ordinary frequencies, wave form, and stray magnetic field; the fact that it may be calibrated on direct current, and that shunts may be used with the ammeter for alternating currents.

106. Thermo-ammeter.—For the purpose of measuring very small currents, there has recently been devised an instrument whose operation depends upon a combination of the electromagnetic, hot-wire, and thermo-electric principles.

It is well known that if two dissimilar metals, such as iron and copper, are connected so as to form a closed circuit, and if one of the junctions be heated, an electric current will flow through the circuit. The current flowing will be approximately proportional to the difference of temperature between the two junctions.

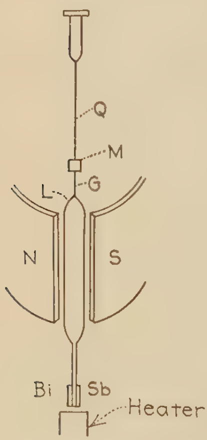


FIG. 71.

The method of applying a combination of this principle with the other two is shown in the diagram of Fig. 71.

A single loop of silver wire L is suspended by means of a quartz fiber Q between the pole pieces NS of a permanent magnet. The loop is surmounted by a glass stem which carries a mirror M, while its lower ends are connected to a bismuth antimony, thermo-couple (Bi, Sb). The heating resistance or "heater," consisting of a fine filament of high specific resistance, is fixed immediately under the thermo-couple. One end of the heater is connected to the frame of the instrument to avoid electrostatic forces. The current to be measured, or a definite part of it, is sent through the heater. Part of the heat generated in the heater is radiated and carried by convection to the thermo-junction raising its temperature. The resulting current flowing through the loop L causes it to turn in the magnetic field. The resulting deflection can then be read off by means of a lamp and scale. In the Duddell thermo-ammeter, however, the loop L is mounted in jewel bearings and a pointer is substituted for the mirror. In the usual pattern of this instrument the full scale deflection is produced by a current of 10 milliamperes, either continuous or alternating, and by constructing heaters of higher or lower resistances the sensibility to current may be increased or reduced as required. The working elements of this ammeter are shown in Fig. 72.

The instrument as at present constructed is not suited for central station use. For measuring current in telephone lines, or for other high frequency currents it is of considerable importance.

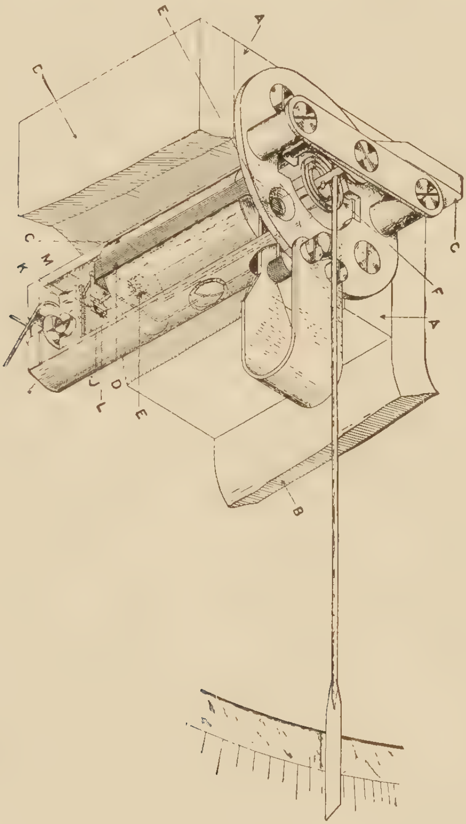


FIG. 72.

107. Magneto-constriction Type or Mercury Ammeter.—Recently there has been developed and placed on the market an ammeter for the accurate measurement of large currents which operates on a principle entirely different from those already explained. This principle was first observed and described by Paul Barry in 1901. The investigations leading up to the development of the instrument are mainly due to Dr. E. F. Northrup who performed many experiments for the purpose of determining the full significance of the principle. The simplest

one of his experiments was the following: A wooden box was constructed as shown in Fig. 73. This box contained two rectangular compartments, or chambers, which were connected by a narrow channel. One side of the channel was made of a sheet of transparent mica, so that the height of the liquid in the channel could be observed. In two opposite ends of the box were fastened brass electrodes.

The two chambers and connecting channel were filled to a depth of about 2 in. with a liquid alloy of sodium and potassium. The remaining space was filled with kerosene. When a current of 180 amperes was passed between the electrodes through the alloy, a V-shaped depression about 1.75 in. was formed in the channel. This is clearly shown in Fig. 73. So long as the cur-

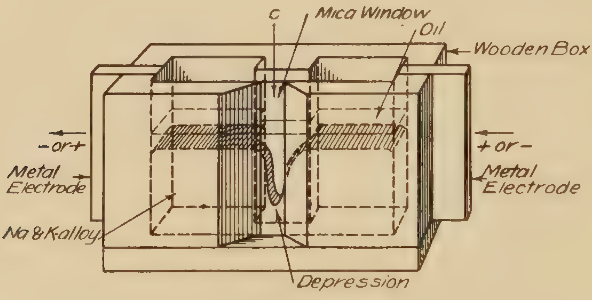


Fig. 73.

rent remained constant the depression remained fixed, but varied with the current. This showed that with a given liquid conductor the depth of the depression is a function of current only, and independent of the direction of current.

This constricting effect of a current upon a liquid conductor is the underlying principle upon which this type of ammeter operates.

108. Force Causing Contraction of Liquid Conductor.—Since explanations of this principle have not as yet found their way into texts on Physics, a rather full explanation will be first given.

In Article 10, it was pointed out that a conductor carrying a current is surrounded by a magnetic field. The character of this field is shown in Fig. 4. These magnetic lines are, however, not wholly outside of the conductor, but inside as well. The distribution of the lines within and without the conductor are not the

same, as will presently be shown. The force causing the constriction of a column of liquid conductor is due to the magnetic field within the conductor. Let us first determine the character of this field:

109. Magnetic Field Inside of a Liquid Conductor.—Let Fig. 74 represent the cross-section of a conductor whose radius is R. With a radius equal to $OP=r$ draw a circle through P, a point within the conductor. Both theory and experiments show that there is no magnetic field inside of a hollow cylindrical conductor and, therefore, the field at P must be due wholly to

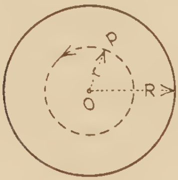


FIG. 74.

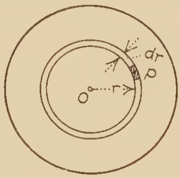


FIG. 75.

current inside of circle whose radius is r. Calling this current I_p the field at P is

$$H=\frac{2I_p}{r}$$

I_p is, however, such a part of the whole current as the area of circle of radius r is of cross-section of wire, or

$$I_p : I : : r^2 : R^2$$

whence
$$I_p=\frac{r^2 I}{R^2}$$

Substituting this value of I_p in expression for H we get

$$H=\frac{2Ir}{R^2}$$

Since I and R are constants, the field strength within the conductor increases directly as the distance of the point from the center.

According to the discussion of Article 14, a conductor carrying a current, when introduced into a magnetic field, is acted upon by a force which acts at right angles to the field and also at right angles to the current. A conductor may be con-

sidered as consisting of an infinite number of small fibers very close together. Each fiber forming the conductor will carry an infinitesimal current, the total current being equal to the sum of the infinitesimal currents. One of these fibers or elements of conductor at P, Fig. 75, will be acted upon by a force directed from P toward O. The intensity of this force per unit length of conductor is equal to the product of current in element or fiber and strength of field.

According to our assumption of uniform current density over cross-section of conductor, the current in an elemental area P, is such a part of the whole current as the area is of the whole cross-section. In Fig. 75 let P be situated half way between two circumferences of radii r, and $r+dr$. Then the current in shaded area whose depth is dr and whose length is unity, equals:

$$\frac{I \times dr}{R^2}$$

The radial force on this element per unit length is then

$$df = \frac{HI \times dr}{R^2}$$

But $$H = \frac{2Ir}{R^2}$$

Hence $$df = \frac{2I^2r\,dr}{R^4}$$

The total force at P is the sum of the forces on all such current elements radially outside of P. This sum is obtained by integrating the above expression from r to R. Integrating we get

$$f = \frac{I^2}{R^4}(R^2 - r^2)$$

When $r = o$, or at center of conductor the pressure is

$$f = \frac{I^2}{R^2}$$

This shows that with a given current strength the radial pressure at center of the conductor varies inversely as cross-section of the conductor.

This pressure is found in solid as well as liquid conductors, but owing to the rigidity of the solid conductor the effect of the pressure is not apparent.

The manner in which the foregoing principles are applied in practice will be understood from Fig. 76, A and B. In this figure B is a vertical section of a compound conductor of mercury and copper, and A is a cross-section of the same conductor along line a of Fig. 76B. The shaded portion represents mercury.

When a current is passed through the conductor in the direction of the arrows, or in the opposite direction, a hydrostatic pressure is created between the outer and inner circumferences of mercury in the chamber containing the mercury. This pressure

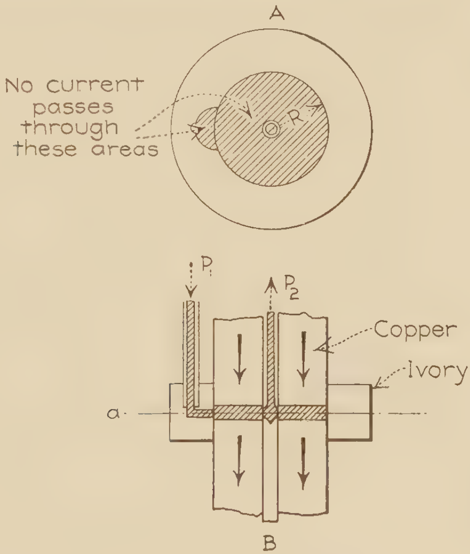


FIG. 76A, and 76B.

will force the mercury up the tube P_2, at the same time the mercury in tube P_1 will sink. The mercury in P_2 will rise until the difference between the elevations of mercury in tubes P_2 and P_1 creates a sufficient pressure to counteract the constricting pressure of current on mercury.

The difference in level between the mercury in tubes P_2 and P_1 is thus a measure of the pressure produced by the current, and consequently of current. It has been shown that, with a given conductor, the constricting pressure depends only upon the square of the current.

110. Cells in Series.—While theoretically to measure the current it is only necessary to determine the difference in pressure

between the center and circumference of a single cell of mercury, practically, however, an instrument is more accurate if the pressure differences in individual cells are added, thus increasing the difference in elevation for a given current strength. The manner in which this is carried out in practice is shown in Fig. 77. This figure represents a cross-section of a 2000-ampere meter.

The current enters the meter through one copper terminal and passes through the cells downward. The cell, as the word is here used means the cavity containing the mercury. Tubes extend from the center of one cell to the circumference of the next one above. By this means, the pressure developed, at the center

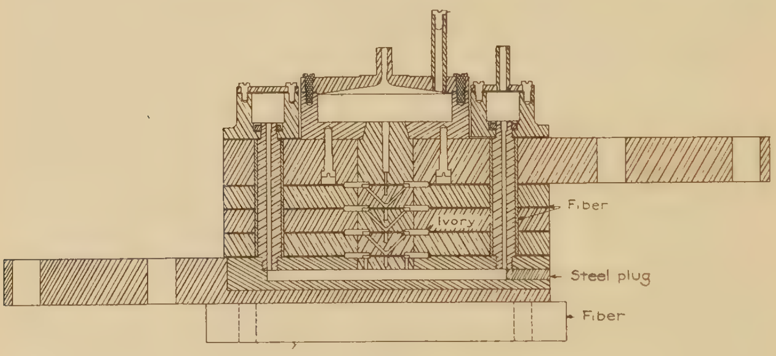


FIG. 77.

of one cell is transmitted to the circumference of the next cell; the resulting pressure in the chamber under the scale is then the sum of the pressures developed in the separate cells. This is the principle of construction of the recent form shown in Fig. 78. The mercury reservoirs are placed one above the other and hence the readings of the meter are independent of the level of the instrument, within reasonable limits.

The comparatively large density of mercury and large temperature coefficient of expansion, prevent its use as an indicator. Colored water is, therefore, used for this purpose. A very small change in the elevation in the mercury will thus make a large change in the scale or indicator tube.

The scale, which is 50 cm. long, consists of two brass strips which lie to the right and left of the index tube. On the right

hand strip is laid off a scale reading in amperes, while the other scale is subdivided into millimeters. This last may be used when one wishes to read to high precision. The scale has a vertical adjustment to take care of changes in the zero due both to tipping and to temperature changes.

Since the constricting pressure is proportional to the square of current, and the height to which the liquid in the tube rises is

FIG. 78.

proportional to the pressure, it at once follows that the square of the current is proportional to the height of liquid in the scale tube. Algebraically this may be expressed by

$$I^2 = K \times h.$$

Where K is a constant and h is the height to which liquid rises. This relation between scale and current is evidently the same as that for the Kelvin balance.

111. Advantages.—Among the chief advantages may be mentioned the following:

1. It has very low energy loss.

2. Its accuracy is high and is entirely independent of variable quantities, such as torque of springs, friction, permanence of magnets, etc.

3. It has a relatively long scale, which is graduated strictly in conformity with the following relations: $I^2 = K \times h$.

4. It may be used interchangeably on direct and alternating current.

5. The instrument is nearly dead-beat and the liquid will rise from zero to full scale in from four to five seconds.

6. Its main advantage is the fact that very large currents can be measured as easily and accurately as small ones.

CHAPTER X

POWER MEASURING INSTRUMENTS

112. Wattmeters.—The instrument most commonly used for measuring power is called a wattmeter. Wattmeters are of three classes, electrostatic, electrodynamic, and electromagnetic. The electrostatic wattmeter is not in common commercial use, and, hence, will not be discussed. Wattmeters in common use are either of the electrodynamometer and induction types.

113. Electrodynamometer Type.—The electrodynamometer type is very common and may be considered the standard indicating wattmeter. The essential features of such an instrument are shown in diagram, Fig. 79, and consist of two coils, one

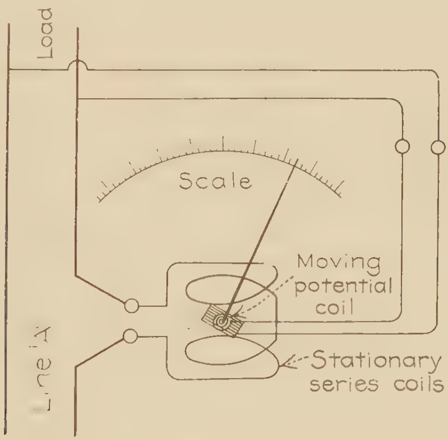


FIG. 79.

fixed and the other movable, as in the electrodynamometer ammeter. In fact, the electrodynamometer ammeter can be used as a wattmeter, if suitable resistance is connected in series with one of the coils, preferably the moving one.

In the diagram shown, the heavy line connected in series with the line A is the stationary or current coil and consists of two parts, each of a few turns of heavy wire. The movable coil, which is mounted in the same manner as the movable coil of the electrodynamic voltmeter, consists of many turns of fine wire.

The manner of connecting such a wattmeter to a circuit is shown in Fig. 81. L represents the lamp, or receiving circuit, whose power consumption it is desired to measure.

114. Theory of Electrodynamometer Wattmeter.—In discussing the electrodynamometer ammeter, and Kelvin's balance, it was stated that the force of attraction between the fixed and

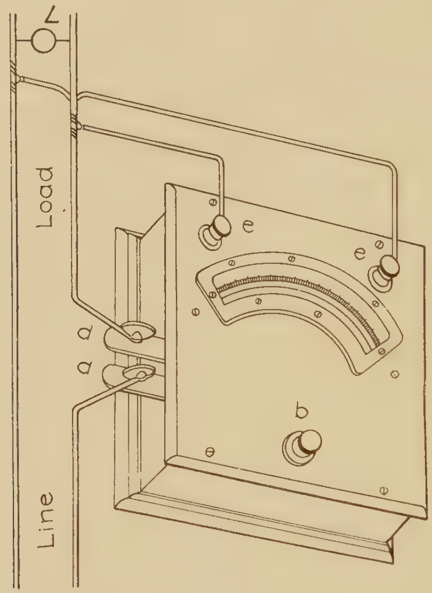


FIG. 81.

movable coils is proportional to the product of the currents in the two coils. In that case, however, the same current flowed through the two coils. In the wattmeter under discussion the force of attraction is likewise proportional to the product of the currents in the fixed and movable coils, but the currents are not the same. The fixed coil carries the current supplied to the load, but the movable coil carries a current which is proportional to the electromotive force across the load.

On direct-current circuits the deflecting force may be represented by

$$F = KI \times i$$

where I is the load current, i the current in pressure coil, and K a proportionality constant.

If R is the resistance of the pressure coil, and E the pressure across the receiving circuit terminals, $i=\frac{E}{R}$. Substituting this value of i, we get

$$F=\frac{K}{R}EI.$$

Since R is also constant, the expression may be written

$$F=K_oEI.$$

This expression shows that the deflecting force, and hence the torque is proportional to the product of current and pressure, *i.e.*, to power consumed in the load or receiving circuit.

Assuming the inductance of the pressure and current coils to be negligible, the electrodynamometer wattmeter also gives the average power on alternating-current circuits. At any instant the torque is proportional to the product of current and pressure at that instant. The average deflecting torque will then be proportional to the average of the product of current and pressure.

Representing the instantaneous pressure and current by

$$e=E_m \sin \omega t$$

and

$$i=I_m \sin (\omega t-\theta)$$

the torque causing a deflection will be

$$\text{Torque}=K \text{ average of } e\times i$$

or

$$T=K \text{ average } E_mI_m \sin \omega t\times \sin (\omega t-\theta)$$

$$=KE_mI_m\times \text{av.} [\sin \omega t (\sin \omega t \cos \theta-\cos \omega t \sin \theta)]$$

$$=KE_mI_m \text{ av.} (\sin^2 \omega t \cos \theta-\sin \omega t \cos \omega t \sin \theta)$$

$$=KE_mI_m (\text{av.} \sin^2 \omega t \cos \theta-\text{av.} \sin \omega t \cos \omega t \sin \theta)$$

The average of $\sin^2 \omega t$ is $1/2$, and the average of $\sin \omega t \cos \omega t$ is 0. Hence

$$T=K\frac{E_mI_m}{2} \cos \theta$$

$$=KEI \cos \theta$$

where E and I are effective values. Thus the wattmeter automatically corrects for power-factor. The constant K, depends upon the windings of the instrument but, as instruments of this type are direct reading, it need not be considered.

Since the reaction of a coiled spring furnishes the controlling force in instruments of this type, it would seem that such an instrument would have uniform scale. This may or may not be the case depending upon the operation of the meter. In one form of this type of wattmeter the movable coil is brought back to its zero position by means of a torsion head as in the

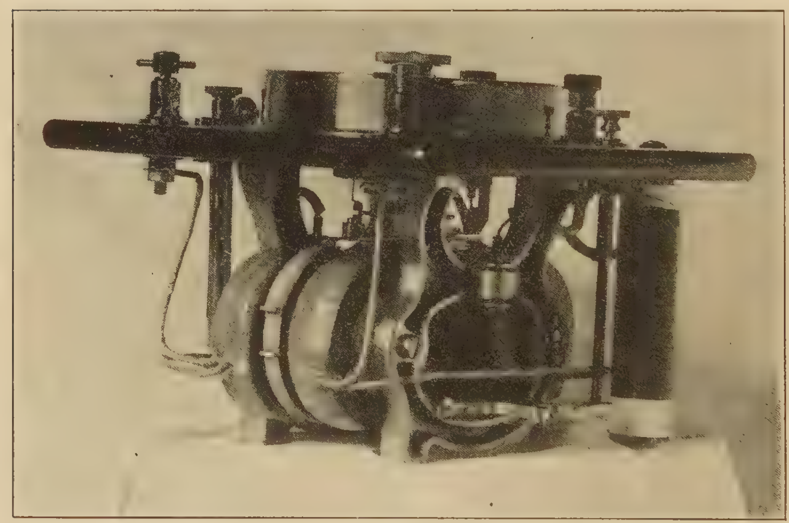


Fig. 82.

case of the dynamometer ammeter already discussed. Such an instrument will naturally have a uniform scale.

In the other form, the pointer is attached to the shaft of the movable coil. Under these conditions the scale is no longer uniform. The graduations at the upper end of the scale are crowded together on account of the fact that when the deflection becomes great the deflecting force is not exactly proportional to the deflection, but more nearly to the sine of the angle of deflection.

The essential parts of two makes of torsion head wattmeters are shown in Figs. 58 and 82. The principles used in the con-

struction of the Roller-Smith wattmeter, Fig. 58, are evidently those of the Siemens electrodynamometer, with the exception that the movable coil is mounted on a shaft, thus permitting the omission of mercury cups. Meters constructed in this way are subject to the influence of external magnetic fields. Fig. 58 is the mechanism of a voltmeter, but the same principles are applied in the construction of wattmeters.

The Westinghouse precision wattmeter, shown in Fig. 82, is also of the electrodynamometer type but is constructed according to the principles of the Kelvin balance.

As has already been pointed out, such instruments are astatic, that is, not subject to the influence of external magnetic fields.

FIG. 83.

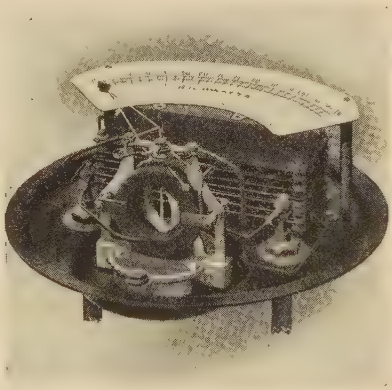


FIG. 84.

The middle, movable coils are mounted on a light frame work carrying suitable jewels mounted on ball bearings.

Both the General Electric and Weston portable wattmeters belong to the second form, *i.e.*, the shaft of the movable coil carries the pointer. The deflection of the pointer is thus a measure of the angle through which the coil has been turned.

The General Electric Company uses the inclined coil principle, as is shown by Fig. 83.

The Weston electrodynamometer wattmeter is shown in Fig. 81. The principles of construction of this instrument are the same as those for the voltmeter, Fig. 57. Fig. 81 also shows the manner of connecting such a meter to a circuit. The large term-

inals d–d are connected in series with the power line, and the small terminals e–e are connected across the line to the terminals of the load. The button b, when pressed down, serves to close the pressure circuit. This is necessary to prevent heating of the pressure coil when no readings are being taken. The scale is graduated in watts or kilowatts, depending upon the range of the instrument. The instruments are normally made for a maximum of 150 volts, but the range can be varied by the use of suitable multipliers and shunts.

For switchboard use, instruments are manufactured upon exactly the same principles. Figs. 84 and 85 show the principles

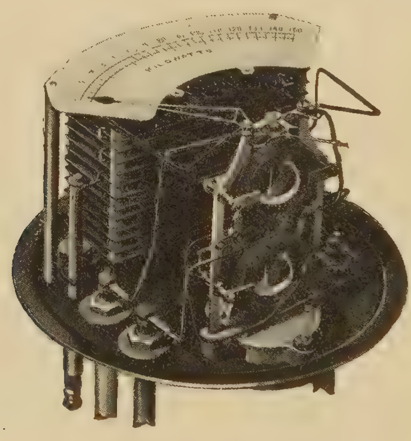


Fig. 85.

of construction of Weston single-phase and polyphase switchboard wattmeters. Although these are direct reading deflection instruments, nevertheless, the refinement of construction and adjustment makes possible the use of uniform scales.

115. Compensation for Power Consumed in Instrument.—Since both the current and pressure coils of a wattmeter possess resistance, some power will be consumed in the instrument itself. The amount of this power is not large, yet it would be unfair to charge it up to the receiving or load circuit. From the diagram of Fig. 79 it is evident that the current through the series coil is the sum of the pressure and load currents. On direct-current circuits this sum is equal to the algebraic sum of the currents, but in alternating-current circuits, the inductance of the potential

coil will never or seldom be the same as that of the receiving circuit, and hence, the two currents will not be in phase. In such a case, the current through the series coil will be equal to the vector sum of the pressure and load currents.

The power consumption of the pressure coil is $i^2R=\frac{E^2}{R}$, where i is the current in the movable coil and R the resistance of the voltage coil. If the load current is I, and the resistance of the series coil r, the power consumption of the series coil will similarly be I^2r. It is necessary to make correction for only one of these losses as can easily be shown. The relation between deflection and power, when current and pressure are in phase, is,

$$D=K\times I\times E.$$

If the wattmeter is connected to the circuit as indicated in Figs. 79 and 81, the current I is equal to the load current plus the pressure-circuit current.

Let I_L = load-circuit current
and i_e = pressure-circuit current

Then the line current is equal to the sum of I_L and i_e or

$$I=I_L+i_e$$

and the deflection is

$$\begin{aligned} D&=KE(I_L+i_e)\\ &=KEI_L+KEi_e \end{aligned}$$

The deflection is thus due to two quantities, one KEI_L, which is proportional to the power taken by the load, and the other KEi_e, which is proportional to the power spent in the pressure coil. The correction must then be made only for the power spent in the movable or pressure coil. On constant voltage circuits this quantity is constant and corrections can easily be made by measuring the voltage, and movable coil resistance.

When the pressure coil is connected across the line side of the circuit, for accurate work correction must be made for the power used in the series coil.

When such a connection is used, the current through series coils is only that demanded by the load. The pressure at the terminals of the voltage coil is, however, a trifle higher than that

at the load terminals, due to the pressure drop, Ir, across the series coil. The correction to be applied is then equal to I^2r. The power spent in the series coil is usually less than that spent in the pressure coil and, hence, the second method of connection is to be preferred when non-compensated wattmeters are used and no corrections are made. When corrections are to be made, the usual method of connecting a wattmeter to a circuit is that indicated in Fig. 81, and corrections are made for the power consumed in the voltage coil. The reason for this is that the resistance of the voltage coil can be determined more easily than the resistance of the series coil. The resistance of the series coil is very small, and the resistance of the contacts d–d is an appreciable part of this resistance. This, of course, is a variable quantity; it, therefore, would be extremely difficult to make any correction if that were to be considered.

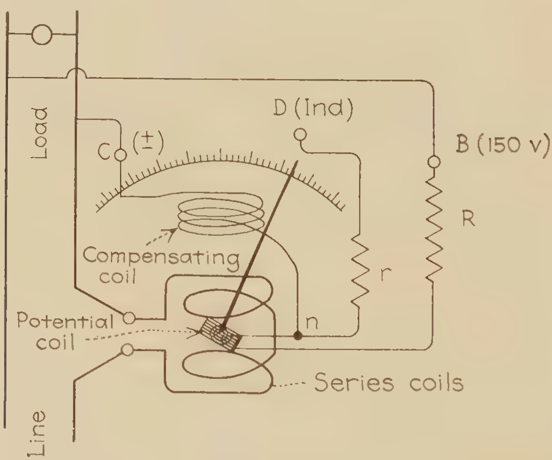


Fig. 86.

The portable wattmeters manufactured by the Weston Electrical Instrument Company, are provided with a compensating coil as shown in Fig. 86. It will be observed that the compensating coil is connected in series with the pressure coil, its winding, however, being reversed with reference to the series coil. The number of turns in this compensating coil is carefully adjusted so that the counter torque, due to the pressure-circuit current, is just equal to the direct torque due to this same current when

flowing through the series coil. Such a compensation makes automatic correction at all voltages, since the current through the compensating coil is always the same as that through the series coil at no-load and same voltage. The complete pressure circuit thus consists of the voltage coil proper, the compensating coil, and the resistance R, which, in some cases, may be separate from the instrument. Connection is made to the power circuit through binding posts C and B.

The binding post D is connected to a resistance r of the same value as the resistance of the compensating coil, and is to be used in calibrating the instrument by means of two different sources of current. Also, in measuring power in high potential cirucits, when the series coils are connected to a series transformer and the voltage coil to a potential transformer, the independent terminal D is to be used.

116. Influence of the Inductance of the Voltage Coil.—In the previous discussion, we assumed the inductance of the voltage coil to be negligible. While this is accurate enough for practical purposes and on circuits with large power-factors, on circuits of low power-factor the errors introduced may be appreciable, and the inductance of the voltage coil must be considered.

When the receiving, or load, circuit has considerable inductance, the current through the series coil will lag behind the electromotive force. In such a case, the voltage current, when the voltage coil is non-inductive, is in phase with the electromotive force, and the deflecting force is proportional to

$$IE \cos \theta,$$

where θ is the phase angle between electromotive force and load current.

When, however, the voltage coil has inductance, the current in voltage coil will lag behind the load pressure and thus the angle between load current and voltage current will be less than between the load electromotive force and current. This is shown in the vector diagram, Fig. 87, where α represents the difference in phase between the voltage current and load pressure. The wattmeter indications under such conditions will be

$$W = KEI\ [\cos (\theta - \alpha)]$$

$$= KEI\ (\cos \theta \cos \alpha + \sin \theta \sin \alpha)$$

but the correct reading should be

$$W' = KEI \cos\theta$$

The difference between W and W' will give the error, which is

$$W - W' = KEI\ (\cos\theta\cos\alpha - \cos\theta + \sin\theta\sin\alpha)$$

The inductance of the potential coil is always small, so the angle α is very small. This being the case, $\cos\alpha = 1$, nearly, and $\sin\alpha = \alpha$, nearly. Substituting these values we get

$$W - W' = KEI\ (\cos\theta - \cos\theta + \sin\theta \times \alpha)$$

$$= KEI \sin\theta \times \alpha.$$

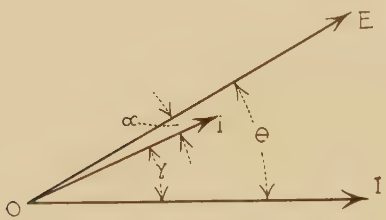


FIG. 87.

From this it is evident that the error is proportional to $\sin\theta$, or the sine of the phase difference between pressure and current in load. When θ is 0, $\sin\theta$ is zero and the error is zero. When the angle is large, that is, when $\cos\theta$ is small, or on circuits of low power-factor, $\sin\theta$ is comparatively large and the error is no longer negligible.

The percentage error is

$$\frac{W - W'}{W'} = \frac{KEI \sin\theta \times \alpha}{KEI \cos\theta} = \tan\theta \times \alpha$$

This, likewise, increases with the inductance of the load circuit. Tan θ increases with θ and becomes infinite when θ approaches 90°.

117. Correction Factor.—Electrodynamometer wattmeters are usually calibrated on direct current, and when so calibrated the inductance has no effect. If R is the resistance of the voltage

coil and i is the voltage current, the energy spent in the coil when used on alternating-current circuits is,

$$Ri^2 = Ei \cos \alpha$$

$$\text{whence } Ri = E \cos \alpha$$

$$\text{and } i = \frac{E}{R} \cos \alpha$$

$$\text{But } W = EI \cos \theta$$

$$\text{and Torque} = KiI \cos \gamma,$$

Substituting $\frac{E}{R} \cos \alpha$ for i, we get

$$\text{Torque, } T = \frac{KEI}{R} \cos \alpha \cos \gamma$$

$$\text{and } \frac{T}{W} = \frac{K}{R} \frac{\cos \alpha \cos \gamma}{\cos \theta}$$

$$\text{whence } W = T \times \frac{R}{K} \frac{\cos \theta}{\cos \alpha \cos \gamma}$$

On direct current the watts are directly proportional to torque, or to $\frac{R}{K}T$, hence, the indication of a wattmeter, correct on direct-current, must be multiplied by $\frac{\cos \theta}{\cos \alpha \cos \gamma}$ to get the correct power on alternating-current.

If the voltage and current are sine curves,

$$\gamma = \theta - \alpha$$

$$\text{and } \cos \gamma = \cos (\theta - \alpha) = \cos \theta \cos \alpha + \sin \theta \sin \alpha$$

In practice α is small, hence without appreciable error $\cos \alpha = 1$, $\sin \alpha = \alpha$. Under these restrictions, the correction factor reduces to

$$\frac{\cos \theta}{\cos \theta + \alpha \sin \theta}$$

When $\theta = 0$, this reduces to 1. If α is appreciable, the correction factor is less than 1 when θ is greater than 0, and the correction

factor decreases with increase in θ. Hence, a wattmeter with inductance in the voltage coil, if calibrated on direct current, will read too high on alternating currents. If the phase difference between pressure and load current is due to capacity, the current will lead the pressure. The relation between the angles is then given by $\gamma=\alpha+\theta$. Under these conditions, the correction factor reduces to

$$\frac{\cos \theta}{\cos \theta-\alpha \sin \theta}$$

When $\alpha \sin \theta=\cos \theta$ the torque which is proportional to $\frac{\cos \theta-\alpha \sin \theta}{\cos \theta}$ is zero, and the deflection is zero. When $\alpha \sin \theta$ is greater than $\cos \theta$ the deflection is negative. When the power-factor, $\cos \theta$, is small, $\cot \theta=\frac{\cos \theta}{\sin \theta}$ is likewise small; hence, under these conditions the wattmeter does not give trustworthy readings. The errors increase with frequency, for α increases with the frequency.

118. Range of Wattmeters.—The range of the electrodynamometer type of wattmeter may be changed by connecting multipliers in series with the voltage coil, or shunts in parallel with the series coil on direct-current circuits and low voltage, low frequency, alternating-current circuits. When the wattmeter is to be used on high voltage alternating-current circuits, transformers are used.

119. Induction Type Wattmeters.—Induction type wattmeters use in their operation the principles of the rotating and revolving magnetic fields. To understand the general application of these principles, consider the case illustrated in Fig. 88 which is merely illustrative.

The essential parts of an induction meter are a pivoted disk or drum D_1, a pressure coil VC, and a current or series coil CC. The copper or aluminum disk, D, is pivoted at its center and carries a pointer not shown in the figure. The motion of the disk is counteracted by suitable spiral springs. The voltage coil is highly inductive so the current in the coil lags approximately 90° or one-quarter of a period behind the pressure. The current in pressure coil produces a magnetic field which is in phase with it. The variation of this flux through the disk induces eddy currents which are one-quarter of a period out of phase with the flux.

The current coil CC is non-inductive. The flux, due to this current through the disk is in phase with it. Hence, the eddy currents, due to pressure current, and flux, due to load current, reach maximum values together and the reaction between them will be a maximum under these conditions. This reaction causes the disk to rotate.

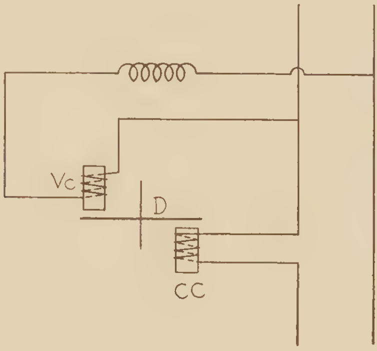


Fig. 88.

The eddy currents are proportional to pressure current, which in turn is proportional to the pressure at terminals of load, or algebraically

$$i = K_1 E,$$

where i represents the effective eddy currents, and E the effective pressure. Similarly

$$\Phi = K_2 I$$

where Φ is the effective flux due to effective load current I. The torque is proportional to the product of i and Φ, hence

$$\text{Torque} = K_0 i\ \Phi = \mathrm{K}_1\ K_2 EI.$$

Or in other words, the torque is proportional to $E\ I$, the power.

It can readily be shown, by the same process of reasoning, that when the current and pressure are not in phase, the torque is proportional to $EI \cos \theta$, where θ is the difference in phase. Hence, the induction wattmeter, when properly adjusted, gives correct indication on circuits whose power-factor is other than unity.

A diagrammatic sketch of the magnetic circuit and windings of the Westinghouse induction wattmeter is shown in Fig. 89. It is evident that the magnetic circuit is of the same pattern as that of the induction ammeter manufactured by the same company. The winding is, however, modified to meet conditions of power measurements.

The windings consist of two principal coils *V–C* and *C–C*. The coils *V–C* are connected through a resistance across the line,

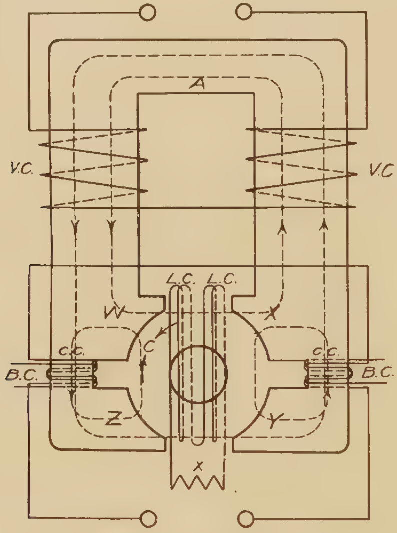


Fig. 89.

and the coils *C–C* in series with the load. The coils *V–C* have relatively high inductance and consequently the flux due to the voltage current is nearly in quadrature with the pressure.

120. Operation.—The pressure current makes (*W–Z*) and (*X–Y*) opposite in polarity. The load current through coils *C–C* makes (*W–X*) and (*Z–Y*) opposite in polarity, and as the flux due to load current is in phase with the current, the resulting field rotates. As already explained this rotating field acting on the drum armature produces a torque which causes a deflection. The motion of armature is opposed by spiral springs as in the case of the ammeter. The auxiliary coils *B–C* are used to equalize any difference in the windings of the voltage coils *V–C*. A complete instrument of this type is shown in Fig. 90.

In the foregoing discussion the assumption is made that the current in the voltage coil is exactly one-quarter of a period out of phase with the voltage at its terminals. In practice this relation cannot be secured with the arrangements of coils shown. This is due to the fact that the voltage coil possesses resistance as well as inductance, and also that eddy currents and hysteresis losses in the core are inevitable. Likewise, the current in current coil is not exactly in phase with the resulting flux. Hence, it follows that it is not easy to insure that the flux due to the voltage coil shall differ by exactly one-quarter of a period from that due to the current coil. The effect of any such a discrepancy

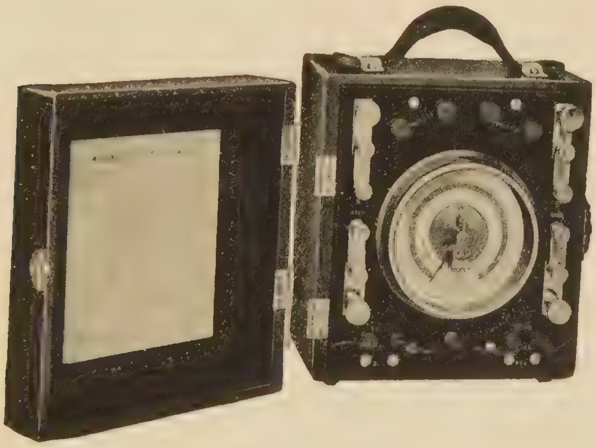


FIG. 90.

upon the indications of induction instruments is similar to the effect of inductance and capacity in the voltage coil of the electro-dynamometer type of wattmeter; that is, the error is small when the power-factor of load is high; but it increases as the power-factor decreases.

121. Lagging Induction Wattmeters.—The term lagging, as here used, means adjusting the meter so that it will read correctly on inductive and non-inductive loads. This is accomplished by means of auxiliary coils whose effect is to produce exact quarter phasing between fluxes due to voltage and current coils. Usually the coils for making this adjustment are connected in the voltage circuit.

One method of securing exact quarter phasing is represented diagrammatically in Fig. 91. This represents the voltage circuit as consisting of three parts, A, B, and C. A is a highly inductive coil, B also possesses some inductance but less than A, while C, which is connected in parallel with B, is a pure resistance. The current in the coil B produces the actuating flux. The principles involved are illustrated in Fig. 92. Representing the voltage across coil B, Fig. 91, by E_B, Fig. 92, it is evident that I_C, current in coil C, is in phase with E_B. Owing to the inductance of coil B, I_B will lag behind E_B by an angle less than 90°. I_B may then be drawn in the direction of OY. I_A, or current in coil A, is the

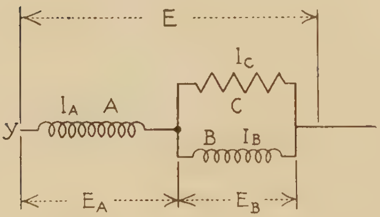


Fig. 91.

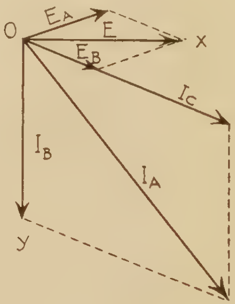


Fig. 92.

vector sum of I_C and I_B. Vector I_A represents this current. Since coil A is highly inductive E_A will lead I_A by nearly 90° and, hence, may be represented by vector E_A. E, the resultant of E_A and E_B will under these conditions be represented by vector E or OX. The phase difference between E, E_A, E_B, I_A, I_B and I_C depend upon the relative values of inductances and resistances of coils A, B, and C. It is thus evident that by a suitable adjustment of the inductances and resistances of these three coils, I_B can be made to lag exactly 90° behind E.

Perhaps the simplest and most commonly used method of lagging a meter consists in winding the core of the voltage coil with, or interposing within the path of the voltage flux, a short-circuited coil whose resistance is adjustable. The short-circuited coil acts as the secondary of a transformer, of which the regular voltage coil is the primary. This is the method used in the Westinghouse wattmeter shown in Fig. 89 in which L. C. represents the lagging coil.

The influence of the lagging coil upon the phase relation between pressure coil and current coil fluxes, is much the same as that of the secondary of a transformer. The phase relation is shown in Fig. 93. Φ represents the magnetic flux linking both the pressure and lag coils. It induces in the lag coil an electromotive force E'_2 and in the voltage coil an electromotive force in the same direction, but of different magnitude, determined by the number of turns. Representing the exciting current in voltage coil by I, the ampere turns necessary to produce the flux Φ is represented by N_1I, and is made up of two components, N_1I_1 and N_2I_2. The electromotive force applied to the voltage coil terminals may be separated into three components; the first E'_1 balances

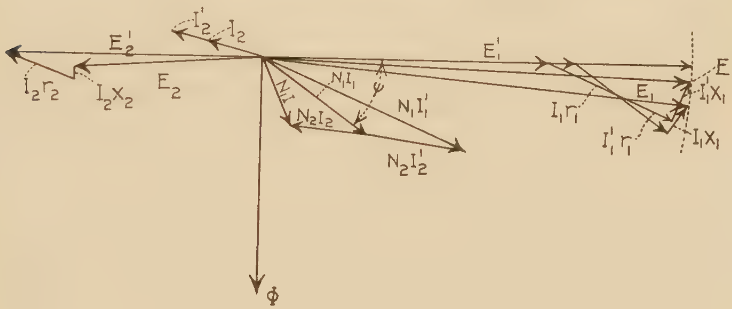


FIG. 93.

the induced e.m.f. due to the flux Φ; the second, represented by I_1X_1, balances the e.m.f. produced by any leakage flux which links with the voltage coil, but not with the lag coil; the remainder sends current through the voltage coil and is represented by I_1r_1. The terminal pressure E_1 is the vector sum of these three components. If the lag coil be open circuited, E_2 becomes identical with E'_2; N_1I_1 becomes identical with N_1I; I_1r_1 becomes smaller and farther from E'_1 in phase. As the current in the lag coil is increased due to a decrease in the resistance in series with it, I_1 must increase to maintain I; angle Ψ decreases, I_1 becoming more nearly in phase with E_1; I_1r_1 increases, making E_1 more nearly in phase with E'_1, the condition desired. An increase in I_1X_1 also aids in bringing about the desired relation. Since E'_1 is always in quarter phase relation with Φ, adjusting the lag coil resistance secures the proper quarter-phase relation between Φ and E_1.

122. Scale.—Since the motion of the movable element is usually controlled by a spiral spring, the scale will be uniform when the torque is exactly proportional to the power in watts. This exact proportionality is not absolutely essential in indicating instruments as the whole scale can be calibrated. The scales of the best instruments are, however, practically uniform and extend over nearly 300 degrees. It is evident that induction instruments can be used only on alternating-current circuits.

CHAPTER XI

PHASE RELATION AND FREQUENCY INSTRUMENTS

123. Introduction.—The indications of two classes of instruments are determined by the phase difference between current and pressure in the same circuit, or the difference in phase between pressures in different circuits.

Instruments of the first class are called power-factor meters, or indicators, and of the second class synchroscopes.

123A. Power-factor.—The term power-factor has been defined in two ways. According to one definition, power-factor is the cosine of the phase difference, the phase difference being the angle between the points at which the curves of current and voltage cut the axis in the same sense. Thus, in Fig. 32 the cosine of the angle represented by the distance between points where e.m.f. and current waves cross axis in the same sense, is called the power-factor. The other definition has no relation to phase difference, but is based upon the relation

$$\text{True power} = \text{volts} \times \text{amperes} \times K$$

whence,

$$K, \text{ or power-factor}, = \frac{\textit{watts}}{\textit{volts} \times \textit{amperes}}.$$

If the current and pressure curves are true sine waves, the values of the power-factor will be the same in the two cases. When, however, this is not the case, and especially when the current and voltage curves have different forms, the power-factor K, as determined in accordance with the second relation will not be equal to the cosine of the phase angle. For practical purposes, the power-factor is usually determined in accordance with the second relation given above. Indirectly, the power-factor of a circuit can be obtained from the indications of an ammeter, voltmeter, and wattmeter. The product of the ammeter and voltmeter readings gives the apparent power; the wattmeter gives the true power, and thus the wattmeter reading divided by the product of the ammeter and voltmeter readings will give the power-factor. It is often advisable, however, to have a separate

instrument for indicating the power-factor, so that no computations need be made.

124. Power-factor Meter.—The essential features of one type of single-phase power-factor meter are shown in Fig. 94. The principle of this instrument is much the same as that of the dynamometer type wattmeter. The instrument, however, is provided with two movable coils in place of one. These coils are mounted at right angles to each other, and the controlling spring is omitted. The current is led into the moving coils by means of two strips which exert practically no torque. The operation of the instrument is then as follows:

The main or load current passes through the coils *CC*, while the voltage current is divided, one part passing through resistance *R* and coil *B*; and the other part through the inductance *L* and

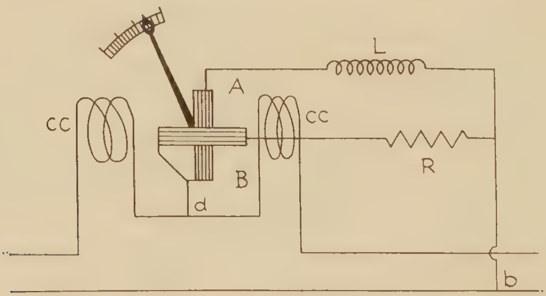


FIG. 94.

coil *A*. The current in resistance *R* and coil *B* will be in phase with the voltage across the load terminals, while that through *L*, which is highly inductive, will be about 90° out of phase with the current in *R* and *B*.

When the load current and voltage are in phase, the reaction between the coils *CC* and coil *B* will be a maximum, and that between coils *CC* and *A* a minimum. As a consequence, the coil *B* will set itself in such a position that its plane will be parallel to the plane of the coil *CC*; or in other words the flux due to the current in coils *CC* will pass straight through coil *B* and in the same direction as the flux due to coil *B*. Under these conditions the pointer, which is attached to shaft of coil *B*, will indicate unity power-factor.

When, however, the load current and voltage are out of phase, the reaction between the coils *CC* and *B* will be less and there

will be an added reaction between the coils CC and A. This added reaction will compel the coils A and B, which are mounted at right angles to each other, to take a new position. This new position is determined by the phase difference between load current and pressure. The deflection of the movable coils is independent of the magnitude of the main current, but it does depend partly upon the ratio of the currents in coils A and B as well as upon the phase difference between load current and pressure. The value of the current in L depends very largely upon the frequency and wave form of the applied voltage, and, consequently, the indications are also modified by the frequency. By carefully designing the coil L, it is possible to keep the ratio of currents in coils A and B practically constant for moderate variations in the voltage and frequency.

The instrument may be calibrated so as to indicate either phase difference in degrees, or power-factor.

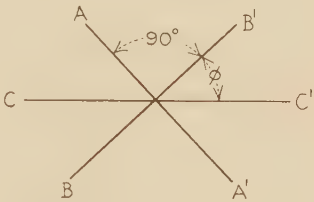


FIG. 95.

125. Analytical Proof of Principles.—That the foregoing explanation of principles is correct can readily be shown by mathematical analysis. In Fig. 95, let AA', BB', and CC' represent the relative positions of the axes of coils A, B, and CC', respectively, at any instant. Furthermore, assume that the current in coils CC is in phase with the voltage. If thec urrent in series coils is represented by

$$i_c = I_m \cos \omega t$$

the currents in A and B may be represented by

$$i_a = I_m \sin \omega t$$

and

$$i_b = I_m \cos \omega t.$$

The coils A and B are designed so that their magnetic field strengths are equal, and as the magnetic field will be in phase with the current producing it, we may represent the instantaneous field strengths of coils A and B by

$$h_a = H_m \sin \omega t$$
$$h_b = H_m \cos \omega t.$$

Now the torque or reaction between either coil and coils CC is proportional to the product of this field strength by current in

coil CC, and sine of angle between the axes of coils. Hence the average torque exerted on coil B is

$$\begin{aligned} T_b &= \text{average of } Kh_b i_c \sin\phi \\ &= \text{av. } KH_m I_m \cos\omega t \cos\omega t \sin\phi \\ &= \text{av. } KH_m I_m \cos^2\omega t \sin\phi. \end{aligned}$$

But the average of $\cos^2\omega t$ is $1/2$, hence the average torque on B is

$$T_b = \frac{KH_m}{2} I_m \sin\phi.$$

This is zero when ϕ is 0 and maximum when ϕ is 90°. That is, torque is 0 when plane of coil B is at right angles to plane of coil C. Likewise the average of torque on coil A is

$$\begin{aligned} T_a &= \text{ av. } h_a i_c \sin(90° + \phi) \\ &= \text{ av. } H_m I_m \sin\omega t \cos\omega \cos\phi. \end{aligned}$$

But the average of $\sin\omega t \cos\omega t$ is zero, hence the average torque on A is zero. The movable element will thus be deflected so that the reaction between coils CC and B is zero. This means that ϕ is zero, or that the plane of coil B is parallel to plane of coil C.

When the current and voltage are out of phase, the same method of finding the torque can be used. Let the pressure lead the current by the angle θ; then the instantaneous currents in the several coils will be

$$\begin{aligned} i_a &= I_a \cos(\omega t - 90°) \\ &= I_a \sin\omega t \\ i_b &= I_b \cos\omega t \\ i_c &= I_m \cos(\omega t - \theta). \end{aligned}$$

The instantaneous values of field strengths due to coils A and B are again

$$h_a = H_m \sin\omega t$$

and

$$h_b = H_m \cos\omega t.$$

The average torque on coil A is

$$\begin{aligned} T_a &= \text{ av. } Ki_c h_a \cos\phi \\ &= \text{ av. } KI_m H_m \sin\omega t \cos(\omega t - \theta) \cos\phi \\ &= \text{ av. } KI_m H_m \cos\phi\,(\sin\omega t \cos\omega t \cos\theta + \sin^2\omega t \sin\theta) \\ &= \frac{K}{2} I_m H_m \cos\phi \sin\theta. \end{aligned}$$

since the average of sin ωt cos ωt is zero and of $\sin^2 \omega t$ is 1/2. Similarly the average torque on coil B is

$$\begin{aligned} T_b &= \text{av. } K i_c h_b \sin \theta \\ &= \text{av. } K I_m H_m \cos \omega t \cos (\omega t - \theta) \sin \phi \\ &= \text{av. } K I_m H_m \sin \Phi (\cos^2 \omega t \cos \theta + \cos \omega t \sin \omega t \sin \theta) \\ &= \frac{K}{2} I_m H_m \sin \phi \cos \theta. \end{aligned}$$

The total torque at any instant will be equal to the sum of T_a and T_b, or

$$T = T_a + T_b = \frac{K}{2} I_m H_m (\cos \phi \sin \theta + \sin \phi \cos \theta)$$

and $$T = \frac{K}{2} I_m H_m \sin (\phi + \theta.)$$

This torque must be zero when the movable system comes to rest, or when $\phi + \theta = 0$.

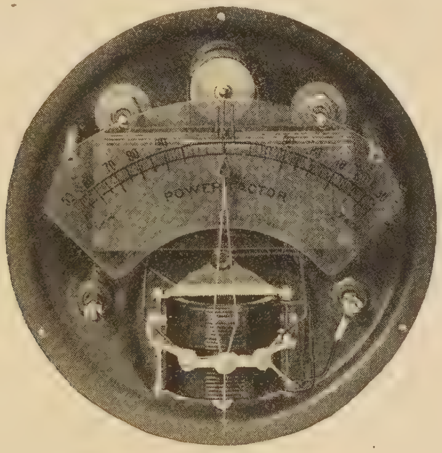


Fig. 96.

The scale may be graduated either in terms of the angle θ, or cos θ, the power-factor. An instrument in which these principles are practically applied is shown in Fig. 96. The movable coils are not visible in this figure but the manner in which they are mounted on the shaft is shown in Fig. 97.

126. Polyphase Power-factor Meter.—The indications of the single-phase power-factor meter cannot be relied upon when the meter is used on circuits whose frequency is different from that

for which the meter is designed. To obviate this objection to single-phase meters when used on polyphase circuits, a meter has been designed which utilizes the actual phase displacement of a

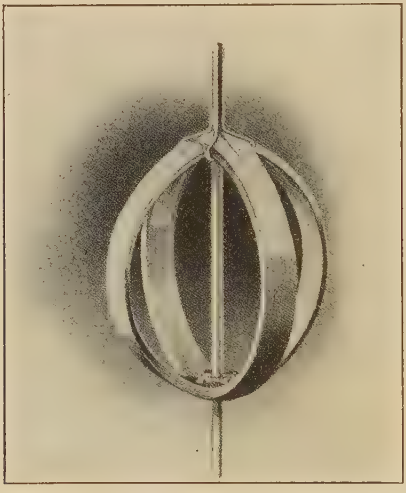


Fig. 97.

polyphase system to obtain the voltage coil magnetic field. A diagram showing the connections, of a three-phase meter is shown in Fig. 98. The connections, as there shown, are intended for a balanced system. The three coils *A*, *B*, and *C* are mounted

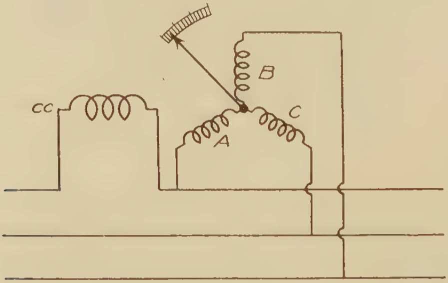


Fig. 98.

on the same shaft in the same manner as the coils of a single-phase meter. The planes of the three coils are 120 degrees apart, and one end of each is connected, through a suitable

resistance, to one of the three mains, the other ends being connected together.

The principles of operation are exactly the same as those of the single phase instrument just described, but as there is no inductance in the voltage coils, the indications of the meter are independent of frequency, wave form, or voltage variations.

For unbalanced three-phase circuits, the instrument is made with three current coils, which may be connected to the separate circuits. The power-factor of each phase, or the average of the whole system, can thus be obtained.

126A. Westinghouse Power-factor Meter.—In place of rotating coils the Westinghouse power-factor meter contains a pivoted iron vane or armature as shown in Fig. 98a. The coils

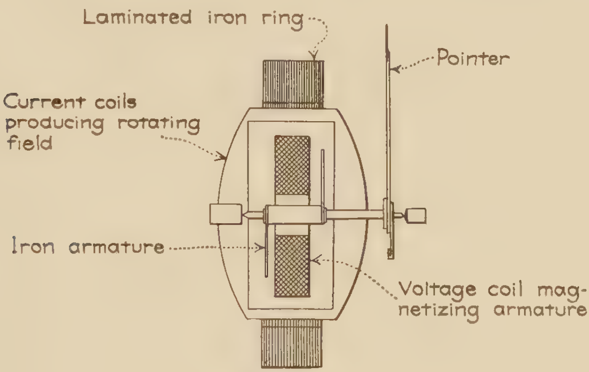


FIG. 98a.

through which the currents flow are angularly spaced coils which are fixed in position. The iron vane is magnetized by a current in phase with the voltage and passing through a coil whose axis coincides with the shaft of vane. The principles of operation are the same as those of the synchroscope shown in Fig. 110 and explained in Article 135, to which the student is referred. The laminated iron ring shown in Fig. 98a provides a return circuit for the flux of the pivoted armature.

The motion of the pointer is damped by means of an aluminum disk moving in the field of two permanent magnets.

In the three-phase meter, the rotating field is produced by current coils spaced 60 degrees apart; in the quarter-phase instrument, by two current coils spaced 90 degrees; in the single-

phase meter, the position of the current and voltage coils is interchanged and the rotating field is produced by means of a split phase winding.

127. Frequency Meters.—Frequency of an alternating current has been defined as the number of cycles per second, where a cycle is considered as consisting of a complete set of positive and negative values. If the magnetic field of an alternator consists of p poles, and has a speed of n revolutions per second, the frequency is given by

$$f=\frac{pn}{2}$$

An instrument designed to indicate the frequency is called a frequency indicator or meter. Frequency indicators are of two types, that is, they make use of two distinct principles in their construction.

128. Resonance Frequency Indicator.—The resonance principle is of considerable importance, not only in its application to frequency indicators, but in many other ways. The principle can perhaps be understood from the following illustrations: In ringing a large church bell, the pull on the rope must come at regular intervals. A small impulse, if imparted at the right instant, and oft repeated may result in considerable motion. If two tuning forks of the same pitch be placed some distance apart, and one be caused to vibrate, in a short time the other will be sounding. The first fork sends out regular impulses of the same frequency as that of the second fork. These impulses are transmitted through the air, and, coming at regular intervals, cause the second fork to vibrate. The sounding board of a piano, and the column of air in the organ pipes are also set into vibration by resonance, the former by the impulses from the wire, and the latter by air impulses from the lip of the pipe.

The application of this principle for indicating the frequency is clearly shown by Fig. 99. Steel strips of different lengths are fastened at one end and free at the other end in much the same manner as the reeds of an organ. These strips have different periods of free vibration, and can readily be caused to vibrate by outside impulses whose frequency is the same. The impulses are magnetic and are supplied by the alternating current in the electromagnet which is connected to the circuit whose frequency

is to be determined. If the free period of vibration of the reed is equal to half the period of the current, the magnetic impulses will set the reed into vibration as follows: As the current in the electromagnet increases, the reed is attracted toward the magnet, and springs away as the current falls to zero; as the current increases in the opposite direction the reed is again attracted. The amplitude is thus increased with each alternation of current until the energy dissipated in the reed by molecular and air friction just equals that imparted to it by the electromagnet. If the period of the alternating quantity differs but slightly from this critical value, the impulses due to the electromagnet will not occur at favorable moments. They will occur sometimes too

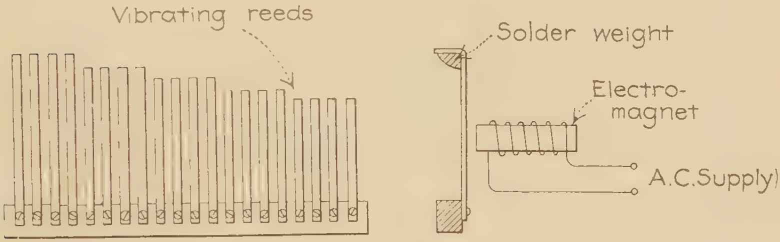


FIG. 99.

early and sometimes too late, so that instead of reinforcing the motion of the reed, some of the impulses will oppose the motion and thus reduce the amplitude. A very slight difference between the frequency of the reed and that of the alternating current is very noticeable in a diminution of the amplitude of the reed. Experiments show that a change of 1/2 of 1 per cent in the frequency diminishes the amplitude by about 50 per cent.

129. Campbell Frequency Meter.—One of the earliest instruments to make use of the foregoing principle was designed by Mr. Albert Campbell. The essential features of such an instrument are shown in Fig. 100. This meter was made with only one reed S, whose free length was variable. One end was fastened rigidly to a sliding rack, while the other or free end projected in front of an electromagnet M. When in use, the rack is moved either to right or left until the maximum amplitude of vibration is obtained. The corresponding frequency is then indicated by the pointer upon a suitable dial.

130. Hartmann and Braun Frequency Meter.—The essential features of the Hartmann and Braun and Siemens and Halske indicators are the same as those shown in Fig. 99. Instead of one reed whose free length can be varied, these indicators are made with many reeds, one for each frequency.

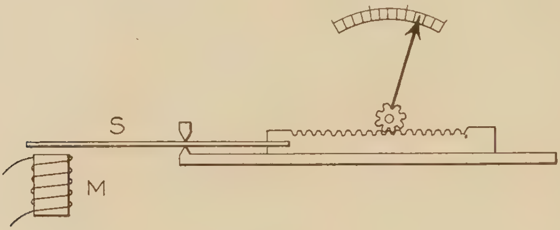


Fig. 100.

In another form the reeds are mounted on the outside of a cylinder, which can be rotated about a central axis. The electromagnet is mounted on an arm pivoted at the axis of the cylinder, and projecting a little beyond or outside of the cylinder. By rotating the cylinder, each reed may be successively brought within the influence of the electromagnet. Every reed is tuned

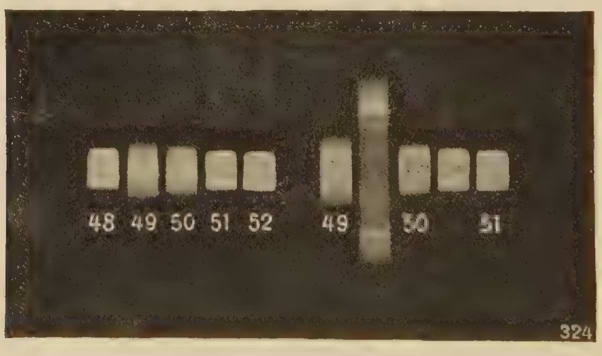


Fig. 101.

to correspond to a different frequency, and, as stated above, its vibration will be reinforced by a current whose frequency is one-half that of the reed. Thus, when the electromagnet is brought up to a particular reed, it is set in vibration and emits a distinct sound only if the conditions are as stated. The frequency of the

current to which the reed responds can be read on a dial fixed above it.

In still another form of meter, the electromagnet and reeds are both fixed. The electromagnet is oblong in form and extends

FIG. 102.

over several reeds. The reeds have their free ends whitened and their vibration is shown as a white band, Fig. 101. These are suitable for switchboard use, Fig. 102.

Yet another form is supplied with two pairs of electromagnets,

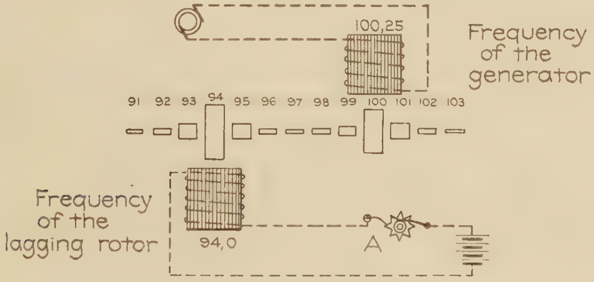


FIG. 103.

one pair in front and the other back of the reeds. By this means, two frequencies from different sources can be determined at the same time.

Fig. 103 shows in diagram the application of such a meter for determining the speed of an induction motor and frequency of

driving generator at the same time. As indicated in the figure, the lower electromagnet produces impulses which are due to interruptions of direct current at the point A. The interrupter is connected to the shaft of the machine whose speed is to be measured, and a battery is connected to the frequency indicator through the interrupter as shown. The number of interruptions or vibrations depends upon the speed of the shaft, and the speed can thus be determined when the frequency of the reed is known.

If alternating frequencies higher than those for which the instrument is made are to be measured, a magnetic superposition arrangement is required. This is accomplished by adding a few turns of a second winding upon the electromagnet core. Through this second winding is sent a direct current which, to a certain extent, polarizes the magnetic field due to the alternating current. By this means the scale readings have double values. The explanation of this is as follows: The electromagnet is alternately positive and negative with the fluctuations or alternations of the current in the coil. A non-polarized reed is attracted by both positive and negative magnetism and, hence, the reed will vibrate twice as fast as the frequency of the current. When, however, the reed is polarized, that is, made a permanent magnet, it will be attracted by only one of the magnetic impulses. For instance, if the north pole of the reed is near the electromagnet, it will be attracted when the electromagnet is negative, and repelled when the electromagnet is positive. Since the alternating magnetization is superposed upon a permanent magnetization, the latter is merely increased and decreased, but not reversed. Under these circumstances the frequency of the reed will be the same as that of the current. The same reed may thus be used to measure two frequencies, the lower frequency will be indicated when the reed is unpolarized, and the higher frequency when it is polarized.

Instruments of this type are very permanent and accurate in their indications.

131. Induction Type Frequency Meter.—An instrument of this type is manufactured by the Westinghouse Electric and Manufacturing Company. The essential features of such an instrument are shown in Fig. 104. The instrument may be described as consisting of two induction voltmeters, the electromagnets of which tend to cause the disk to rotate in opposite directions. The electromagnet M_1 of one of the voltmeter elements is con-

nected in series with a non-inductive resistance R, and the electromagnet M_2 of the other voltmeter element is connected in series with a relatively high inductance L. The current through M_1 is thus independent of frequency while that through M_2 will vary inversely as the frequency, other conditions remaining constant. The coils are so adjusted that any change in voltage causes the torque due to the two electromagnets to vary in the same ratio. The indications of the instrument are thus independent of voltage variations, but depend solely upon variations in frequency. The aluminum disk D is shown as a circle. In

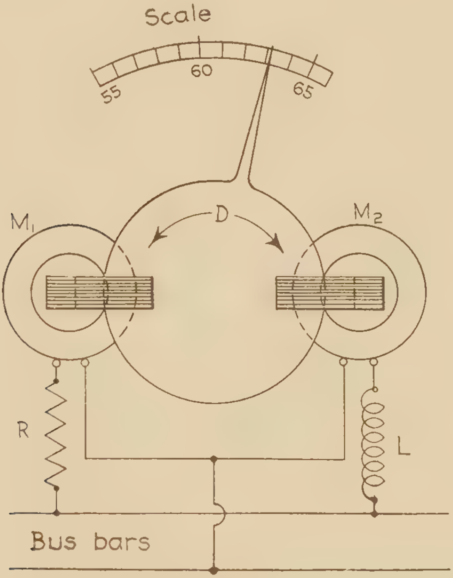


FIG. 104.

actual construction this is not the case. If the disk were a true circle any change in frequency would produce continuous rotation. The left-hand edge of the disk which moves under M_1 is practically the arc of a circle whose center coincides with the shaft. The right-hand edge, which moves under M_2 is practically the arc of a circle whose center is slightly above the shaft. With this arrangement the amount of metal in the air gap of electromagnet M_1 is practically constant, while the amount of metal in the air gap of electromagnet M_2 varies with the position of the disk. When the frequency decreases, electro-

magnet M_2 becomes stronger than M_1 and the disk turns counter clockwise. The part of the disk in the air gap of M_2 decreases until the torques of the two electromagnets balance, when the disk stops; when frequency increases, the torque of M_2 is decreased and the disk rotates clockwise; a greater part of the disk gradually enters the air gap until the two torques are again balanced. For every frequency, there is a definite point at which the meter comes to rest. The exact shape of disk is obtained by experiment; such an arrangement avoids the necessity of controlling springs.

This type of instrument is very popular in this country and is well adapted for switchboard use.

132. Weston Frequency Meter.—The frequency meter of the Weston Electrical Instrument Company operates on somewhat

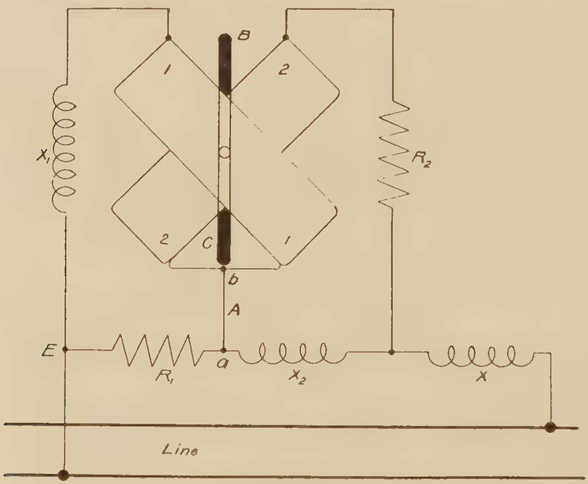


FIG. 105.

the same principle as the movable core type of ammeter and voltmeter. There is, however, this difference; in the ammeter and voltmeter the magnetic field varies in intensity but not in direction, while in the frequency meter the field remains constant so long as pressure is constant, but its direction varies with frequency. The direction of the field is determined by the ratio of the currents in two coils which are mounted at right angles to each other. The relative intensities of the currents are determined by an ingenious arrangement of inductance and resistance coils.

Fig. 105 is a diagram of the internal connections of the instrument. The two fixed coils are marked (1–1) and (2–2) respectively. As is plainly evident from the diagram, coil (1–1) is connected in series with reactance coil X_1 and in parallel with resistance coil R_1; and coil (2–2) is connected in series with resistance coil R_2 and reactance coil X_2. The two coils are also connected in series. At the center of two coils mounted on a shaft is the movable iron core BC. To the shaft is also attached the pointer. The actual construction of the instrument is shown in Fig. 106. At a particular frequency, the fall of potential along X_1 and coil (1–1), Fig. 105, to point b is the same as that across R_1

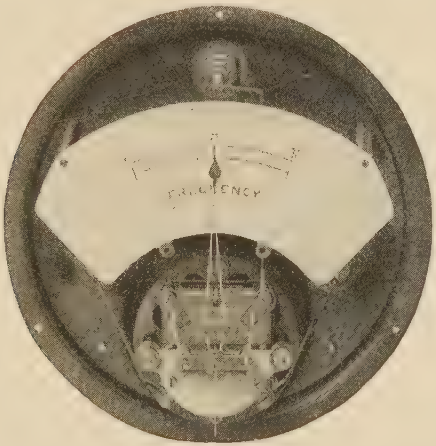


FIG. 106.

to a. Under these conditions the current through coil (1–1) is the same as that through (2–2) and in phase with it. The resultant magnetic field in this case will be parallel to CB in the diagram. This position of the magnetic field will remain fixed so long as the frequency remains constant. This is shown in Fig. 107, where H_1 represents the position and maximum value of field due to coil (1–1), and H_2 represents the maximum field due to coil (2–2). Since the intensities of the two fields change together, the resultant field will be represented by H, both in magnitude and direction. The resultant magnetic field will thus coincide in direction with OB, and only its intensity will change when voltage alone changes. The position of the soft iron core is determined by the position of the resultant magnetic field.

Any change in frequency will change the ratio of the currents in the two fixed coils. For instance a higher frequency will decrease the current through X_1 and also through X_2. Part of the current through R_1 under this condition passes through A and through coil (2–2) in addition to that which passes through coil (1–1). The magnetic field of coil (2–2) is thus stronger than that of coil (1–1) and the resultant magnetic field is shifted in the direction of H_2, Fig. 107. Thus every change in frequency is accompanied by a shifting of the space position of the resultant field, and this shifting causes a deflection of the pointer.

133. Synchronizing Devices.—Although instruments that indicate whether two generators are in synchronism are not properly meters, nevertheless their practical importance justifies a discussion of them in this text.

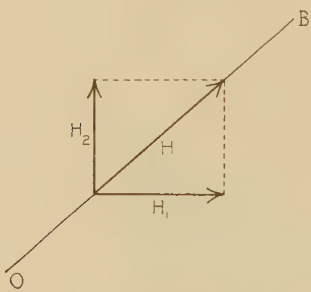


FIG. 107.

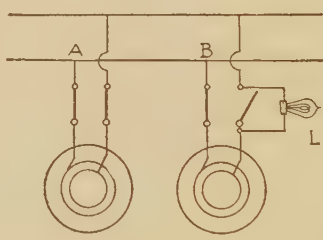


FIG. 108.

When two alternators, or two synchronous motors, are to be operated in parallel, some device is necessary to show whether the two machines are in synchronism, that is, whether at the same frequency, and whether the terminals of the separate machines are positive and negative together. The simplest form of a device for this purpose is an incandescent lamp connected across the contacts of a single pole switch as shown in Fig. 108. When the points A and B are at the same potential, no current will flow through the lamp L and, consequently, it will not light up. In order that this condition be fulfilled the electromotive force of one generator must equal the electromotive force of the other generator and the two electromotive forces must be in phase. Owing to the fact that it requires an appreciable difference of potential to cause an incandescent lamp to light up, there is considerable indefiniteness in the use of such an indicator.

In well-appointed central stations the synchronizing lamps are rapidly being displaced by special devices called synchroscopes or synchronism indicators. An indicator of this type should perform three distinct functions, as follows:

1. It should indicate the difference in speed between the two generators to be synchronized.

2. It should indicate which machine is running the faster, and finally, the time of exact synchronism.

3. It should indicate phase difference when frequencies are equal. Modern synchronism indicators perform these functions well.

The principles of operation of synchronism indicators are practically the same as those of the power-factor meters already discussed. Thus, in Fig. 94, if the coils CC are wound with fine wire and connected to the terminals of one alternator while the two ends marked a and b are connected to the terminals of the other alternator, the pointer will indicate the phase difference between the electromotive forces of the two machines.

In practice, the stationary coils CC are connected to the line, or terminals of machine running, while the moving coil is connected to the generator to be synchronized. The resistance R is usually an incandescent lamp. The inductance L and resistance R are used for "splitting" the phase of the current through the rotating element so as to produce a revolving field.

The field through the stationary coils pulsates with a frequency equal to that of the "running" generator while the field in rotating coils, due to the incoming generator, revolves. If the frequencies of both machines are the same, there is a certain position of the armatures where no torque will be exerted upon it. If, however, the frequencies are different, the field of one set of coils is constantly changing its phase with reference to the other, and, consequently, there is a torque exerted upon the armature causing it to rotate. The speed of the armature is equal to the difference of the frequencies, the armature making one revolution for each complete cycle gained by one generator over the other. The direction of rotation will also depend upon the relative speeds of the two generators.

134. Weston Synchroscope.—A synchroscope, working on the foregoing principles, would evidently rotate in one direction if the incoming generator were too fast and in the opposite direction if too slow, and the rotation would be continuous unless some

retarding force were used. The Weston synchroscope uses spiral springs to counteract this motion. The movement of the pointer is thus limited. The connections of the various operating parts of this synchroscope are shown in the diagram of Fig. 109. The incoming machine is connected to terminals A and B, while the machine with which it is to be synchronized is connected to busbars at C and D. If the two machines are not running at the same frequency, the phase displacement will continuously shift and with it the torque on the movable coil will vary from zero to maximum in one direction back to zero and to maximum in the other direction. This variation in torque will cause the pointer to move back and forth over the dial. If

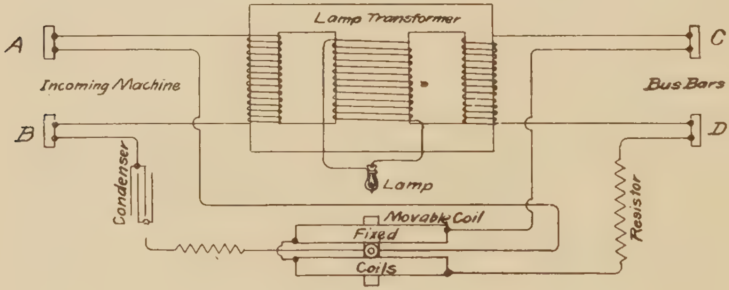


Fig. 109.

the machines have the same frequency, but are not in phase, the pointer will come to rest at one side or the other of the middle point of the scale, the position being determined by the average of the torque.

Remembering that the average torque is zero when the current in movable coil is 90° out of phase with the current in stationary coil, it is evident that the pointer will stand in a vertical position when the two machines are in synchronism; for at that time the current in the movable coil leads the other current by one-quarter of a period. This phase displacement is brought about by the condenser in the movable coil circuit.

When the two generator currents are not in phase, the currents in the movable and stationary coils will no longer be in quadrature. When this is the case, a torque will be exerted upon the movable coil causing a deflection. The direction of the deflection will be to the left when the incoming machine is slow and to the right when it is running too fast.

The synchronizing lamp which illuminates the dial is connected to the low voltage secondary of the transformer. An examination of the wiring of the winding on the transformer will show that when A and C are of the same polarity, the flux through secondary wiring is a maximum and lamp is brightest. That is, the lamp is brightest when the pointer indicates exact synchronism.

135. Westinghouse Synchroscope.—The essential principles of another form of soft-iron movable core synchroscope are shown in Fig. 110. As shown, the winding consists of three fixed coils M, N, and C.

The axis of the coil C coincides with the shaft to which the pointer is attached. Upon this shaft is mounted a cylindrical iron core

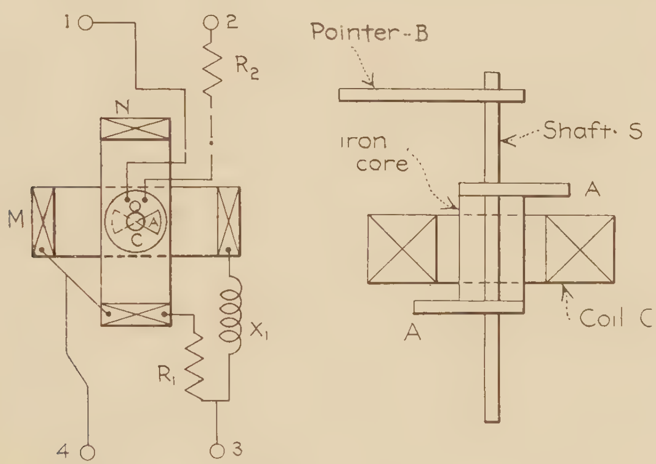


FIG. 110.

which carries two projections A–A. The other two coils have their axes 90° apart, but in the same plane; this plane, however, is at right angles to the shaft. The axes of the three coils thus correspond to the three rectangular axes of coördinate geometry.

In series with coil M is connected an inductive reactance, while in series with N is connected a non-inductive resistance. The two coils are connected in parallel across the busbars. The coil C is connected through a non-inductive resistance R_2 across the mains of generator to be synchronized.

Analysis of the principles involved will show that the principles of operation are nearly identical with those explained in Article 72. There it was shown that a movable core would rotate when sub-

jected to the influence of two coils which are mounted at right angles to each other and through which currents in quadrature flow. When alternating current flows through C the projections A–A are alternately positive and negative. If at the same time a current be flowing through coil N, the movable iron core will be deflected so that the projections A–A are parallel to the field of the coil N. If the frequencies of the two currents are equal, the current in coil C will reverse with that in N, and hence the movable core will remain stationary. If now, current in quadrature with that in C be passed through coil M, its average torque on armature will be zero, according to explanation of Article 125. When, however, the frequency or phase of current in coil C is not the same as that of current in coil N, the magnetic field in coil M will have some effect in causing a deflection. The demonstration for this is identical with that given in Article 125 concerning the electrodynamometer type of power-factor meter. That is, the pointer will come to rest when $\phi+\theta=0$, where ϕ is the deflection and θ the phase difference. This expression also shows that ϕ is constant so long as θ remains constant, and ϕ varies as θ varies. Thus, when the frequencies of the two machines are different, θ will be a varying quantity, and the pointer will rotate.

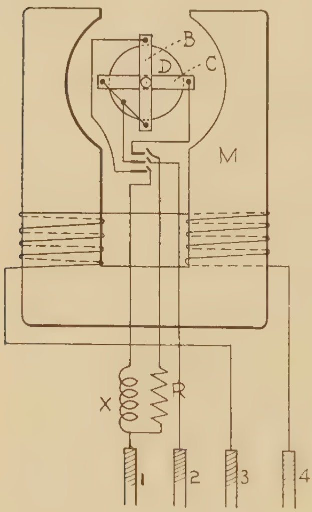


FIG. 111.

136. Lincoln Type Synchroscope.—The Westinghouse Electric and Manufacturing Company makes still another form of synchroscope known as the Lincoln type. A diagram of the internal connections of this type is given in Fig. 111. The essential parts of the Lincoln type of synchroscope are a bipolar laminated field M, the winding of which is connected to the busbars, and thence to the machine in operation. On iron core D, which is mounted on a shaft in such a way that it can rotate freely, are wound two coils, B and C, at right angles to each other. These two coils are connected in a "split-phase" relation through a non-inductive

resistance R, and an inductive reactance X to the incoming machine terminals.

The theory of the operating principles of this form of synchroscope is identical with that of the power-factor meter, Article 125, and hence need not be repeated. Since the relation between deflection of pointer and phase difference is given by $\phi+\theta=0$, it can easily be shown that the angular speed of the pointer is proportional to the difference in the frequencies of the two machines. If the two currents start in phase but have frequencies f_1 and f_2 after an interval of time t, they will be out of phase by

$$\begin{aligned}\theta &= 2\pi f_1 t - 2\pi f_2 t\\ &= 2\pi t(f_1-f_2)\end{aligned}$$

Hence $\phi = -2\pi t(f_1-f_2)$

or $\dfrac{\phi}{t} = \omega = -2\pi(f_1-f_2)$; $\dfrac{\phi}{t}$ or ω is the speed of rotation of the pointer.

CHAPTER XII

RECORDING OR GRAPHIC METERS

137. Introduction.—By recording meter is meant an instrument which makes a continuous record, on a properly ruled chart, showing the instantaneous values as well as fluctuations of a quantity whose magnitude changes with time. Quantities whose instantaneous values as well as fluctuations lend themselves to such a record are current, voltage, power, power-factor, and frequency, or in fact, any quantity whose instantaneous value may be given by an indicating instrument.

Nearly all electrically operated apparatus and machinery requires, for efficient operation, either constant potential or constant current. Thus the ordinary carbon filament lamp will change about 25 per cent in candle-power with a 5 per cent fluctuation of voltage. A knowledge of the fluctuations in electrical quantities is thus of great importance. This information is most readily obtained by the aid of recording, or graphic meters. These meters may be roughly divided into two general classes—direct acting, and relay.

138. Direct Acting.—It is very evident that from purely theoretical considerations a recording meter can be made by attaching a pencil or pen to the pointer of most of the indicating instruments so far discussed, and also attaching a properly graduated chart, moved by clockwork, upon which the pen or pencil can trace a line.

Simple as such an arrangement appears, it is by no means easily carried out in practice. The chief practical difficulty is the elimination of pen friction. The friction of the pen is considerable and, to overcome this, considerable force must be exerted by the movement. This necessitates an expenditure of additional energy, else the accuracy of the instrument is decreased. The pen must also contain a large quantity of ink, enough at least for several days' use. As the ink is used up, the weight at the end of the pen varies. The pen must be so designed as to be filled easily and the ink must not be spilled in the event of sudden movements of the pointer.

The main difficulties or drawbacks to most of these meters may be classed as follows:

1. The attention required in winding the clock, changing the paper, and filling the pen.

2. The inaccuracy in readings occasioned by the friction of the pen.

3. The increased consumption of energy to overcome the effect of friction.

4. On account of the friction and weight of pen, the instrument is liable to lack sensitiveness.

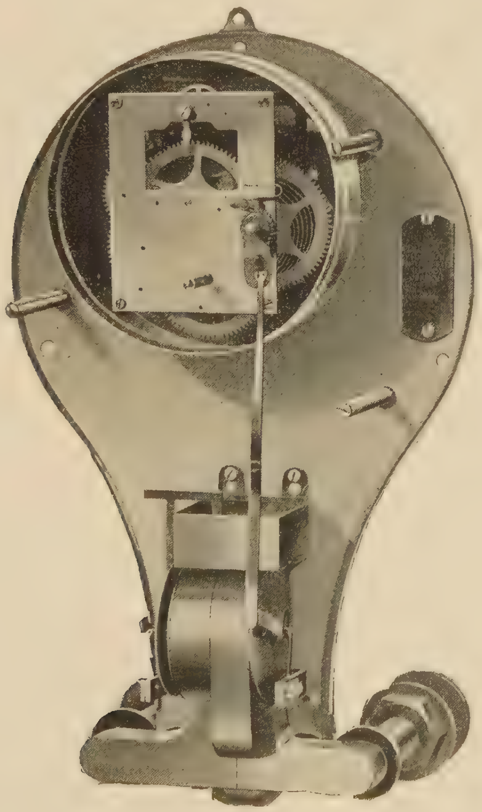


FIG. 112.

138A. Bristol Recording Instruments.—One of the oldest and simplest forms of direct acting graphic meters is that of the Bristol Company. The ammeters of this company make use

of the electromagnetic principle for their operation, while the voltmeters and wattmeters operate on the Kelvin balance or electrodynamometer principle. Fig. 112 shows the construction of a high-current capacity ammeter. The current coil is stationary, being attached to the back of the instrument. The moving element consists of a combination of disk armature

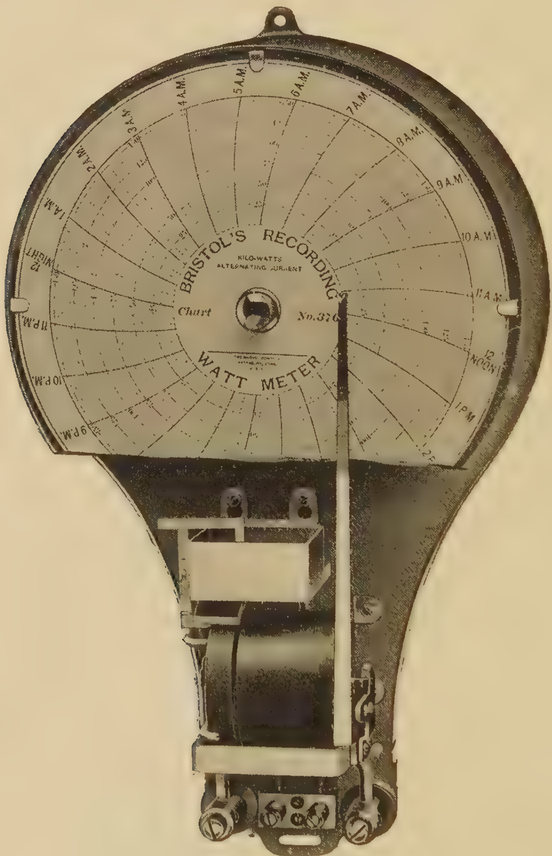


Fig. 113.

mounted on a non-magnetic shaft extending through the current solenoid. Both ends of the shaft are supported upon vertical steel springs.

When current flows through the current coil the armature is attracted toward the stationary coil. The motion of the armature, which is proportional to the current, is opposed by the

vertical knife-edge springs. The pen arm is attached directly to one of these springs. It is clear that the motion of the pen is many times that of the disk armature. In another form the pen is mounted on a knife edge below the axis of coil. The pen is actuated by the iron core as shown in Fig. 113. By means of such a device the motion of the pen is multiplied. There are no jewels, permanent magnets, make and break

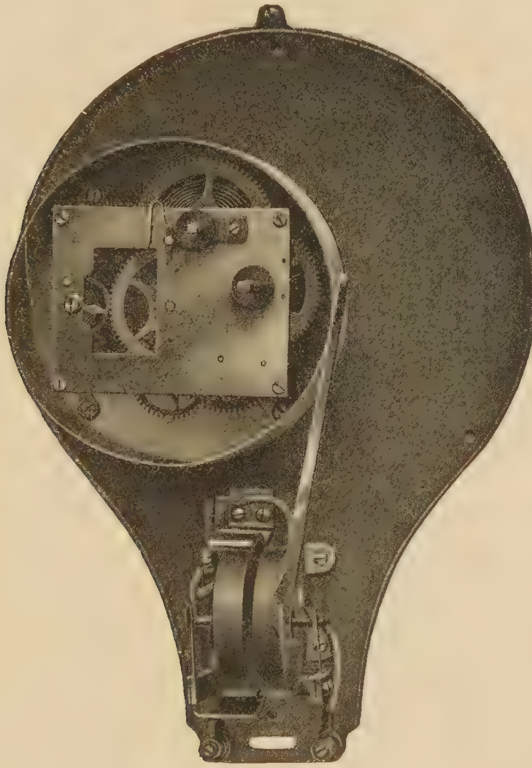


FIG. 114.

contacts, or spiral control springs. The sensitiveness of the meter is mainly determined by the friction of the pen on the paper.

Where current rapidly fluctuates it is advisable to have some damping device. In this particular case this is secured by attaching one end of an arm to the disk armature shown at left of coil and the other end to a vane which is submerged in oil in the box *F*.

Since the voltmeter and wattmeter both operate on the same principle an explanation of one will be sufficient.

Fig. 113 shows the construction of a wattmeter. The current coil is stationary, while the voltage coil, is mounted upon a shaft the ends of which rest upon knife edges of the spring supports. The terminals of the voltage coil are connected to the positive and negative conductors, and the magnetic effect of the current through this coil of high resistance will be dependent upon the voltage, while the magnetic effect of the main current through the current coil which is of low resistance, will depend upon the number of amperes passing. The mutual attraction of the coils will be the product of these magnetic forces and proportional to the number of watts. The marking arm is attached directly to the knife edge supports of the movable coil and partakes of its motion. One of the knife-edge supports is made with a double bearing. By this means the motion of the movable coil is multiplied, permitting the location of the voltage coil near the current coil. Such a construction makes it possible to use an evenly divided scale on alternating-current instruments as the magnetic field is quite constant over the short distance that the coil moves.

The construction of the voltmeter is in principle the same as that of the wattmeter. The voltage coil is divided, one part being rigidly attached to the meter frame, and the other part is mounted in the same way as the voltage coil of the wattmeter. The graduations on the dial are, of course, in volts instead of watts. Fig. 114 shows a Bristol Recording Voltmeter.

139. General Electric Recording Meters. Another form of the direct acting type of recording meters is that made by the General Electric Company. The appearance of a polyphase wattmeter with cover removed, is shown in Fig. 115. The movable element of an ammeter is shown in Fig. 116, by the aid of which the operation of the instrument will be most readily understood. The two fixed coils *AA* are connected in series with each other and with the line. The current in these coils sets up a magnetic field which acting on the iron armature *B* produces a turning moment on the shaft *D*. The consequent movement, which is opposed by the spiral springs *E*, is transmitted through pen arm supporting frame *G* to pen arm *H*. The resulting motion of the pen *K* traces on the chart *L* a curve whose distances from a zero line are proportional to the current. As

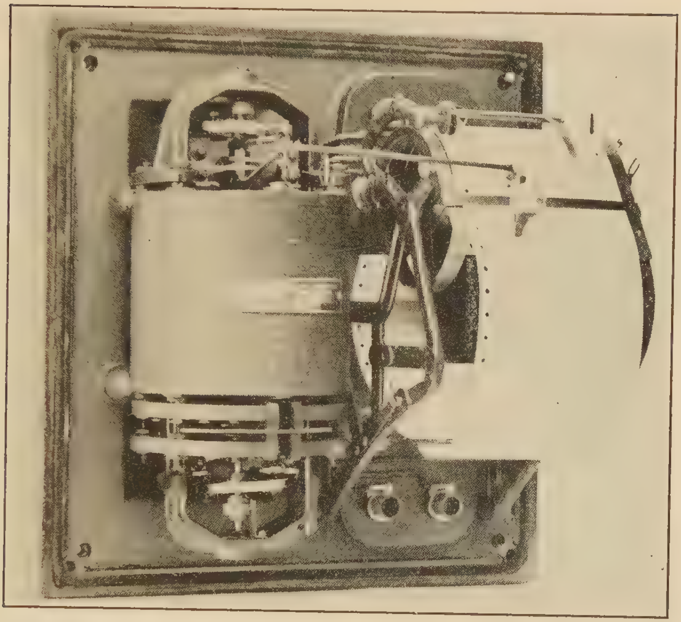


FIG. 115.

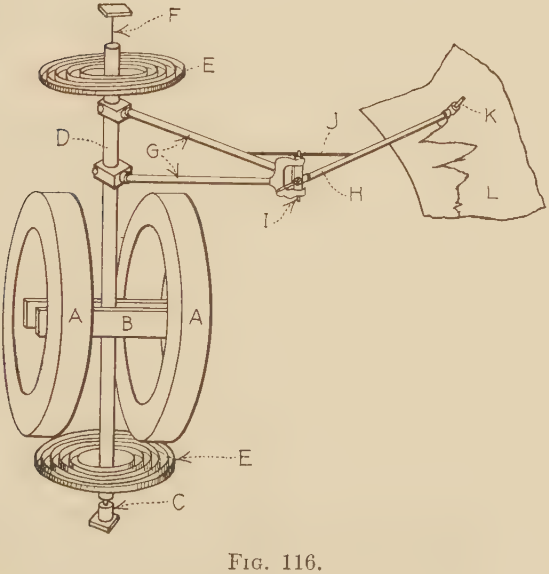


FIG. 116.

shown in Fig. 115, the motion of the pen is restricted to a straight line, and hence the chart may be ruled in rectangular coordinates, which is an advantage in many respects.

The movable element is suspended from the top support by means of a steel piano wire. The lower end of the shaft D, is accurately centered by a small steel pivot passing through a sapphire jewel. The friction due to supports is thus nearly eliminated.

The comparatively heavy weight of the movable element, including the pen, necessitates a strong torque to give sufficient sensitiveness.

Fig. 117.

The pen, a cut of which is shown in Fig. 117, holds enough ink to operate one week without refilling. The pen depends for its operation upon capillary action. The point consists of an iridium tube of very small bore, hard, durable, non-corrosive, and capable of receiving a high polish. This point is sealed into the end of a very small glass tube which in turn is placed inside a larger glass tube. The lower end of the small capillary tube is submerged in ink, which is carried to the point by capillary attraction.

The record is made on a band of specially ruled paper which is fed at the rate of 3 in. per hour by means of clockwork.

140. Damping.– To prevent undue swinging of the pen, its motion is damped by means of an aluminum disk rotating between the poles of permanent magnets.

141. General Electric Recording Voltmeters and Wattmeters.—In so far as the recording device is concerned, the voltmeters and wattmeters of the General Electric Company are practically identical with the ammeter.

The voltmeters and wattmeters operate on the dynamometer principle, employing fixed and movable coils. This principle has already been fully explained.

142. Relay Type of Recording Meters.—The objections enumerated at the beginning of this chapter against recording meters are, to some extent, eliminated in meters operated on the relay principle.

In this type of meter, the moving element of the meter proper operates merely a set of contacts, which close an auxiliary circuit. This auxiliary circuit energizes the solenoids which operate the pen. A comparatively large amount of energy is not objectionable in this case nor does friction in any way impair the accuracy and sensitiveness of the instrument.

The clock mechanism which moves the paper is made electrically self-winding and does not require attention. The recording pen is made to move across the paper in a straight line, and the record is obtained on a continuous sheet of paper ruled with rectangular coordinates.

143. Principles of Operation.—A complete set of recording instruments embodying these features is built by the Westinghouse Electric and Manufacturing Company. The voltmeters, alternating-current ammeters, wattmeters, and frequency meters, operate on the electrodynamometer or Kelvin balance principle. The operating principle of the power-factor meter is that of the magnetic vane or movable iron core. The measuring elements of the direct-current ammeters operate on the principle of the permanent magnet moving coil type. In order to diminish the influence of the earth's or other external magnetic field, two coils, astatically arranged, are pivoted within the magnetic fields of two permanent magnets.

144. Construction.—The Westinghouse Recording Voltmeter, with cover removed, is shown in Fig. 118. The electrically operated measuring element is shown in the upper part of the cut. The similarity between the Kelvin balance and this is at once evident.

A schematic diagrm of the connections is shown in Fig. 119, which shows quite clearly the manner in which the meter operates.

The fixed coils A, B, C, D, and the two movable coils E and F are connected in series in the same manner as those of the Kelvin balance. The movable coil E is provided with a relay contact J, located between the stationary relay contacts H and I of the solenoid circuits of the recording element.

The recording element comprises the pen-actuating solenoids K and L, their iron plungers K^1 and L^1, which are supported by the T-shaped lever arm M, pivoted at N; the pen arm O is connected to M by pin bearing P and provided at the upper end with a pin

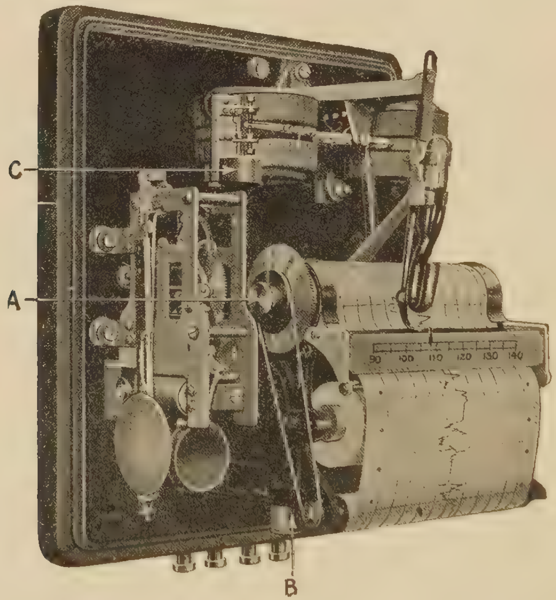


FIG. 118.

R, which moves in the stationary guide slot V; and the recording pen S, arranged to pass across a suitable record paper T moved by clockwork not shown in the diagram.

The control spring consists of the helical spring U, mechanically connecting the movable coil system of the meter element with the movable pivoted supporting arm M of the recording element.

The solenoid coils K and L are connected to the stationary relay contacts H and I, respectively, as shown, with their junction brought out to binding post No. 2. The contact J of the

movable coil system of the meter element is connected to binding post No. 1.

Leads from the control circuit are brought to binding posts Nos. 1 and 2, and leads from the circuit to be metered are brought to binding posts Nos. 3 and 4.

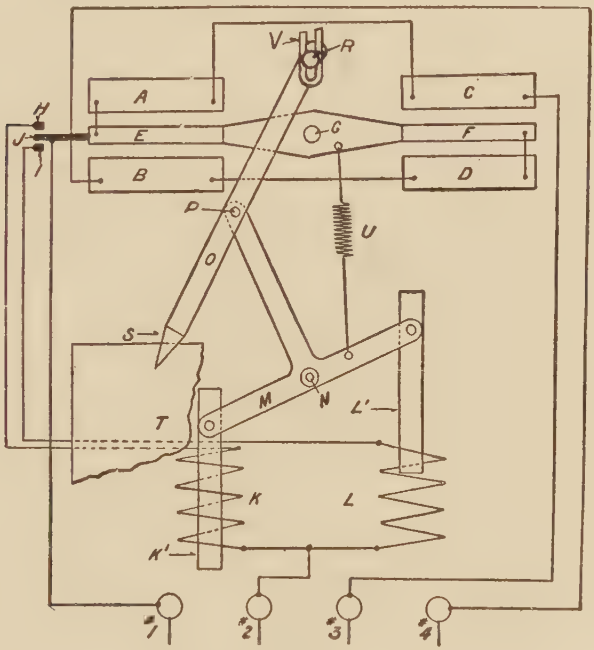


FIG. 119.

145. Operation.—The measuring coils are wound in such a direction that when the current in them increases, coils E and F are attracted by coils B and C and repelled by coils A and D respectively. This attraction will close the relay contact J on the solenoid terminal I, thus closing the recording circuit through solenoid L, energizing it and causing it to pull the plunger L^1 downward. The downward motion of L^1 rotates the T arm M about the pivot N. This movement of M moves the pen toward the right across the chart.

The downward motion of L^1 will continue until the tension of spring U is just sufficient to counteract the attraction and repulsion between fixed and movable coils. When the torque of the movable coil system is balanced by the controlling spring, the contacts I and J are pulled apart, opening the solenoid circuit.

The dimensions and weights of the various parts of the meter and the control spring are so proportioned that the entire moving system, including solenoids, pen-actuating arms, and measuring coils, remains stationary in the position occupied when the solenoid circuit was broken. In the meantime, the clock continues to move the record paper forward, thereby causing the stationary pen to draw a line lengthwise on the chart. This line represents the quantity which is being metered.

If the quantity being metered rises, the contact J is again forced down against the contact I, and the entire operation already described is repeated until the increased tension of the control spring U again balances the increased torque of the moving coils and opens the solenoid circuit. The recording system will then remain stationary until another change takes place in the current in the measuring coils.

Where the quantity measured decreases in value, the electrodynamic torque decreases and control spring U depresses the coil spring F bringing contact J against contact H. The diagram clearly shows that this operation opens the circuit of solenoid L, but closes the control circuit through K. The electromagnetic effect of current in K pulls K^1 downward, thus turning the supporting arm M to the left and causing the pen arm to move the pen toward zero or minimum scale value. This movement continues until the arm M has been sufficiently tilted to relieve the tension in the spring U, thus restoring the balance between the actuating forces of the meter element and the spring, causing the contact J to leave the contact H and breaking the circuit through the solenoid K. Thus any variations in the quantity measured cause the contact J to move up or down, making or breaking the circuit through either one or the other of the pen actuating solenoids. The corresponding oscillating motion of the pen, combined with the uniform motion of the clock-driven record paper, results in the drawing of a line, the distances of which from the zero line represent the magnitude of the quantity in the metered circuit.

146. Damping.– The motions of all the moving parts of the meter and recording elements are rendered dead beat by means of suitably arranged pistons working in glycerine dash pots. The movements of the solenoid plungers are damped by the action of pistons attached to their lower ends and working in dash pots located below and partly within the solenoid coils. One of these is shown at B, Fig. 118. The action of the pistons relieves the

plungers of excess momentum, thus preventing them from overshooting and hunting. The magnitude of this control can be readily varied by changing an adjustable opening in the washers located just below the pistons. Quick pen action is readily obtained by increasing the size of the opening, or by using a light grade of oil, while the use of a heavy grade of oil will give extreme slowness of action on badly fluctuating loads. In general it will be found most satisfactory to have the pen travel across the paper in from 15 to 20 seconds.

In all meters except power-factor meters, a piston working in the dash pot shown at *C*, Fig. 118, damps the motion of the movable coils of the meter element, thereby preventing the movable contact from vibrating against the stationary relay contacts.

147. Sensibility.—The sensibility of the meter may be readily controlled by varying the distance between the stationary relay contacts. With the contacts adjusted close together, the line drawn on a rapidly fluctuating load will be very irregular. A more regular curve can be obtained, however, by increasing the distance between the stationary contacts.

148. Westinghouse Recording Ammeters, Voltmeters, and Wattmeters.—The foregoing principles are applied to both A. C. and D. C. voltmeters and wattmeters and to A. C. ammeters. The only difference being in the character of the windings. The fixed and movable windings of the voltmeters and ammeters are connected in series. The voltmeter windings are of fine wire and those of the ammeters are of wire large enough to carry five amperes. The range of the instruments may be changed by the use of multipliers and shunts on D. C. circuits, and voltage and current transformers on A. C. circuits. The single-phase wattmeters have fixed coils identical with those of A. C. ammeters, and are operated from current transformers. The movable, or voltage coils, are wound with fine wire and connected in series with each other and in series with an external resistance. The D. C. wattmeters are similar to A. C. wattmeters, except that the current coils are designed to carry the total current. The D. C. ammeters differ in that no fixed coils are used. In place of fixed coils two permanent magnets are used. The movable coils and permanent magnets are arranged astatically.

149. Westinghouse Recording Frequency Meters.—These meters are of the same type of construction as voltmeters, except that the coils are wound differentially in two circuits, one circuit being

connected in series with a non-inductive resistance and the other with an inductive reactance. The two circuits are then connected in parallel across the line, so that any variation in the frequency will change the current in the inductive circuit, and hence the torque on the movable coil will change with the frequency. The recording element operates in the same manner as that of the other meters.

150. Westinghouse Recording Power-factor Meter.—The construction of the relay type of graphic power-factor meter is shown in Fig. 120. It is plain that the recording element is

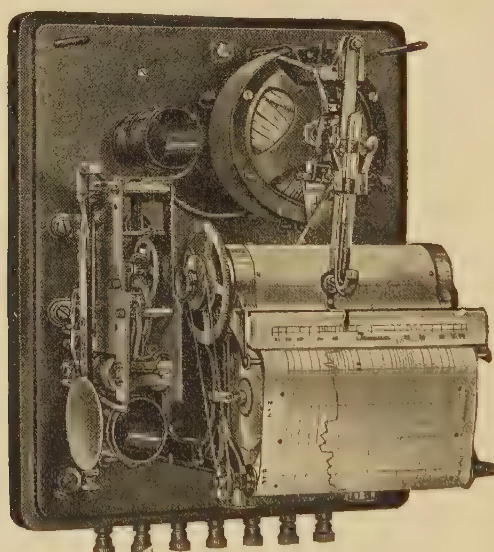


Fig. 120.

identical with that of the other meters. The meter element is the same as that of the Westinghouse indicating power-factor meter, explained in the previous chapter. The only novel feature is the manner in which the circuit through the controlling solenoids is closed and opened. This feature can readily be explained by reference to Fig. 121. To the shaft of the iron armature G is connected a light arm J which plays between contacts H and I on the arm O'. The contact arm J takes the place of the pointer on the indicating power-factor meter. No controlling springs are employed.

151. Operation.—The direction in which the light arm J moves is determined by the power-factor. If the phase difference between the voltage in coils A–B and current in coil C is such that J makes contact with I the recording circuit is closed through solenoid L. The resultant pull on plunger L' will move pen, pen-

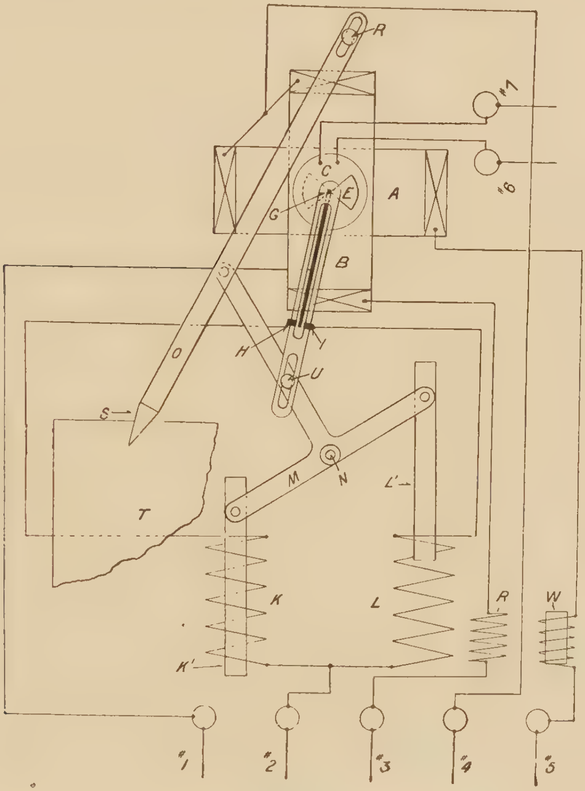


FIG. 121.

arm, and arm U to the right. This motion will continue until the arm J has moved a distance which on the indicating meter would represent the phase difference. The final position of pen S will then be determined by the power-factor, and any change in the power-factor will cause the position of the pen to change. The line traced will thus represent the power-factor.

152. Right Line Pen Movement.—To obtain accurate records on charts having rectangular ruling, the pen must move in a straight line at right angles to the motion of the paper. This

right line, or parallel motion, is obtained by making $PN = PS = PR$, Fig. 119. R is a pin rigidly attached to the arm O and sliding in slot V. With such an arrangement, the pen S moves in a straight line perpendicular to a line through R and N. In the older form of these instruments the pin R was fixed and the slot was in the pen arm O. The arm PR thus varied in length and the motion of the pen was only approximately a straight line.

The chief advantage of a recording meter lies in the fact that it makes a continuous record of the value of and variations in the electrical quantity. An examination of the record may thus be made at any time. This examination may be the means of detecting faults or disclosing characteristics which need improvement, and which would otherwise be overlooked.

The disadvantage in their use is lack of sensitiveness, which defect is due mainly to the weight and friction of pen. The objections to the form of pen used on the Bristol meter are the evaporation of the ink due to the exposure of a large surface, and the small capacity of the V-shaped trough. The effect of the pen friction, which is a variable quantity, impairs the accuracy of the record. The weight of the pens on the General Electric and Westinghouse meters makes accurate record impossible when the quantity fluctuates rapidly. Then, since the ink is fed by capillary action, the capillary tube will, sooner or later, become clogged, and it is cleaned with considerable difficulty.

Recording voltmeters are always more difficult to employ satisfactorily than the other instruments, because they are of very little use unless they are both sensitive to small changes of voltage and also remain in accurate calibration to within one volt or less.

The records of such instruments are often useful not merely for indicating the range of fluctuation of voltage in a distributing system, but also for indicating the nature of the fluctuations, as to suddenness, protractedness, and frequency. The cause of the fluctuations can often be determined from an examination of the charts with reference to these points of behavior, with reasonable expectation either of removing or of minimizing such as may be serious. The degree of damping is an important consideration in the operation of the recording instruments. They should be strictly aperiodic as far as possible, neither overshooting the mark on the one hand, nor undershooting and lacking in promptitude on the other.

CHAPTER XIII

INTEGRATING METERS, WATT-HOUR METERS

153. Introduction.—Integrating meters are instruments that register the sum of the electric quantity measured over a period of time. Thus if the intensity or strength of a quantity varies with time, the registration of an integrating meter will be proportional to the sum of the several products formed by multiplying together the quantity at a given time by the time during which it remained constant. For instance, if I_1, I_2, and I_3 are currents in a circuit for times T_1, T_2, and T_3 respectively, the integrating meter will register a quantity which is proportional to $I_1T_1+I_2T_2+I_3T_3$. It is thus clear that the element of time, as well as the electrical quantity determines the registration of the meter. In practice it is necessary to know the electrical energy and quantity of electricity that has been utilized, and accordingly we have watt-hour meters and ampere-hour meters.

154. Watt-hour Meters.—The definition of the unit of energy, the watt-hour, is given in Article 25. An instrument whose registration is proportional to the energy impressed or utilized, is called a watt-hour meter, often incorrectly called "recording wattmeter" or simply "wattmeter." It is gratifying to note that makers are beginning to recognize this confusion in names and some of them call the meter by its true name.

The distinction between a watt-hour meter and other meters such as wattmeters, both indicating and recording, is very clear, and the reader should keep this distinction in mind. The indication or registration of a watt-hour meter is determined by the energy that has passed in a given time, while the indication of a wattmeter is determined by the rate at which that energy is passing.

An analysis of the principles of operation of watt-hour meters will show that they may be classed as electrodynamometer and induction types. The former type is usually used only on direct-current circuits, although instruments of this type may be used on alternating-current circuits. The induction type can be used on alternating-current circuits only.

155. Electrodynamometer Type (without iron).—The diagram of Fig. 122 shows the essential features of this type of meter. The similarity between the essential parts of this type of meter and those of an electrodynamometer is very evident. The electrodynamometer contains fixed and movable coils. The watt-hour meter likewise contains fixed and movable coils. The stationary, or series winding, consists of two coils *FF* through which all or a proportional part of the line current passes. The movable coil, or armature *A*, consists of several coils of fine wire

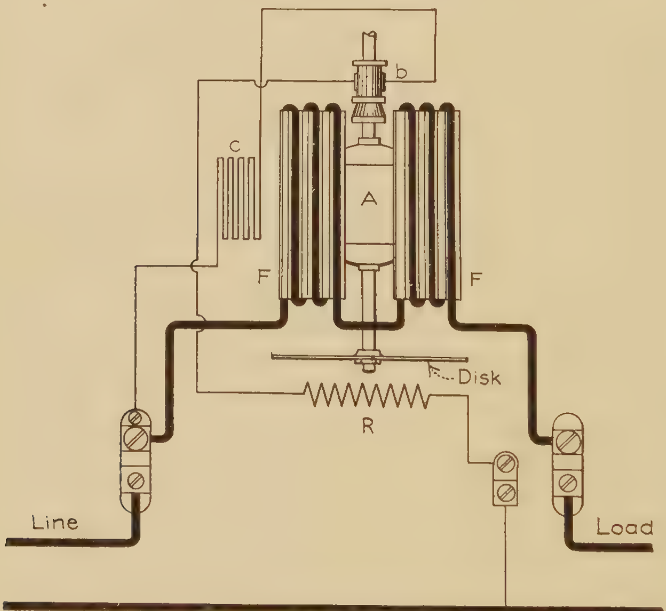


Fig. 122.

and is connected in shunt with the load through the resistance *R*, and compensating coil *C*, whose function will be explained later. The main difference between the electrodynamometer and this type of watt-hour meter consists in the permissible rotation of the movable coil. The motion of the movable coil of an electrodynamometer is opposed by a spiral spring, and the coil is thus restricted in its rotation. On the other hand, the movable coils of the watt-hour meter are free to rotate continuously. This is accomplished by mounting upon the shaft a commutator to which

the ends of the several coils are connected. Current is led into the armature coils by means of brushes which rest upon the commutator. For this reason this type of meter is usually called the commutating type to distinguish it from another form which does not require a commutator. The movable system is then mounted between supports, the ends of the shaft resting on jewels. On account of the manner in which it is connected to the circuit, this type of watt-hour meter is sometimes compared with the shunt motor. This similarity is very evident. There is one distinction, however, and that is the fact that neither the field nor armature of the watt-hour meter contains iron. It has been shown that the attraction between the stationary and movable coils of the electrodynamometer is proportional to the product of the currents in the two coils. The torque causing the deflection is thus proportional to the product of these currents. The torque on the armature of a watt-hour meter is likewise proportional to the product of currents in armature and field coils. The current in the armature is proportional to the voltage across load terminals, and the current through field coils is equal, or proportional, to the load current; hence, the torque on the armature is proportional to the product of the load voltage and current. That is, the torque is proportional to power.

156. Counter-torque.—In order that the driving torque may remain proportional to power, there must be present a counter-torque whose value increases and decreases with the load. Such a counter-torque is obtained by mounting upon the armature shaft a disk of aluminum which rotates between the poles of two permanent magnets. These magnets and the disk are shown in Fig. 123, which is a view of the Westinghouse direct-current watt-hour meter. The development of the retarding torque and its relation to driving torque is as follows: The magnetic flux between the poles of the permanent magnets is constant, and hence the eddy currents induced in the disk, as it rotates, are proportional to the speed of the disk. The counter-torque is proportional to the product of the eddy currents and magnetic flux between the magnet poles. Since the currents are proportional to the speed, the counter-torque must be proportional to the speed. The counter-torque thus increases and decreases with the speed, that is, with direct torque on the armature. When the load increases, the speed increases until the counter-torque just balances the torque on the armature. When the load

decreases, the speed decreases until the two torques are again equal. Thus, neglecting friction, the speed of the armature is proportional to the load, and the meter should register correctly at all loads.

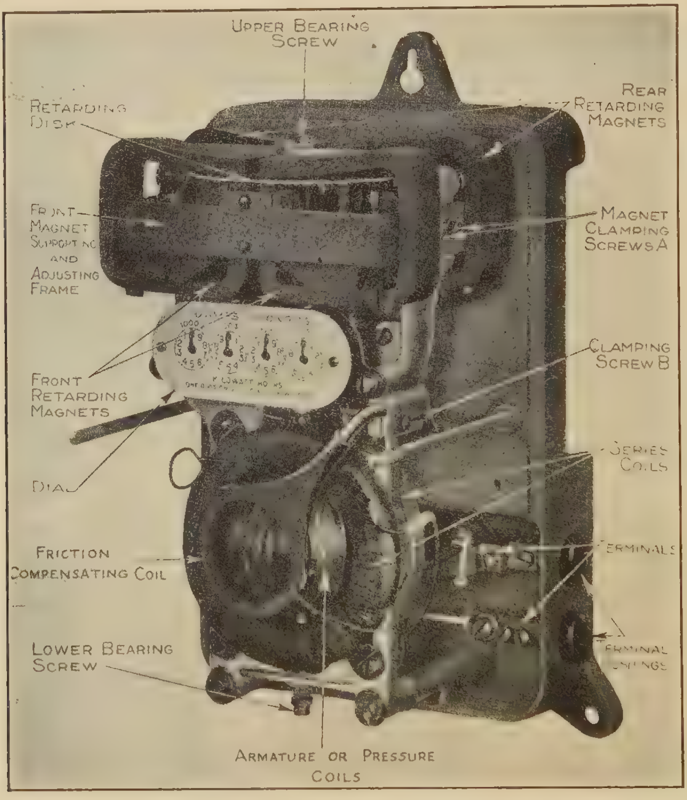


Fig. 123.

157. Summation of Power.—The torque acting upon the rotating element at each instant is proportional to the power being consumed by the load at that instant. For simplicity, assume the load to consist of a fixed number of incandescent lamps, and that the voltage E is constant. Under these conditions, the power will be constant and equal to IE. The driving torque is likewise constant and equal to KIE. Since the counter-torque increases with the speed, the driving torque will increase the speed until the two torques just balance each other. When

this condition is reached, the speed remains constant, that is, the disk makes the same number of rotations each minute. Under these conditions the number of rotations of disk in a given time is strictly proportional to the time. We may thus write

$$\text{Torque} \times \text{time} = \text{work}$$

but
$$\text{Torque} \times \text{time} = KIE \times \text{time}.$$

It has already been shown that $IE \times$ time is electrical energy, hence the torque $\times$ time is proportional to electrical energy pass-

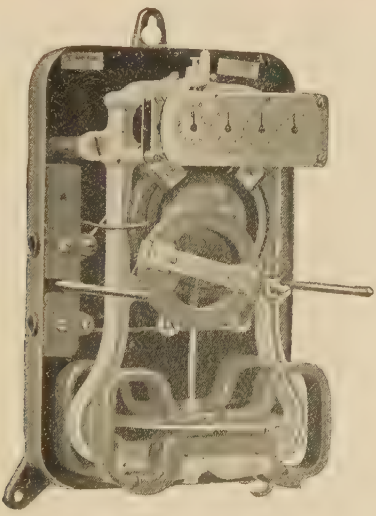


FIG. 124.

ing through the meter. The product of torque by time evidently determines the total number of rotations of disk, hence

$$Kn = \text{electrical energy.}$$

The total number of rotations n of disk is then a measure of the energy transmitted to the metered circuit. The number of rotations of the disk or armature is transmitted through a suitable train of gears to the dials. The operation of the registering mechanism is quite simple and needs little explanation. The upper end of the shaft is finished with a worm, or small gear wheel, the teeth of which mesh with the first of a train of gears. The number of teeth on the gears is such that when the one operating the pointer on the first dial has made ten revolutions, the one operating the second has made only one revolution, etc.

The dials are graduated in units of electrical energy such as the watt-hour or the kilowatt-hour.

The speed of the meter is usually adjusted so as to make the meter direct reading. This is taken care of in the design of the instrument and by adjusting the position of the permanent magnets.

The general principles here explained are applied in the Westinghouse, General Electric and Duncan, Columbia, and other direct-

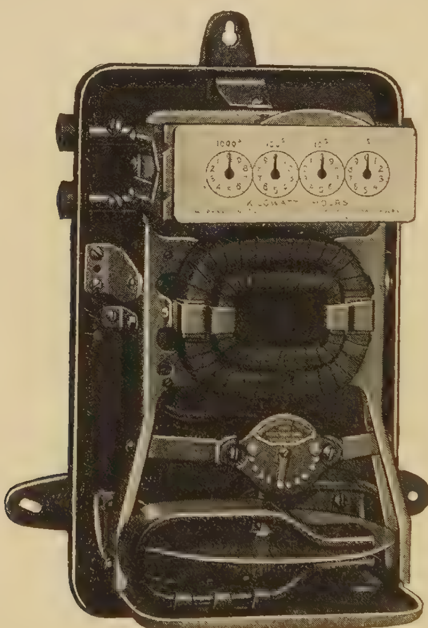


Fig. 125.

current meters, interior views of which are shown in Figs. 123, 124, 125, and 126.

The series watt-hour meter is not well adapted for measuring energy on circuits carrying heavy currents. The main difficulty lies in the construction of the series coils. To meet these difficulties, several watt-hour meters of the shunted type have been placed on the market within the past few years. One of the latest forms of the shunted type is shown in Figs. 127 and 128. The construction of the meter is in general very similar to the series form. The field consists of four comparatively large coils of large conductors surrounding a rather elongated armature.

The conductors have to be very large in order that their resistance may be low enough to permit sufficient current to flow with only a small voltage drop across the shunt. Furthermore, in order that the magnetic field may be strong enough to give a relatively high torque, the field coils must contain many turns.

158. Electrodynamometer Type (with iron).—There has lately been placed on the market a watt-hour meter which differs in some respects from those already discussed. This is made by the Columbia Meter Company.

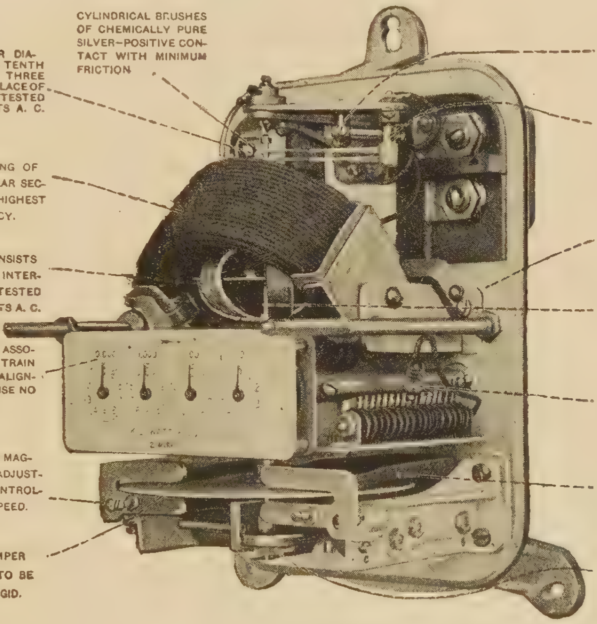


FIG. 126.

The main difference between this form of Columbia direct-current watt-hour meter and other D. C. watt-hour meters lies in the design of armature or rotating element. This difference will be brought out more clearly by reference to Fig. 129, which shows the rotating element of the Columbia instrument.

The armature windings, as shown, are a group of six cylindrical coils arranged between two aluminum disks, close to the central shaft and parallel to it.

Within each coil is a thin strip of silicon steel whose ends are bent at right angles to the axis of the coil, and extend radially

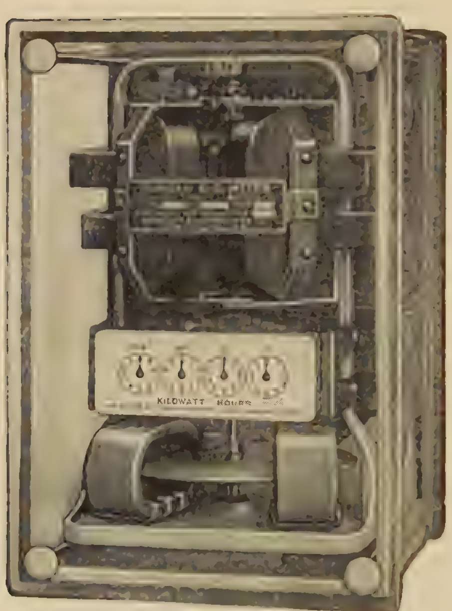


Fig. 127.

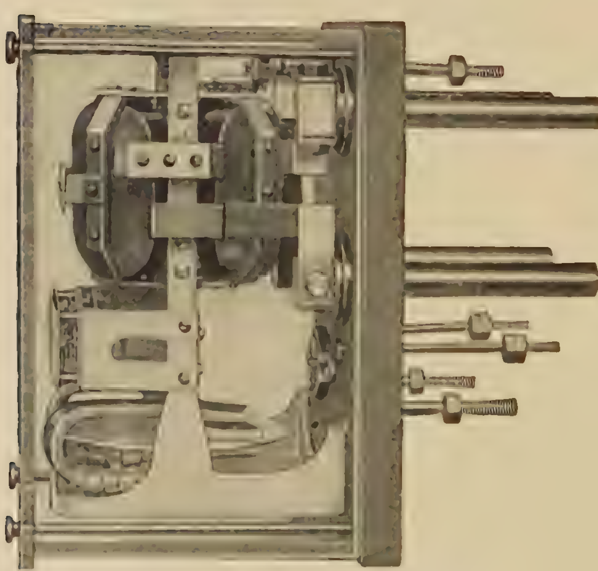


Fig. 128.

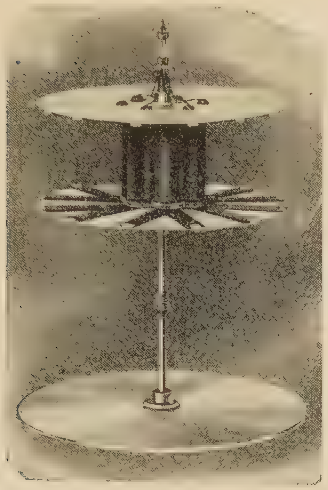


FIG. 129.

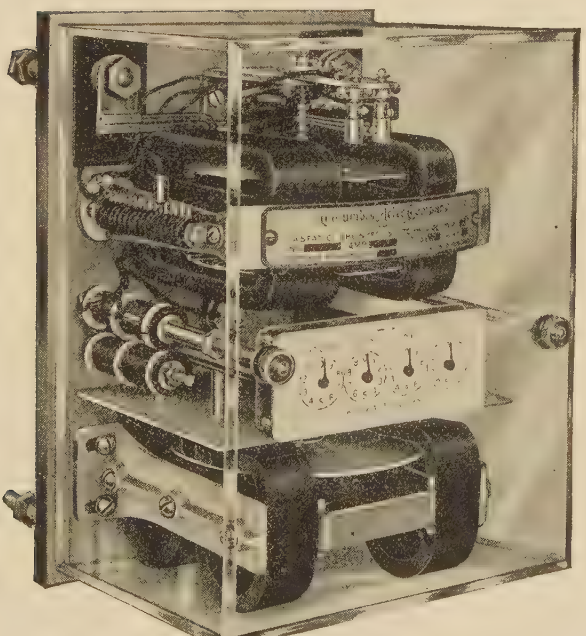


FIG. 130.

along the lower surface of the upper, and upper surface of the lower disk. These radial extensions are split so as to distribute the flux more uniformly around the circumference of the disk. The series winding consists of four coils arranged astatically. That is, the coils are connected in such a way as to cause the magnetic flux to flow through each element of a pair in opposite directions. By such an arrangement, the influence of a stray field on one coil is neutralized by its influence on the other coil of the pair. The positions of the various parts of the meter are shown in Fig. 130.

Another characteristic difference between the Columbia and other D. C. watt-hour meters is the use of shunts, which use is made possible by employing iron in the armature. The use of iron in the armature makes it possible to secure sufficient torque to operate the meter with a much smaller field current. Accordingly, they adjust all their meters so as to take exactly five amperes in the current coils at full-load.

159. Friction Compensation.—No matter how carefully the meter is constructed, all friction cannot be eliminated. Some energy is thus always required for the operation of the meter. In a well-designed meter this amount of energy is very small, yet if some means are not provided for overcoming this frictional torque, the meter will not register on a very light load. With the introduction of high efficiency incandescent lamps, the necessity for accurate compensation is much greater than previously. Nevertheless, the compensating torque should not be so great as to overcome excessive or unnecessary friction. It is usual to design the coil so that on about 5 per cent of full-load the maximum possible compensating torque will give an excessive speed of about 10 per cent. If the frictional torque is greater than this, the cause should be discovered and removed.

The compensating torque is obtained by connecting a coil in series with the armature or voltage coil, the plane of the coil being parallel to the current, or series coil. Such a coil is shown at C, Fig. 122. The strength of the compensating torque can be adjusted in either one of two ways. In the General Electric and Westinghouse watt-hour meters the position of the compensating coil with reference to the armature is changed until the proper degree of compensation is secured. The arrangement of the compensating coil of the Westinghouse meter is shown in Fig. 123, where it is called friction compensation. By releasing

the clamping screw B, the arm supporting the coil is released and may be moved up or down, nearer to or farther from the armature, thus changing the torque it exerts.

The Duncan Electric Manufacturing Company uses a somewhat different method of varying the torque. In the Duncan meter the compensating coil is firmly fixed within the front series coil, and the intensity of the compensating field is varied by changing the number of active turns on the coil. This is accomplished by moving a small contact lever either to the right or left, as the case demands, over multipoint contacts. Fig. 131 shows this compensating coil with the lever at its middle position.

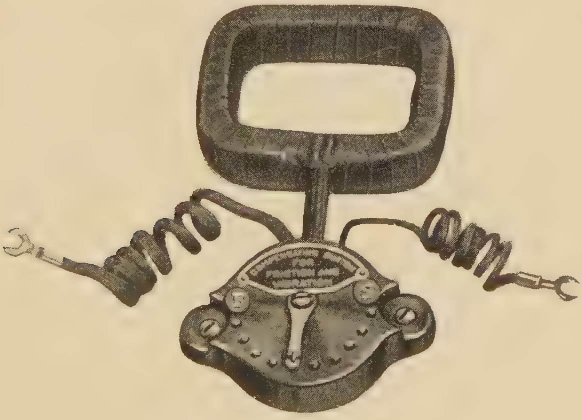


FIG. 131.

The Columbia Meter Company uses in principle a like method. The only difference is that the proper adjustment is obtained by changing the position of the hard-rubber plug with its enclosed brass spring bushing along the projecting terminals of a series of small resistance coils visible to the left of the dial-plate, Fig. 130. Changing the position of the plug changes the number of active turns and, hence, the compensating torque.

160. Creeping.—The armature circuit, when the meter is in service, is connected to the mains all the time. Thus, there is a current at all times through the armature. When the field due to the compensating coil is such as to furnish torque just sufficient to overcome the friction when the meter is installed where there is no vibration, it may creep when installed in a place subject to jar. The jarring reduces the friction of the bearings, and at the

same time the armature is partially relieved of its weight while the vibration lasts. Under these circumstances the inital torque may be sufficient to cause the meter to revolve slowly.

Another cause of a meter's creeping may be due to a higher voltage than that for which the meter was adjusted. If the compensating torque on a certain voltage is nearly great enough to cause the meter to register on no-load, an increase in voltage will increase the armature current and, since the compensating coil is in series with the armature, the compensating current will increase. The compensating torque, which is proportional to the product of the armature current and compensating current, will also increase. The two currents being the same, the compensating torque is then proportional to the square of the armature current. We may write this

$$T = KI^2$$

but

$$I = \frac{E}{R}$$

where E is the electromotive force between mains and R the resistance of armature circuit, including compensating coil. Then

$$I^2 = \frac{E^2}{R^2}$$ and substituting for I^2, we get

$$T = \frac{K}{R^2} \times E^2$$

K and R both being constant, the expression shows that the compensating torque is proportional to the square of the voltage between mains. It is thus evident that a compensated meter correct on light loads will register on no-load when the voltage is raised. The average conditions in practice are met by adjusting the meter so that on light load it is from 2 to 5 per cent slow.

161. Brushes.—Some of the chief objections to the commutator watt-hour meter are friction of brushes, sparking at the brushes when the commutator becomes oily or dirty, a change in speed due to improper position of brushes, and additional weight of moving parts due to commutator. The use of a commutator thus introduces difficulties which cannot be wholly eliminated, but their effects can only be minimized by careful design and construction. Brushes must therefore be made out of material whose elastic properties do not change with time. To meet this and other requirements, it is common practice to make the

stems of the brushes out of phosphor bronze wire or strips, and to provide the contact ends with silver tips. It has been found that brush friction is considerably reduced by making the tips round instead of flat.

The pressure of the brushes upon the commutator is governed by either the tension of a spring or the force of gravity. Where spring control is used, the elasticity of the stem of the brush supplies the necessary tension. A typical device of this kind is shown in Fig. 132, an examination of which will give an understanding of its operation.

FIG. 132.

In the gravity method of control, uniform pressure is secured by attaching the brushes to one end of an arm which is pivoted and carries a counter-weight at the other end. The distance of the counter-weight from the fulcrum may be changed, thus changing the tension.

162. The Commutator.—In order that the frictional torque may be reduced to a minimum, the commutator must be of very small diameter. The smallest diameter that can be successfully used on a 110- to 220-volt meter is about 1/10 in. For higher voltages the diameter of the commutator must be greater to permit of proper insulation.

The commutator is usually made by forcing a piece of silver tubing over a fiber bushing on the shaft. The tube is then sawed into the proper number of segments which are held in place by fiber or metal rings. The metal rings are, of course, insulated from the segments. It is customary to use silver for both the commutator segments and brush tips, because it is the cheapest metal that can be used which does not readily oxidize. The commutator of the Duncan shunted type is made of gold. Some makers use fiber to insulate the segments from each other, while others leave merely an air space.

163. Armature.—The distinctive characteristics of some meter armatures have already been briefly pointed out. The armatures

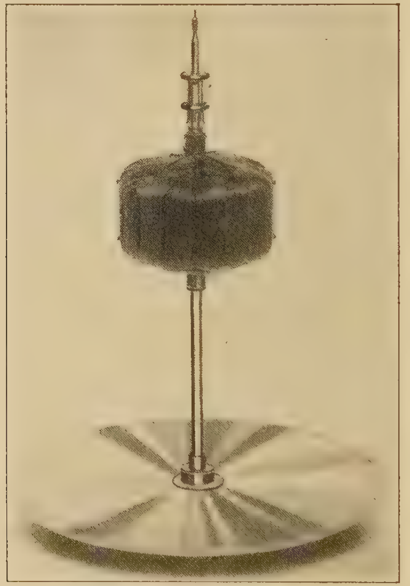


FIG. 133.

of watt-hour meters without iron in their magnetic circuits are mainly of two forms, spherical and cylindrical. The spherical form permits of a more compact construction, thus minimizing the magnetic leakage and correspondingly increasing the torque; or, what amounts to the same thing, securing maximum torque with a given weight of armature and given energy consumption in armature and field. For the cylindrical form, the advantage

is claimed that it can be repaired much more readily. The cylindrical form is shown in Fig. 133.

The windings of both forms of armature are of the drum type. The coils of the spherical armature are usually wound upon a light fiber shell which is mounted directly upon the shaft, the coils being held in place by grooves pressed in the shell. The supports for the cylindrical armature are two light hard-wood spiders firmly fastened to the shaft. This permits of a light and open construction which is conducive to good ventilation.

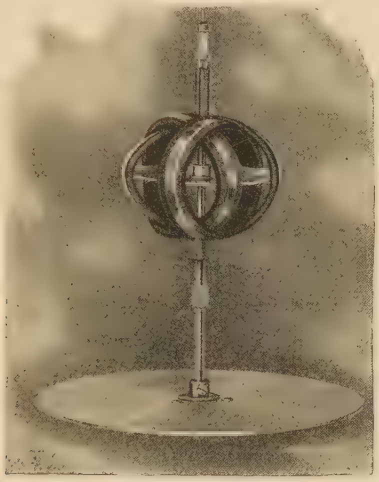


FIG. 134.

The coils of the armature are wound with wire of pure copper and of smallest gauge consistent with mechanical strength, usually not larger than No. 40 B. & S. gauge. Armatures wound for 110 to 220 volts have as a rule eight coils of 1000 turns each connected to the eight segments of the commutator. The Columbia meter, however, has only three coils which are connected to a three-segment commutator, Fig. 134. For voltages above 220 it is common practice to wind the armature with 16 coils, and the commutator has a corresponding number of segments. The main reason for this is to reduce the voltage between adjacent coils and commutator segments.

The resistance of the armature coils and auxiliary resistance is so adjusted that practically the same current flows in the arma-

ture of meters for different voltages. The armature resistance is practically the same in each case, but for the higher voltages additional resistance is placed in series.

164. Bearings.—The necessity for a very small frictional torque makes the proper design of the bearings of great importance. The function of the top bearing is merely to hold the movable element centered, and is therefore subject to very little pressure, and, consequently, very little friction. One form of top bearing is shown in Fig. 135a. It consists of a steel pin fastened to a removable screw and projecting down into a bushing in a

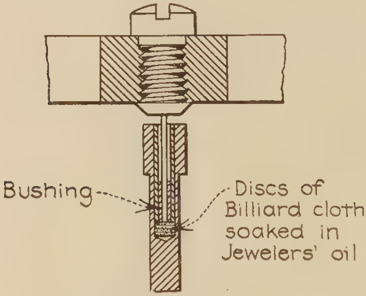


FIG. 135a.

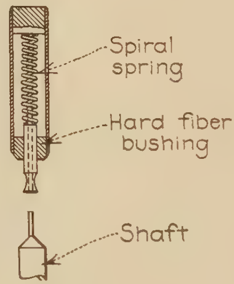


FIG. 135b.

recess drilled in the shaft. The bottom of this recess is filled with billiard cloth, saturated with watch oil. A film of oil is maintained around the pin by capillary action. In another form of bearing the conditions are reversed. The pin is a part of the shaft and the recessed bushing projects from above downward. This form is shown in Fig. 135b.

Owing to its importance, by far the most attention has been devoted to the design of the lower bearing, which has now reached a high degree of perfection. There are, in general, two forms of lower bearing. One form may be called pivot bearing and the other ball bearing. Details of a typical pivot bearing are shown in Fig. 136. The pivot is not an integral part of the shaft, but is made separately and screwed into the lower end of shaft or spindle. It will be observed that the bearing consists of a hollow screw with a helical spring. Surmounting the spring is a plug within the upper face of which is embedded the cupped jewel.

The ball bearing is shown in Fig. 137. The lower end of

the shaft, instead of ending in a conical pivot, has a cup-shaped jewel fixed in it. Another cup-shaped jewel is fixed in the upper face of the plug, and between these jewels is a

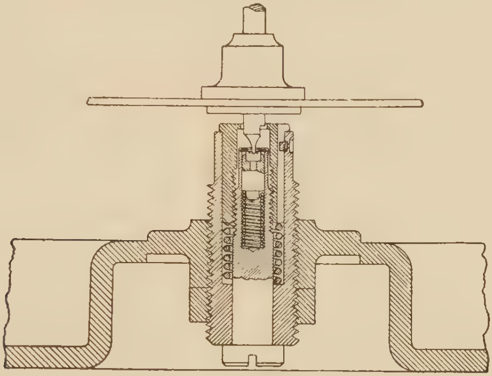


Fig. 136.

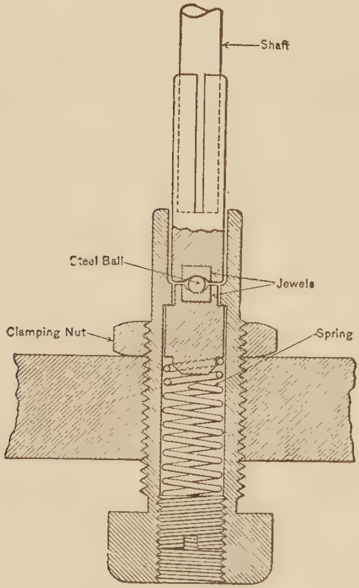


Fig. 137.

hardened steel ball. It is claimed that the ball bearing has the longer life. In so far as friction is concerned there is perhaps not much choice.

165. Jewels.—Until recently it has been almost universal practice to use sapphire for the jewels. Experience has demonstrated the inability of sapphire to withstand, for more than a comparatively short time, the grinding and hammering action of the pivot. The only material that is able to show permanently satisfactory performance is the diamond. The earliest diamond jewels were flat and required a stone ring to maintain the pivot in place, an arrangement which did not give complete satisfaction. Later experiments showed that it was possible to grind the diamonds in the form of a cup, and jewels of this form require no guiding ring. The cupped diamond jewels are now used extensively for meter bearings. Experiments performed on meters with sapphire and diamond jewels show that a higher accuracy on light load was maintained by the meters with diamond bearings.

166. Magnets.—A most important feature of the watt-hour meter is the permanent magnets, for upon their permanency depends to a great extent the performance of the meter. The necessity for magnets whose strength will remain constant can easily be appreciated when it is remembered that the retarding torque is proportional to the product of eddy currents in disk by magnetic flux. The eddy currents at any given speed of disk are, however, proportional to magnetic flux, hence retarding torque is proportional to the square of the magnetic flux, or in algebraic symbols,

$$T = K\Phi^2.$$

It is thus very evident that a small change in Φ will have an important effect in changing the counter-torque and, indirectly, the registration of the meter. In this respect the manufacturers of first-class instruments take special precautions, and strive to produce magnets that will retain their strength indefinitely.

167. Registering Mechanism.—A typical registering mechanism is shown in Fig. 138. This consists of dials, dial train, and reducing train. The dial and dial train of gears are not clearly shown; the reducing train, however, is. Great care is necessary in the manufacture of the various parts in order to eliminate all imperfections. The wheels are usually made of hard brass and gold-plated to prevent corrosion. The entire mechanism is aligned by dowel pins and attached to the frame by screws.

168. Electrodynamometer Type on Alternating Current Circuits.—Since the principles of operation of the commutator form of watt-hour meter without iron are the same as those of the electrodynamometer type wattmeter, most of the theory of the latter instrument as given in Chapter X will hold with reference to the watt-hour meter. In the discussion referred to, it was shown that the deflecting torque is proportional to the power when the voltage coil is non-inductive. This, however, is seldom the case and, hence, adjustments, termed lagging, must be made before the meter will register accurately on alternating-current circuits.

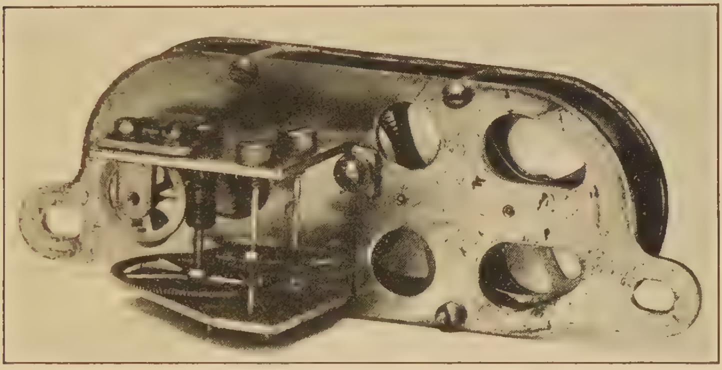


Fig. 138.

It was shown in Chapter X that a wattmeter with an inductive voltage circuit gave too high indications on circuits of low power-factor, and that on circuits where the current leads the pressure, the deflection under certain conditions would be negative. An electrodynamometer wattmeter which has not been lagged is subject to exactly the same inaccuracies, viz., on inductive load the registration will be too high, and on a load such as an over-excited synchronous motor, it may run backward. This will be the case where α is greater than $\cos\theta$, α and θ having the same significance as in Article 116.

169. Lagging.—In order to avoid the foregoing errors it is necessary to adjust or modify the series circuit of the meter in such a way that the angle between the series current vector and the voltage vector shall be exactly the same as that between the armature current vector and the voltage vector on non-inductive load. This is accomplished by shunting a part of the series current through

a non-inductive resistance. Under these conditions, the main current will divide inversely as the impedances of the two parallel circuits. Since the series winding is to some extent inductive, the current in this will lag behind the current through the non-inductive shunt. The main current will then be the vector sum of these two components, and its angle of lag will be determined by the ratio of these components. Thus in Fig. 139, let I_1 be the current through the series winding. Then if R_1 and X_1 are the series coil resistance and inductance respectively, R_1I_1 will be the resistance drop, and X_1I_1 the reactance drop. The series current will be in phase with R_1I_1; and the applied voltage and current through non-inductive shunt are in phase with OE. If OI_1 represents the series current vector, and OI_2 the shunt current vector; then OI is the resultant, or main current vector. The angle β by which the series circuit current lags behind OI can evidently be changed by changing the ratio of OI_1 and OI_2. These in turn depend upon the resistance of the shunt circuit. In a properly lagged meter this angle β is made equal to the angle of lag between applied voltage and voltage circuit current.

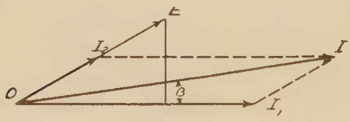


FIG. 139.

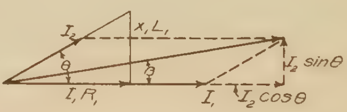


FIG. 140.

170. Value of Shunt Circuit Resistance.—The value of the shunt circuit resistance can be calculated if the resistances and inductances of the voltage and series circuits are known. Thus in Fig. 140 let I_1 be the series current, and I_2 the shunted current. R, R_1 and R_2 are the resistances of the voltage, series, and shunt circuits respectively, and if X, and X_1 are the reactances of the voltage and series circuits, we may find R_2 in terms of R, R_1, X, and X_1, as follows:

$$\tan \beta = \frac{I_2 \sin \theta}{I_1 + I_2 \cos \theta}$$

but $$\cos \theta = \frac{R_1}{(R_1^2 + X_1^2)^{\frac{1}{2}}}$$

and $$\sin \theta = \frac{X_1}{(R_1^2 + X_1^2)^{\frac{1}{2}}}$$

Then

$$\tan \beta = \frac{\dfrac{I_2 X_1}{(R_1^2+X_1^2)^{\frac{1}{2}}}}{I_1 + \dfrac{I_2 R_1}{(R_1^2+X_1^2)^{\frac{1}{2}}}}$$

$$= \frac{I_2 X_1}{I_1(R_1^2+X_1^2)^{\frac{1}{2}}+I_2 R_1}$$

The relation between I_1 and I_2 is given by

$$I_1 : I_2 :: R_2 : (R_1^2+X_1^2)^{\frac{1}{2}}$$

whence $$I_2 = \frac{I_1}{R_2}(R_1^2+X_1^2)^{\frac{1}{2}}$$

Substituting for I_2, we get

$$\tan \beta = \frac{\dfrac{I_1 X_1}{R_2}(R_1^2+X_1^2)^{\frac{1}{2}}}{I_1\ (R_1^2+X_1^2)^{\frac{1}{2}} + \dfrac{I_1 R_1}{R_2}(R_1^2+X_1^2)^{\frac{1}{2}}}$$

This reduces to

$$\tan \beta = \frac{X_1}{R_1+R_2}$$

But the angle of lag of current in voltage circuit is given by the relation

$$\tan \alpha = \frac{X}{R} \text{ and if } \alpha \text{ is to equal } \beta$$

$$\frac{X}{R} = \frac{X_1}{R_1+R_2}$$

Whence $$R_2 = \frac{RX_1 - XR_1}{X}$$

In practice, R is large, being about 1200 ohms, while R_1, X, and X_1 are comparatively small. Hence, it is evident that R_2 will be large and that only a small per cent of the line current will pass through the shunt. No account is taken of the capacity of the armature, since the inductance usually predominates. Likewise, the effect of mutual inductance of the coils is so small that in practice it is negligible.

171. Three-wire Direct-current Meters.—For measuring energy on three-wire direct-current circuits, one three-wire, or two two-wire meters may be used. A three-wire meter differs very little from a two-wire meter in construction, and in principle of operation, not at all. In meters intended for three-wire circuits the two series coils are distinct; the ends of each being brought out to terminals which, when in service, are connected to the load circuits as indicated in Fig. 141. The voltage coil may be connected across mains 1 and 3, or between either 1 or 3 and neutral 2.

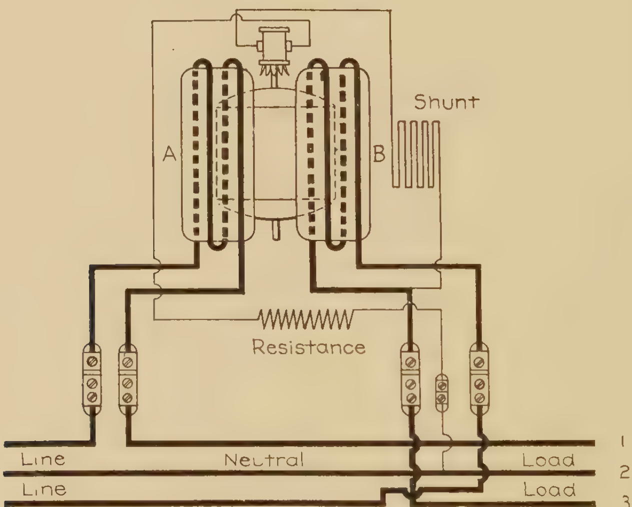


FIG. 141.

When the connection is as indicated in Fig. 141, the torque exerted upon the armature is equal to the sum of the torques exerted by coils A and B separately. When the voltage between mains 1 and 2 equals that between mains 2 and 3, the torque exerted by coil A is

$$T_1 = K_1 I_1 E$$

and the torque exerted by coil B is

$$T_2 = K_2 I_2 E$$

Since the two current coils for accurate registration should exert

the same torque under like conditions, $K_1 = K_2$ and the total torque equals

$$T = T_1 + T_2 = K(I_1 + I_2)E$$

which is evidently proportional to the load.

If, however, the load is unbalanced to such an extent that the voltages are no longer equal, then the torque ceases to have the same ratio to the load. Let E_1, E_2, I_1 and I_2 represent the voltages and currents on the two sides. The total load is then

$$W = E_1 I_1 + E_2 I_2$$

The pressure current is, however, due to E_2 only, hence the meter registration is equal to

$$W_1 = (I_1 + I_2)E_2$$

The difference between W and W_1 is the error. This is

$$\begin{aligned} W - W_1 &= E_1 I_1 + E_2 I_2 - E_2 I_1 - E_2 I_2 \\ &= I_1(E_1 - E_2) \end{aligned}$$

This is zero only when $E_1 = E_2$. When E_1 is greater than E_2, the meter is slow, and when E_2 is greater than E_1, the meter is fast. This error may or may not be appreciable, depending upon the degree of unbalancing.

When the voltage coil is connected across outside mains, that is, 1 and 3, Fig. 141, the error on unbalanced load will always be of the same sign. Under the previously assumed condition the load is again

$$W = E_1 I_1 + E_2 I_2$$

but the registration of meter is

$$W = \tfrac{1}{2}E(I_1 + I_2)$$

where E is voltage between mains 1 and 3.

But $$E = E_1 + E_2$$

hence $$W_1 = \tfrac{1}{2}(E_1 + E_2)(I_1 + I_2)$$

and error is

$$\begin{aligned} W - W_1 &= E_1 I_1 + E_2 I_2 - \tfrac{1}{2}E_1 I_1 - \tfrac{1}{2}E_1 I_2 - \tfrac{1}{2}E_2 I_1 - \tfrac{1}{2}E_2 I_2 \\ &= \tfrac{1}{2}(E_1 - E_2)(I_1 - I_2) \end{aligned}$$

which is zero only when $E_1 = E_2$ or $I_1 = I_2$. In general, on unbalanced load the meter will register too high, for when I_1 is greater than I_2, E_1 is less than E_2 and $W - W_1$ is negative; also when I_2 is greater than I_1, E_2 is less than E_1 and again $W - W_1$ is negative, that is, the registration is higher than the energy supplied.

It must also be noted that on unbalanced load the error is in general greater when voltage coil is connected between neutral and one outside wire than when it is connected between the two outside wires.

On circuits that are subject to considerable unbalancing it is preferable to use two-wire meters.

172. Mercury Watt-hour Meter.—The source, or cause of many commutating type watt-hour meter troubles is the commutator.

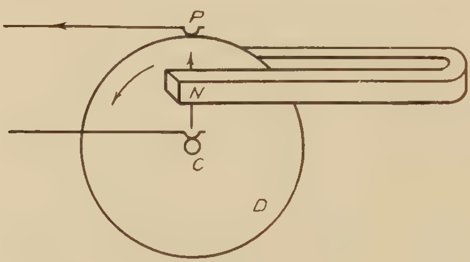


Fig. 142.

mutator. The chief objections to the commutator are: friction of brushes, sparking at brushes when commutator becomes oily or dirty, change in speed due to an improper position of the brushes and additional weight of moving element. Many attempts have been made to lessen the influence of these troubles, but the most radical procedure has been the development of a watt-hour meter which eliminates the commutator entirely.

The principle of operation of the mercury integrating meter was discovered in 1823 by Barlow. Fig. 142 is a diagram of the operating parts of Barlow's invention. As the reader will observe, this consisted of a copper disk D mounted on a horizontal axis so as to rotate freely between the poles of a permanent magnet. When current is passed into the disk through the axle and out at the circumference, the reaction between the current in disk and the permanent magnet field develops a torque, which causes the disk to rotate in the direction of the arrow head. The rotation is in such a direction as to carry the current out of the

magnetic field. This torque will vary with the current strength, for, as has already been shown, the torque is proportional to product of current strength and field. The field being constant, the torque must vary with the current.

The adaptation of this principle to integrating meters will be readily understood by reference to Figs. 143 and 144 which show the essential characteristics of the Sangamo direct-current watt-

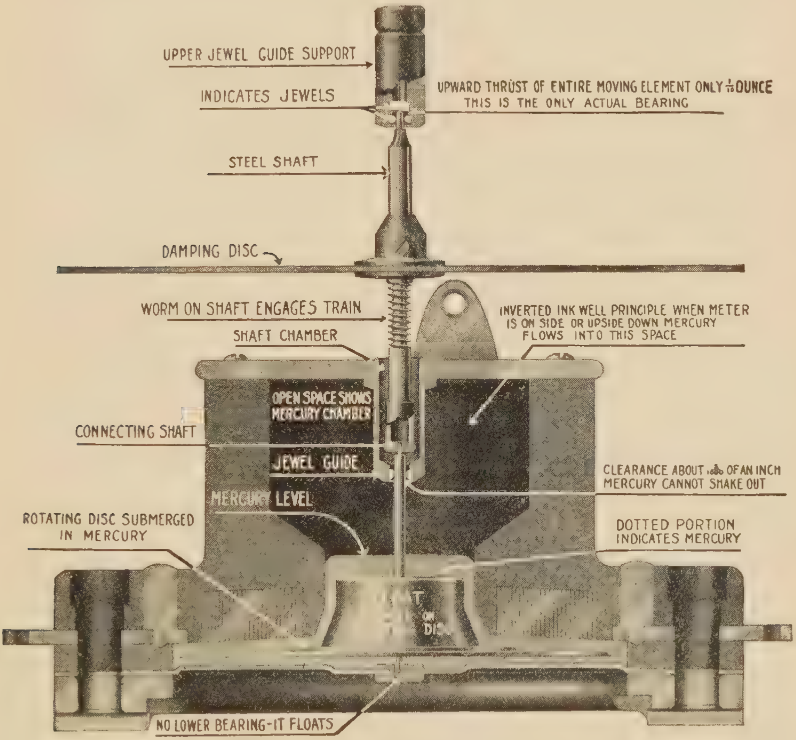


FIG. 143.

hour meter. Fig. 143 shows a cross-section of the motor element with the principal parts designated. The rotating element, or armature, is the copper disk submerged in the mercury. Surmounting the disk is a hardwood float whose weight is adjusted so that the buoyant effect of the mercury will relieve the lower bearing of the weight of the entire moving system. By careful adjustment the downward pressure has been entirely eliminated and a small upward thrust has been produced.

The mercury chamber of the meter is made of insulating material into which have been imbedded two nickel-plated copper terminals and a laminated steel ring. The copper terminals serve to lead current into and out of the mercury chamber, and the steel ring reduces the reluctance of the magnetic circuit, thus compelling the magnetic lines to pass through the armature disk.

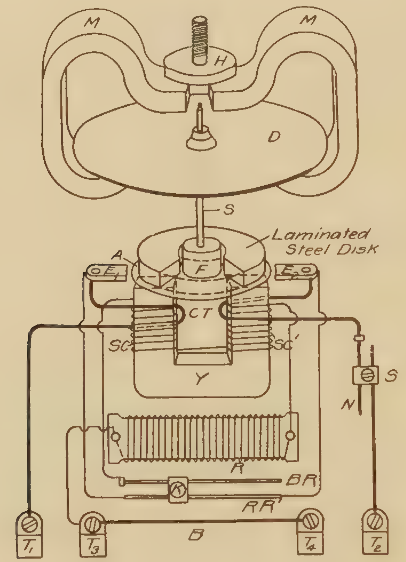


FIG. 144.

173. Operation.—A simplified diagram of the connections and wiring of the meter is shown in Fig. 144. The current enters the meter at the terminal T_1, passes through the heavy conductor and through copper lug E_1 into the mercury and armature disk A; it leaves the armature disk through the mercury to lug E_2, thence through heavy copper conductor to terminal T_2 and back to line. The voltage circuit consists of the fine winding surrounding the magnet core Y, and a resistance coil R'. The current in this circuit develops a magnetic field between the ends of core Y and steel ring P. The intensity of this field is proportional to the current in coil SC' and hence, to voltage across the main line. The torque on the armature is proportional to the product of armature current and magnetic field, and hence to power, which

is the condition necessary for watt-hour meters. This torque is, however, very low, being only about 2 centimeter-grams when five amperes are flowing through the disk. This low torque is due to the fact that the armature may be considered as consisting of only one turn. Since the pressure of the movable element has been relieved, and the friction reduced to a minimum, a very high torque is not absolutely necessary. It is not practicable to pass more than 10 amperes through the mercury and disk, and hence shunts are used on all meters above 10 amperes capacity.

The counter-torque is obtained by the aluminum disk D rotating between poles of permanent magnets M exactly as in all motor watt-hour meters.

174. Compensation for Friction.—The inaccuracy at light load due to friction is corrected in two different ways. On meters of service type for circuits whose voltages are 220 and 500 volts, a thermo-couple compensation is regularly used. The thermo-couple consists of two strips of different metals joined at one end while the other ends are connected to lugs E_1 and E_2. The junction is surrounded by a heating coil connected in series with the voltage circuit. The heating of the junction of the two dissimilar metals develops a low pressure which sends a current through the armature disk. This low current, reacting with the magnetic field, gives the necessary initial torque. The thermo-couple is so arranged that the current set up in it will always pass from lug E_1 to lug E_2 through the armature chamber, no matter which way the heating current passes through the heating coil. Hence, it is evident that meters having thermo-couples for light-load compensation must be connected so that the load current also passes from lug E_1 to E_2. If this is not done, the effect of the compensating element will be to oppose the driving torque and the meter will be very slow on light load.

On low-voltage meters, light-load compensation is secured by passing through the armature disk a fraction of the voltage circuit current. Thus the armature is connected in shunt with a part of the voltage circuit. This method of compensation is shown in Fig. 144. One wire RR' has one end connected to lug E_1 and the other to lug E_2. Another wire, BR, parallel to RR' has one end connected to the voltage circuit and a sliding contact K joins the two wires. Current energizing the voltage coils SC passes from RR' through K to BR and thence to coils. If K is near left end of BR and RR', least compensation is obtained. Sliding

K toward the right end causes a current to flow through armature disk and compensating torque is increased.

On meters using thermo-couples for generating the compensating current, the sliding contact K is moved in just the opposite direction to secure the same adjustment. That is, a movement to the right increases the resistance in the light-load adjusting circuit, causing the meter to run slow; whereas if the clamp is moved to the left the resistance is decreased, thus increasing the compensating current.

The current through mercury and disk tends to demagnetize the permanent magnets. To overcome this the load current is

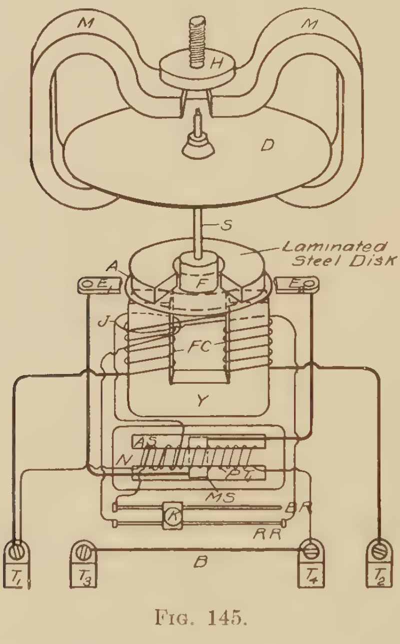


FIG. 145.

passed around the magnet core. As the load current increases, the demagnetizing and compensating actions increase together.

The mercury meter is not made for three-wire circuits. Two meters have to be used where it is desired to measure energy in three-wire direct-current circuits with this type of watt-hour meter.

175. Alternating-current Mercury Watt-hour Meter.—The high inductance of the voltage circuit of the D. C. mercury meter prevents its use on alternating-current circuits. The principles of an

alternating-current meter are shown diagramatically in Fig. 145. A comparison of this diagram with that of Fig. 144 will show that the load and voltage circuits have been interchanged. The load current energizes the magnet Y in the alternating-current meter, and the voltage circuit is connected to the primary, PT, circuit of a transformer. The secondary winding of the transformer is connected in series with the armature circuit to which it supplies current.

176. Operation.—Since the load current energizes the magnet, its magnetization will vary in magnitude and phase with the current. The secondary circuit of the transformer, including the armature circuit, is practically non-inductive. The secondary current is thus in phase with the secondary voltage. As the secondary voltage of a transformer is in opposition (180 degrees out of phase) to the primary, the secondary current will rise and fall with the line voltage. If the load circuit is non-inductive, the load current will fluctuate with line voltage and hence the magnetism that passes into the armature chamber will rise and fall with the current through the armature disk. Thus, on non-inductive load the meter will have maximum torque for a given current and line voltage, as is necessary for correct registration.

When, however, the load current lags behind the pressure, as on inductive load, the time interval between the maximum or zero values of magnetic flux, and armature current is exactly the same as the time interval between the load pressure and current. The torque for a given current and pressure is decreased in the ratio of the power-factor. This can be shown analytically exactly as in Chapter X. The current through the armature is proportional to the voltage. Assuming this to be harmonic, we may write

$$i = K_1 E_m \sin \omega t$$

and the magnetic flux is likewise proportional to the load current which lags θ degrees behind E. Then

$$\phi = K_2 I_m \sin (\omega t - \theta)$$

The instantaneous torque is proportional to product of i and Φ, hence

$$\tau = K_3 i \Phi = K_1 \times K_2 \times K_3 E_m I_m \sin \omega t \sin (wt - \theta)$$

and the average torque will be the average of the right-hand member of this equation. For average of τ, writing T, we get

$$T = K \,\Omega\, E_m Im \times \text{ average } \sin \omega t \sin (\omega t - \theta).$$

It has already been shown that the average of sin ωt sin $(\omega t-\theta)$ is $\frac{1}{2}$ cos θ; hence,

$$T=\frac{KE_m I_m \cos\theta}{2}=KEI\cos\theta.$$ But EI cos θ is the power; hence the torque is proportional to power on inductive circuits as well as on non-inductive circuits.

177. Compensation for Friction.—The core of the transformer is surrounded by an auxiliary secondary winding AS of few turns. One end of this winding is connected to one end of the wire BR and the other end to the middle of an auxiliary coil around Y, the ends of the coil being connected to RR'. The clamp contact K may be moved to the right or left, thus varying the relative resistances of the two parts of auxiliary magnetizing circuit, and indirectly, the current through this coil. When K is near the middle of BR and RR', the resistance both ways through RR' to J is the same, and there is no magnetizing effect. When the sliding contact K is near the left end, the reduction in resistance of one wire to J, and increase of the other, gives a forward torque. If K is near the right end, a backward torque results. For use on three-wire circuits the series circuit is divided, one coil being connected in each of the outer wires. The range of the meter may be extended by the use of current and pressure transformers.

178. Full-load Adjustment.—One other difference between the Sangamo and the other makes of watt-hour meters is worthy of mention. The retarding effect of the permanent magnets is varied by shunting some of the magnetism through the soft iron disk H Figs. 144 and 145, instead of moving the magnets nearer to, or farther from the axis of the disk. The soft iron disk is threaded and may be screwed up or down, thus changing the reluctance of the magnetic circuit. If the disk is screwed down, some of the magnetism passes from one magnet through the magnetic shunt H to the other, thus weakening that which passes through the copper disk. Under these conditions the meter will run faster.

179. Induction Type Watt-hour Meters.—Some of the disadvantages of the electrodynamometer type of watt-hour meter for use on alternating-current circuits have already been mentioned. The advantages of the induction type of watt-hour meter have relegated the electrodynamometer type to direct-current circuits almost exclusively, and hence, the two types are sometimes classified as direct-current and alternating-current watt-hour meters.

The fundamental advantage of the induction type watt-hour meter is the absence of commutator, and in fact of all sliding electrical contacts. The electrical circuits are all stationary, hence the movable element consists merely of a shaft and disk. The disk performs the functions of both the armature and retarding disk in

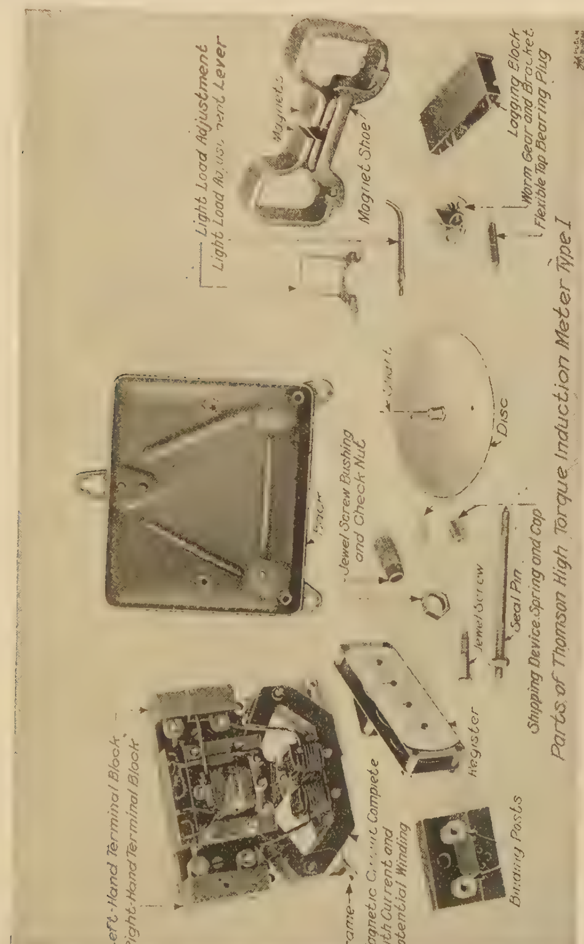


Fig. 146.

the other type. This great reduction of the number of parts greatly decreases the weight of the movable element, and consequently diminishes the bearing friction and jewel wear.

The fact that all windings are stationary permits a much more rugged and cheaper construction, eliminates commutator trou-

bles, decreases the friction, and greatly improves the accuracy of the meter over long periods of time. The induction type meter can, however, be used on alternating-current circuits only. Fig. 146 shows the parts of a General Electric induction type single-phase watt-hour meter. The essential operating parts are a stationary element comprising the electric and magnetic circuits, the rotatable disk, the registering mechanism, and retarding magnets.

180. Operation.—These meters operate upon the principle of the revolving magnetic field already explained in connection with the induction type wattmeters. Although exactly the same general principles apply in the two cases, nevertheless they

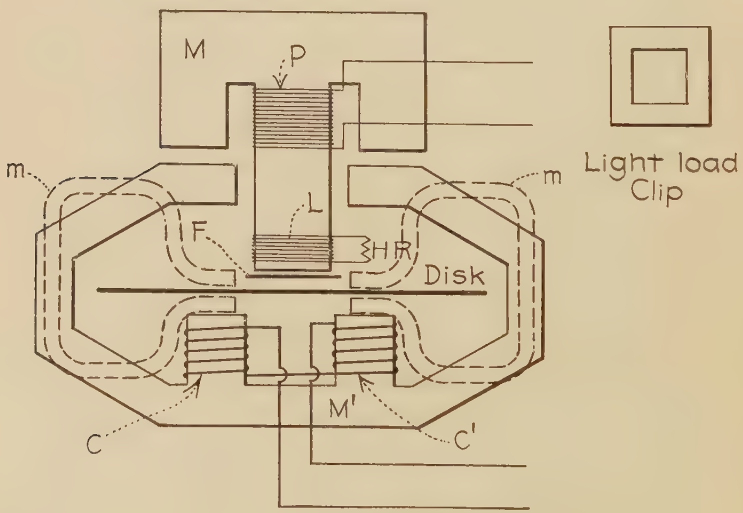


Fig. 147.

are applied in a somewhat modified way and hence a more extended discussion is justified. To make clear these principles there is given in Fig. 147 a simplified diagram of the driving and revolving parts. The element consists of two magnetic circuits, M and M', which are built up of laminated steel punchings. Core M carries the voltage coil P and lag coil L. The series coils CC' are wound upon the two projections of M'. Immediately below the central prong of M is a copper stamping F known as the light-load clip. The disk is shown in position between the central prong of M and upward projecting parts of M'; mm are the retarding magnets.

Fig. 148 shows the distribution of the magnetic lines around the two cores of the series coils. It is very evident that the end of one of these cores is a north pole while the other is a south pole. The distribution of the magnetic lines due to voltage coil is shown in Fig. 149. It will be seen that these lines radiate in all

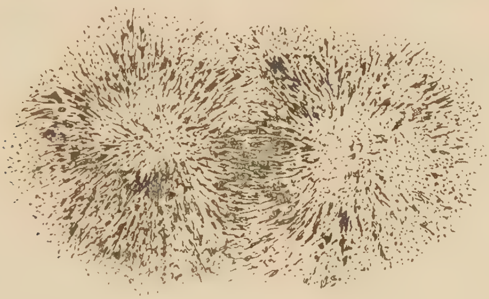
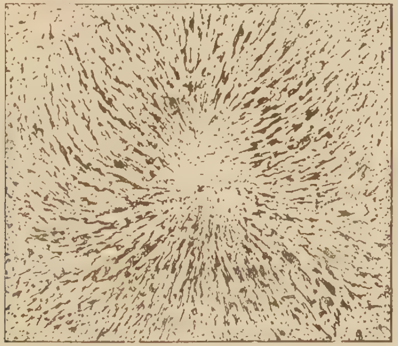


Fig. 148.

directions, chiefly to the right and left, some, however, passing downward.

These figures were obtained by sprinkling iron filings upon sensitized paper, which was laid flat upon the stationary element of the meter, while direct current was passed successively through



Fgi. 149.

the series and voltage coils. The figures thus do not accurately show the field within the air gaps, but outside of the cores as the iron filings shunt the magnetic lines. Illustrative diagrams of the distribution of the magnetic lines between the cores correspond-

ing to Figs. 148 and 149 are shown in Figs. 150, 151, and 154. Fig. 151a. shows the relative position of voltage coil core, B,

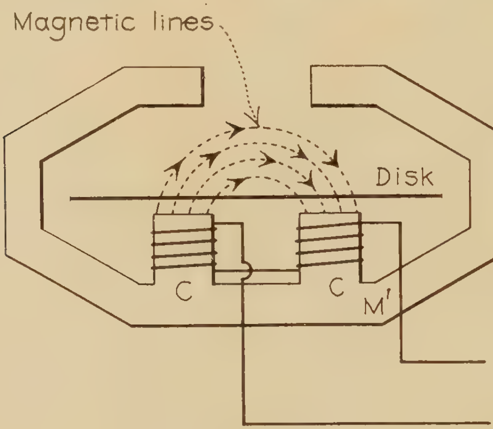


FIG. 150.

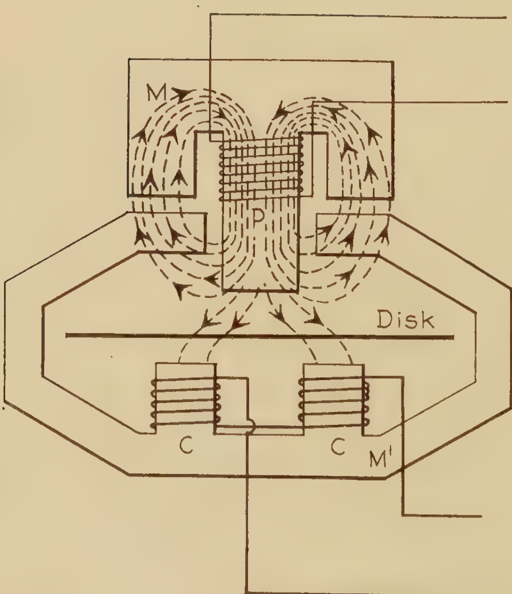


FIG. 151.

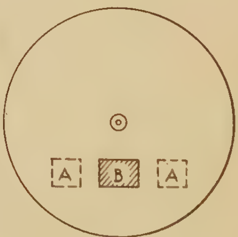


FIG. 151a.

series coil cores, A-A, and disk. It is thus clear that when direct current is used for excitation the lines due to the series

coil leave the end of one core and enter the other while those due to the voltage coil divide and pass upward through the adjacent iron.

When alternating currents are used for excitation, the resultant fields are similar to those produced by direct currents, but, owing to the fact that they are due to alternating currents, they shift either in one direction or the other.

The coil *P*, Fig. 147, consists of many turns of fine wire, the coil is highly inductive, and the current flowing in it is almost one-quarter of a period behind the voltage across its terminals.

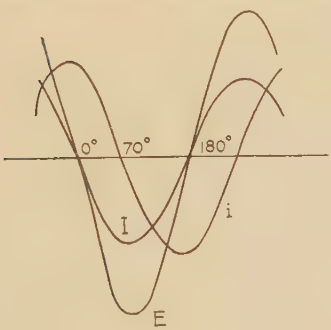


FIG. 152.

On the other hand, the coils *CC′* are nearly non-inductive and the current in them is in phase with the voltage, when the load power factor is unity. The phase relation of these quantities is shown in Fig. 152. This figure is a tracing of an oscillogram, and curve *I* represents the series current, *E* the applied voltage, and *i* the voltage coil current. It is very evident that although the line voltage and current are in phase, the voltage coil current lags only about 72° behind the voltage. The important thing is that the voltage coil flux and series coil flux should be in quadrature, and not that the currents should be. How this is secured will be shown later.

181. Shifting Magnetic Field.—Giving attention to the character and distribution of the magnetic fields only when at their maximum values, it can be shown that the resultant field shifts with reference to the disk, as the currents in the pressure and series circuits fluctuate.

For the time being, assuming a phase difference of one-quarter of a period between series and voltage currents, at the instant the series current has reached a positive maximum value, the voltage current is zero. At this instant the polarities of the iron cores will be as indicated at *A*, Fig. 153.

Considering a north pole + and a south pole −, the polarity of 1 is 0; of 2 it is +; of 3 it is zero; of 4 it is −; and of 5 it is 0.

A quarter of a period later, the current in the series coils has fallen to zero and that in the voltage coil is a maximum. The

polarity of the cores at this instant is indicated in *B*, Fig. 153. In the same way *C* shows the direction of flow of magnetic lines

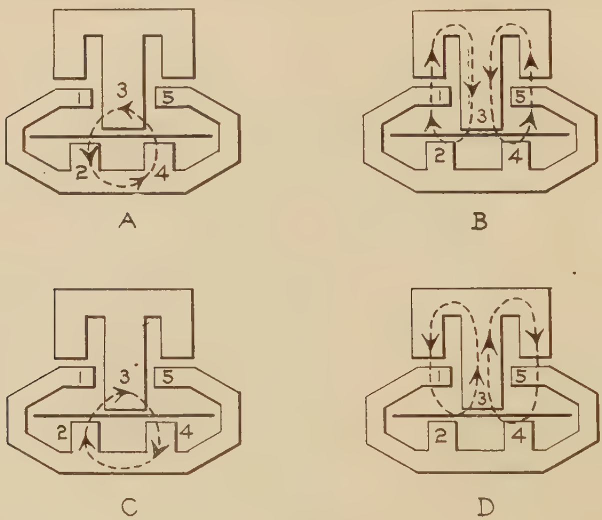


Fig. 153.

at the end of half a cycle and *D* at the end of three-quarters of a cycle. Arranging a table to show the magnetic condition of cores 2, 3, and 4 at the given instants, we get the following:

Table II.

	Poles			
Instants	1	2	3	4
Start	0	+	0	−
¼ period	−	0	+	0
½ "	0	−	0	+
¾ "	+	0	−	0
End of period	0	+	0	−

An examination of the above table shows that, under the conditions assumed, the polarity shifts continuously from left to right. This, of course, is true of the other magnetic conditions.

The shifting of the magnetic field induces currents in the disk, as already explained, and the reaction between these currents and magnetic field causes the disk to rotate.

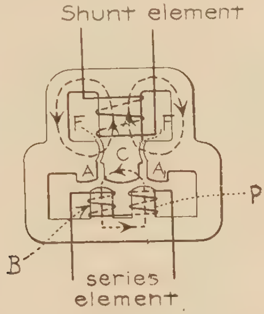


FIG. 154.

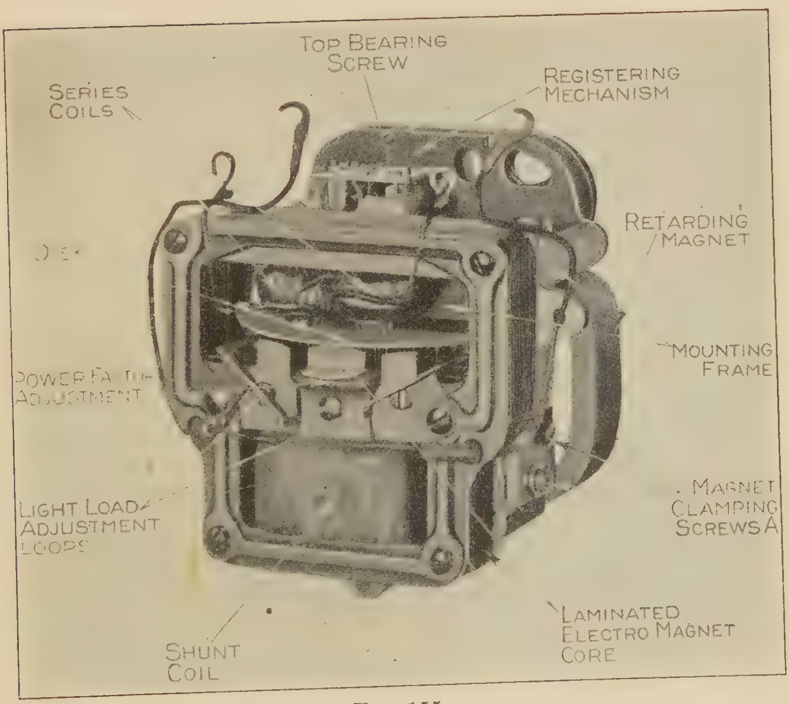


FIG. 155.

182. Practical Construction.—In Fig. 154 is shown the essential parts of the magnetic circuit of the Westinghouse single-phase

induction meter. This shows that the construction differs very little from that of the General Electric meter. The principles of operation are exactly alike in the two instruments. The

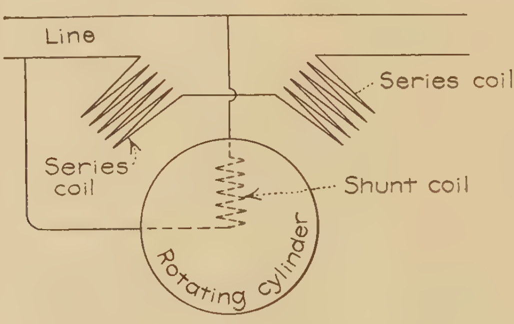


Fig. 156.

complete Westinghouse meter with case removed is shown in Fig. 155. It is very evident that the meter is very compact and rugged.

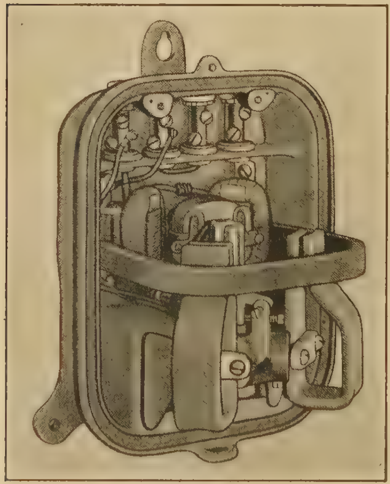


Fig. 157.

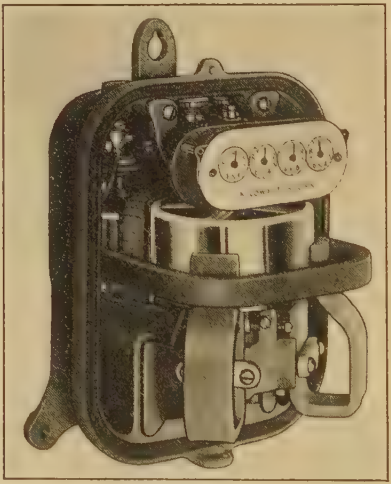


Fig. 158.

The induction meter of the Fort Wayne Electric Works applies the same principles in a somewhat modified manner. The relative position of the series and voltage coils is shown in Fig. 156. In actual construction, in the earlier instruments the coils had

only air cores, but the coils of the later forms have laminated iron cores, just as the other makes. In place of a disk, the rotatable element is an aluminum cylinder. The two figures, 157 and

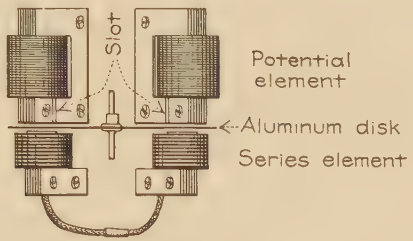


Fig. 159.

158, show clearly the actual construction of one form of this instrument.

Yet another way of securing a rotating or shifting magnetic

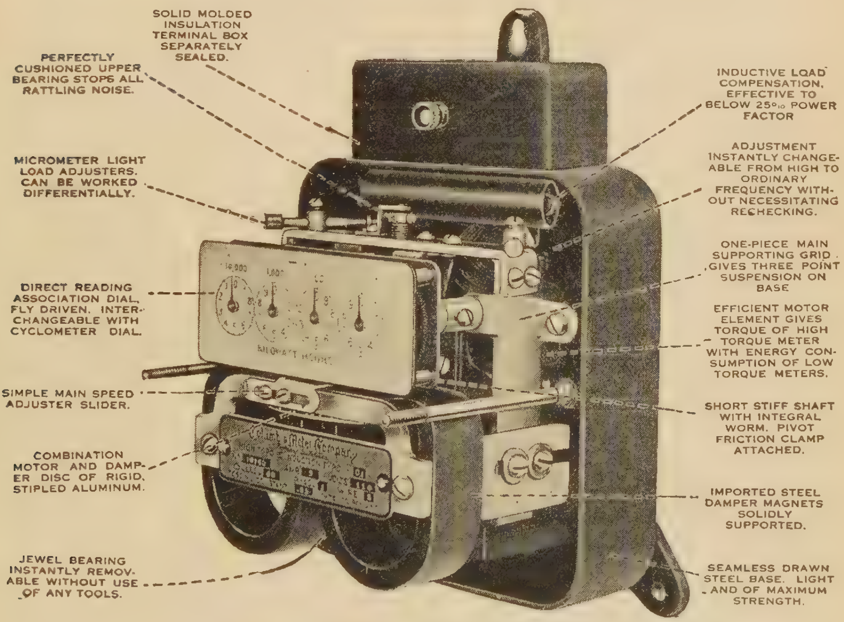


Fig. 160.

field is shown in Fig. 159. The voltage element consists of a pair of fine wire coils. Each of these coils is wound on a laminated sheet steel core of rectangular form as shown. The magnetic

circuit is very nearly all confined to the steel core, but at the bottom, near the disk, is a narrow slot where some leakage will naturally occur. This leakage flux must pass through the disk. In doing so it sets up the desired eddy currents.

The series coils are mounted on short laminated cores which differ from the potential circuit cores in not forming closed magnetic circuits, but instead present exposed ends to the aluminum disk. The series coils are placed below the disk but displaced with reference to the pressure circuit windings. A complete view of the Columbia induction meter is shown in Fig. 160.

183. Full-load Adjustment.—The fundamental principles of operation of these various makes of induction meters are all the same. Similarly the full-load adjustment is performed by varying the retarding effect of the permanent magnets. This in general, is accomplished in two ways, either by changing the position of the magnets with reference to the shaft, or by shunting some of the magnetism.

The full-load speed of the General Electric and Westinghouse meters is adjusted by moving the permanent magnets either away from, or nearer to, the shaft as the case demands.

The retarding torque is proportional to the product of eddy currents and strength of magnetic field. The eddy currents are proportional to the magnetic field strength and speed of disk, hence the torque is proportional to the product of the square of the magnetic field and speed of disk. Mathematically this can be expressed by $T = k\Phi^2 r\ \omega$, where Φ is the flux, r is the mean radial distance from the shaft of the disk to the poles, ω is the angular speed, and k a proportionality constant.

If the magnet is moved outward, the distance r is increased and, hence, the speed must decrease if the torque is to remain constant. If the speed does not decrease, the retarding torque increases in proportion to r.

If the disk is replaced by a cylindrical cup, as in the Fort Wayne meter, moving the magnets up or down does not change the radial distance. In this case a change in retarding torque is secured by changing the flux that passes through the aluminum cylinder. By moving the magnets down beyond a certain point, some of the magnetic lines pass below the edge of the cylinder and have no effect in inducing eddy currents. Under this condition, the retarding torque is less and the meter runs faster.

Moving the magnets upward will obviously have the opposite effect.

A change in torque may also be secured by providing for the magnetic lines a by-pass or magnetic shunt, whose reluctance may be varied. This is the method used on the Sangamo meter. An adjustable armature bridges the gap between the poles of the magnets, and a change in the position of this armature with reference to the magnet poles deflects a greater or smaller part of the flux around the disk.

184. Relation Between Torque and Power.—According to the principles just explained, an expression may readily be derived for the driving torque. For, if we represent the mean of the eddy currents by I, and the average flux by Φ, the driving force at a given position of retarding magnets is given by

$$T = k\Phi I$$

But the eddy currents are proportional to the product of flux and relative speed of magnetic field and disk. Since r is a constant quantity, the driving torque in general is then given by $T = k''\Phi^2\omega_r$ where ω_r is the relative speed. But in Chapter VI it was shown that Φ is the equivalent of two magnetic fields rotating in opposite directions at the same angular speed. If then in Fig. 45, OF_1 and OF represent Φ_1 and Φ_2, the two opposite torques due to these rotating fields are

$$T_1 = k_1\Phi^2{}_1(\omega_1 + \omega)$$

and $$T_2 = k_1\Phi^2{}_2(\omega_1 - \omega)$$

Where ω_1 is the angular speed of rotating fluxes and is equal to $2\pi f$, ω is angular speed of disk.

The retarding torque due to the permanent magnets when at a fixed distance from shaft is likewise given by

$$T_3 = k_2\Phi_3{}^2\omega$$

where Φ_3 represents the flux of the permanent magnets.

The retarding torque, due to the permanent magnets, is in the same direction as T_1, hence $T_1 + T_3$ retard the disk, while T_2 drives it. When constant speed has been reached, the algebraic sum of these torques must be zero, or

$$T_2 = T_1 + T_3$$

That is, $k_1\Phi_2{}^2(\omega_1 - \omega) = k_1\Phi_1{}^2(\omega_1 + \omega) + k_3\Phi_3{}^2\omega.$

Reducing we get $k_1(\Phi_2{}^2-\Phi_1{}^2)\omega_1=k_1(\Phi_2{}^2+\Phi_1{}^2)\omega+k_3\Phi_3{}^2\omega$.

But $\Phi_2{}^2=H_1{}^2+H_2{}^2+2H_1H_2 \sin \theta_1$

and $\Phi_1{}^2=H_1{}^2+H_2{}^2-2H_1H_2{}^2-H_1H_2 \sin \theta_0$ (see Art. 73).

Hence $\Phi_2{}^2-\Phi_1{}^2=4H_1H_2 \sin \theta_0$

and $\Phi_2{}^2+\Phi_1{}^2=2(H_1{}^2+H_2{}^2)$

Therefore, $4k_1H_1H_2\omega_1 \sin \theta_0=2k_1(H_1{}^2+H_2{}^2)\omega+k_3\Phi_3{}^2\omega_1$.

ω_1 is equal to $2\pi f$, and for any given frequency is constant. Replacing $8\pi k_1$ by K_1, we get, $K_1fH_1H_2 \sin \theta_0=2k_1\omega\ (H_1{}^2+H_2{}^2)$ $+k_3\Phi_3{}^2\omega$

The term $K_1fH_1H_2 \sin \theta_0$ represents the driving torque, and the two right-hand members represent the retarding torque. The term $2k_1\omega(H_1{}^2+H_2{}^2)$ represents the retarding effect of the rotating fields, and $k_3\Phi_3{}^2\omega$ is the retarding effect of permanent magnets. When ω is low, the effect of $2k_1(H_1{}^2+H_2{}^2)\ \omega$ is negligible and the retarding effect is wholly due to $K_3\Phi_3{}^2\omega$. Under this condition the relation between driving and retarding torque is

$$K_1fH_1H_2 \sin \theta_0=k_3\Phi_3{}^2\omega$$

Now H_1 is proportional to the voltage, and H_2 is proportional to the series current. We may then write

$$H_1=k'E$$

and $H_2=k''I$, which substituted in above equation give

$$K_1k'k''fEI \sin \theta_0=k_3\Phi_3{}^2\omega$$

or $EI \sin \theta_o=K_o\Phi_3{}^2\omega$, where

$$K_o=\frac{k_3}{K_1k'k''f}$$

That is, the product of current, pressure, and sine of phase difference between magnetic field due to series and voltage currents respectively is proportional to speed of disk. The actual power is however, equal to $EI \cos \theta$, hence if the meter is to register accurately

$$\sin \theta_0=\cos \theta$$

$$\text{or } \theta_0=\frac{\pi}{2}\pm\theta.$$

For correct registration, it is thus imperative that the phase

difference between the two magnetic fields be exactly one-quarter of a period when the power factor of load is unity.

If $2k_1(H_1^2+H_2^2)\omega$ is not negligible in comparison with $k_3\Phi_3^2\omega$, the calibration curve of the meter will not be a straight line. If the meter is correct on light load, it will be slow on full or overload, and *vice versa*. On any given voltage and frequency, H_1^2 is constant; the inaccuracy in registration is then due to $H_2^2\omega$, and in order that this may be negligible both H_2 and ω must be small. This is taken care of in the design of the meter.

185. Lagging Induction Watt-hour Meters.—As has just been demonstrated, for accurate registration on circuits of low power-factor, the flux due to the pressure coil must be one-quarter of a period out of phase with the flux due to the series current when operating on circuits whose power-factor is unity. Mainly on account of the resistance, eddy currents, and hysteresis of the voltage circuit, this phase difference is not obtained without additional adjustment.

The methods of lagging used by the General Electric Company and the Westinghouse Company are identical in principle. The manner in which this is carried out in practice is shown in Figs. 147 and 155. Coil L, Fig. 147, is known as the lag coil, and consists of a few turns of a high resistance wire wound around the end of the voltage coil core. A similar coil is shown in Fig. 155 where it is labelled "power-factor adjustment." The function and operation of the lag coil is explained in Article 121. As there pointed out, part of the flux due to the voltage coil, passes through the lag coil. The resistance of the lag coil is adjusted, until the phase displacement of the resultant flux is exactly 90 degrees in time phase from the flux due to the current coils. The greater part of the pressure coil flux does not pass through the lag coil, but takes the shorter path as indicated in Fig. 151. This fact is not shown clearly in Fig. 149, for in that case direct current was used for excitation and the inductance of lag coil had no effect.

186. The Effect of Over and Under Lagging.—The driving torque of an induction meter is given by

$T=K_0H_1H_2\sin\theta_0$ where H_1 and H_2 are the maximum values of the voltage and current fields respectively, and θ_0 is the time phase difference between them. The power in the load circuit is, however, given by

$$P=EI\cos\theta.$$

In order that the torque may be proportional to power, θ_0 must be equal to $\frac{\pi}{2}+\theta$. In case this relation does not exist, the torque will be either too large or too small and the registration will be in error. For instance, suppose $\theta_0=\frac{\pi}{2}+\alpha\pm\theta$, or that the meter is over lagged. Then $\sin\ \theta_0=\sin\left[\frac{\pi}{2}+(\alpha\pm\theta)\right]=\cos\ (\alpha\pm\theta)$ and the driving torque will be given by $T=K'EI\ \cos\ (\alpha\pm\theta)$.

When θ is positive, or the current leads the pressure, $\cos\ (\alpha+\theta)$ is less than $\cos\ \theta$ and the torque is too small. When θ is an angle of lag, $\cos\ (\alpha-\theta)$ is greater than $\cos\ \theta$, and the torque is too large. Thus, an over lagged meter will under register on leading current, and over register on lagging current.

It can easily be shown that when the meter is under lagged or when $\theta_0=\frac{\pi}{2}-\alpha\pm\ \theta$, the meter will be slow on lagging and fast on leading current. The demonstration is left for the student.

187. Light Load Compensation.—Exactly as in the electrodynamometer type and other types of watt-hour meters, for accurate registration on light load, some means must be provided for overcoming the retarding torque due to friction. Owing to the absence of brushes, and to the fact that the movable elements of induction type watt-hour meters are much lighter, and have a higher torque per unit weight than commutator meters, there is much less necessity for friction compensation.

When compensating devices are used, they are operated by the voltage circuit as in other types of watt-hour meters. The compensation remains constant at all loads with a given adjustment; it may, however, change with the voltage or frequency.

The general principles upon which the various compensating devices of different makers operate are fundamentally the same although the methods of applying these principles differ considerably. The fundamental principle consists in the production of an unbalanced or shifting magnetic field. The compensating device operates so that at any instant the magnetic flux is not uniformly distributed over the pole face of the voltage coil core. The variation in the distribution of the magnetic flux produces the same effect as a shifting field.

This non-uniformity in flux distribution may be produced

by interposing within the air gap a short-circuited conductor, or by modifying the voltage coil core in such a way that the flux density will vary in time from one side of the pole face to the other.

The former method is made use of by the Westinghouse and General Electric Companies. Fig. 155 shows how this method is applied by the Westinghouse Company.

As here shown, the light-load compensation is secured by means of two "Light Load Adjustment Loops" which are in reality two copper punchings forming a closed electrical circuit. One side of each loop is in the air gap of the voltage coil core, and the whole loop is mounted in such a way that it may be turned through a small angle, thus changing its position with reference to the magnet core. This adjustment is accomplished by means of two knurled nuts which are accessible from the front of the meter but not shown in the figure.

It is clearly shown in Fig. 155 that turning either of these loops down or up permits more or less of the magnetic flux to pass through the loop. This flux as it passes through the loop induces a current therein, and the reaction of this current upon the flux retards its development when increasing, and *vice versa*. This retardation unbalances the main flux, and produces the same effect as a shifting field. The result is the development of a torque whose value is modified by any change in the position of the short-circuited loop. By carefully adjusting the position of these loops, a torque just sufficient to overcome the retarding torque of friction may be secured.

A similar method is employed by the General Electric Company, as is evident from Fig. 147. As shown, the light-load compensator F consists of a rectangular copper punching, which is placed under the central prong of the voltage coil core. If this punching is not symmetrically placed with reference to the core, the flux through the disk will vary in intensity from one side to the other. This variation, or one may say shifting of flux, will produce enough torque to overcome that due to friction.

The second method of applying the general principle is exemplified in the meter whose connections are shown in Fig. 161, and the induction meter of the Columbia Meter Company.

In Fig. 161 is shown the auxiliary core (b) which is pivoted in the middle. Loosening screw S_1 and tightening S_2 moves the end that is near the cylinder from left to right. Such a change

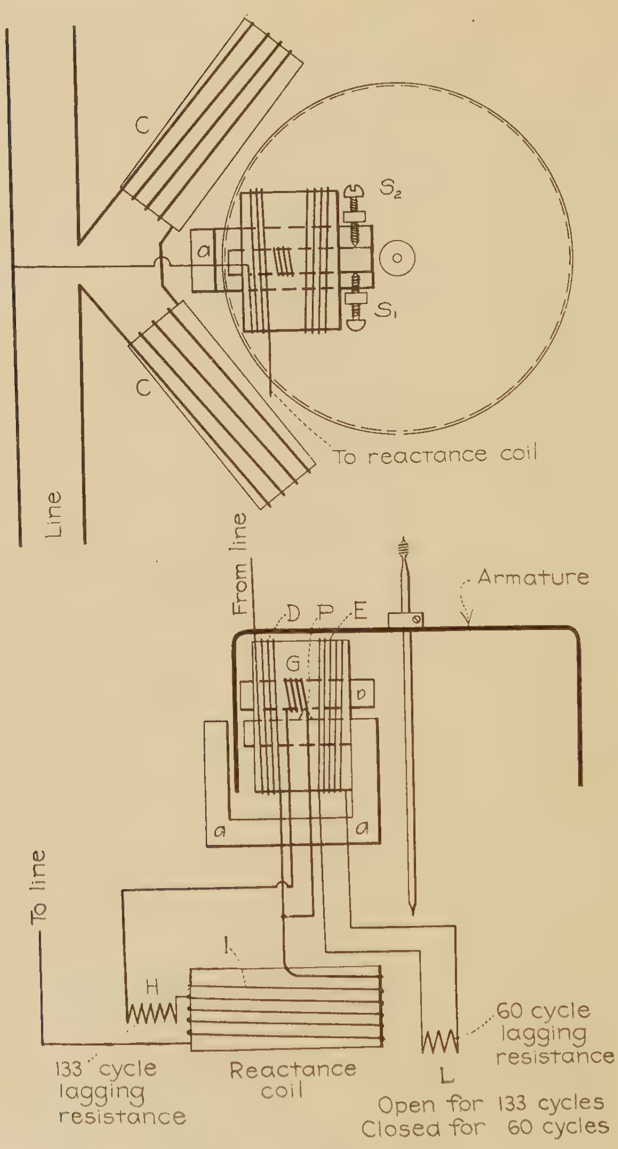

C
S_2
a
S_1
C
To reactance coil
Line
From line
Armature
D P E
G
a a
To line
I
H
60 cycle
lagging
resistance
133 cycle
lagging
resistance
Reactance
coil
L
Open for 133 cycles
Closed for 60 cycles

FIG. 161.

in the relative positions of the auxiliary and main cores causes a shifting of the magnetic flux. This shifting is similar to that produced by the other compensating methods.

188. Flux Shunting Method.—In the older form of the Columbia induction meter the necessary unbalancing is obtained by means of a piece of soft iron adjustably bridging a part of the air gap in the voltage coil core. The bridging piece of iron is held in place by two screws passing through a slot in the extension arm which carries it. By loosening these screws, the position of the bridging piece of iron can be varied. In the more recent designs the shunting of magnetic lines by means of a piece of iron across the air gap is no longer used, but an ingenious scheme of securing the same result is employed. This is shown in Fig. 162. As is evident from the figure, a specially designed piece is mounted in an adjustable manner on top of each voltage coil core. The frame of the meter is made of non-magnetic material and, hence any leakage flux will follow the path of least reluctance. The case of meter is of magnetic material, and as it comes near the back of the voltage coil cores some leakage flux will pass through the case downward and enter the core again from the bottom through the armature disk. When the sliding pieces of iron are symmetrically placed with reference to the cores and case, the effect of the leakage flux from one core balances the effect of the leakage flux of the other core. When, however, the reluctance of the leakage flux path is changed by moving one of the sliding pieces nearer to or farther away from the case, this balancing no longer exists, and the disk will be caused to rotate. If the right-hand piece is nearer the case, the disk rotates from left to right. If the other piece is near the case, the disk rotates from right to left when voltage circuit alone is closed. Hence, by adjusting the positions of the two sliding pieces, any necessary degree of compensation can be produced.

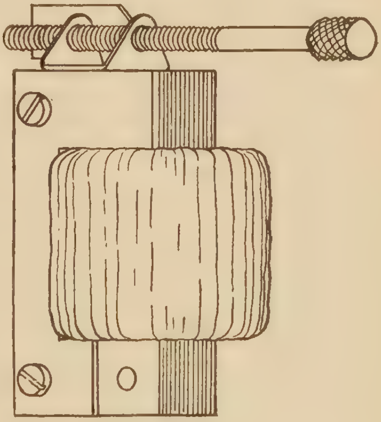


FIG. 162.

189. Influence of Frequency.—The current through the pressure coil of a watt-hour meter is given by

$$I=\frac{E}{(R^2+X^2)^{\frac{1}{2}}}$$

where R is the resistance and X the inductive reactance of the coil. The resistance R changes only with the temperature, the effect of which will be discussed later; but X, which is equal to $2\pi fL$, varies with frequency. Hence, I will increase with decrease in frequency and decrease with increase of frequency. This increase or decrease will cause a corresponding increase in the voltage coil flux, and as the torque is proportional to the product of the maximum value of the voltage coil and current coil flux, it will also vary. Again, the angle of lag of voltage coil current may be obtained from the expression $\tan \beta=\frac{X}{R}=\frac{2\pi fL}{R}\cdot$ That is, the tangent of the angle of lag varies as the frequency. Since without great error, one may assume that the voltage coil flux is in phase with the voltage coil current, it is evident that the angle θ_0, in the expression for torque, viz., $T=K_oH_1H_2 \sin \theta_0$, depends upon the frequency. Any variation in frequency, thus, produces two effects: First, increases or decreases the voltage coil flux; and, second, changes the phase relation of the two operating fluxes.

A decrease in the frequency increases the voltage coil current, but a lowering of the frequency decreases the value of θ_0, producing the same result as under lagging. It has been shown that an under lagged meter tends to run slow on lagging current, hence the increase in torque, due to greater current, will tend to compensate for the phase difference error. On leading current, however, the error will be increased.

Similarly, when the frequency increases, the voltage coil current increases, and the phase difference between two operating fluxes increases. The effect of the difference in phase is the same as though the meter were over lagged. These two effects tend to neutralize each other on inductive load, but are cumulative when the load current leads the pressure.

This double effect will be more readily understood from the vector diagram of Fig. 162a. In this diagram OE represents both the magnitude and direction of the applied voltage and

$O\Phi_1$ = voltage coil flux at normal frequency,
$O\Phi_2$ = voltage coil flux at a frequency below normal,
$O\Phi_3$ = voltage coil flux at a frequency above normal.

The corresponding lag coil fluxes are given by $O\Phi'_1$, $O\Phi'_2$, and $O\Phi'_3$. The meter is assumed to be properly lagged for normal frequency, *i.e.*, θ_1 is 90°. At low frequency the resultant flux

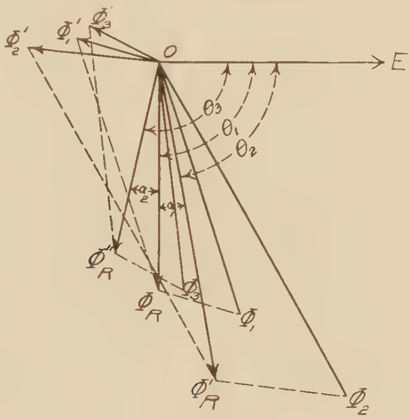


Fig. 162a.

Φ'_R makes an angle θ_2 with the voltage and this angle differs from 90° by the angle α_1. The phase relations are thus the same as though the meter were under lagged on normal frequency.

The resultant flux $O\Phi''_R$, when frequency is above normal is out of phase by an angle $\theta_3 = 90° + \alpha_2$, and plainly the phase

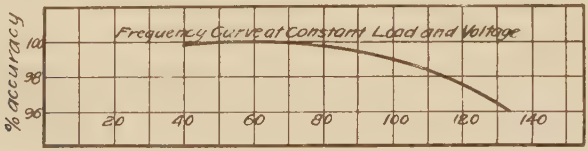


Fig. 163.

effect is the same as though the meter were over lagged on normal frequency.

The effect of variations in frequency on the accuracy of the meter is shown in Fig. 163.

The meter for which the curve is given was lagged for accurate registration on 60 cycles. It is seen that there is a

falling off in accuracy both with increasing and decreasing frequency.

190. Double Lagging.—The foregoing discussion shows that when meters are designed for two widely different frequencies, some provisions must be made for changing the effect of the lag coil. Such a device is called double lagging. A good example of a double lagged meter is the 60- or 133-cycle watt-hour meter of the Fort Wayne Electrical Works. The internal connections of this meter are shown in Fig. 161.

As shown, the voltage coil core consists of two parts, *a* and *b*; the main core (*a*) forms nearly a closed magnetic circuit, while *b* is a laminated piece of rectangular cross-section pivoted at *P*. This auxiliary core *b* is wound with an auxiliary coil, *G*, which is connected in series with a resistance, *H*, and shunted across a few turns of reactance coil, *I*.

The voltage coil consists of two parts: one *D*, which is wound around cores *a* and *b*, and *I* which has a separate iron core. The iron core of *I* forms a closed magnetic circuit and hence its inductance is much higher than the inductance of *D* whose core has an air gap.

The lag of voltage current is due to the influence of both coils, but the magnetism developed by *D* alone is effective in causing rotation of armature, and it alone must be in exact quadrature with flux produced by coils *C–C* on load of unity power-factor. To compensate for any discrepancy in the quarter phase relation mentioned, there are provided two coils, *E* and *G*. On circuits of low frequencies, both coils are operative while on circuits of high frequency coil *E* is opened and *G* alone is effective. In general. a change in the frequency adjustment will affect the speed of the meter. This effect can be corrected by an adjustment of the position of the permanent magnets when the meter is under test.

191. Single-phase Watt-hour Meters on Polyphase Circuits.—It will be shown later that to measure power on polyphase circuits, by means of single-phase instruments—with the exception of the two-phase system—there are needed as many meters as there are phases less one. That is, on a three-phase circuit two meters are sufficient, and by properly connecting single-phase meters to the polyphase circuits the energy will be equal to the algebraic sum of the meter readings.

192. Three-wire Single-phase Induction Watt-hour Meters.—The three-wire system of distribution is much more economical

than a two-wire system when any considerable amount of energy s to be transmitted. To measure the energy, either two single-phase two-wire meters or one three-wire meter may be used. When discussing three-wire meters of the electrodynamometer type, it was shown that the registration was correct only under certain conditions. Somewhat the same limitations apply to three-wire induction watt-hour meters, with the additional fact that these limitations are complicated by the characteristics of alternating currents.

The alternating-current three-wire meter differs verylit tle in construction from the two-wire instrument. Like the direct-current three-wire meter the instrument contains two series or current circuits and one voltage circuit. The series circuits are connected, one in each of the outside lines, and the voltage circuit may be connected either across the outside wires or between neutral and either outside wire. Which method of voltage coil connection is used depends upon the design or make of instrument. The former method is perhaps the most common for reasons that will presently appear.

Assuming the meter to be correctly adjusted, the accuracy of its registration will depend to a considerable extent upon the character of the load and connection of voltage coil. The influence of these different conditions upon the accuracy of the meter may then be considered under the following heads:

I. Voltage coil connected across outside wires
 1. Load balanced
 2. Load unbalanced

II. Voltage coil connected between neutral and one outside wire.
 1. Load balanced
 2. Load unbalanced.

193. Voltage Coil Connected Across Outside Wires.—Fig. 164 is a diagram of an unbalanced single-phase three-wire system. S_1 and S_2 represent the series coils and V represents the voltage coil of the watt-hour meter. The vector diagram of Fig. 165 shows the phase relations of currents and voltages as produced by a most serious case of unbalancing. Very seldom would all the conditions there assumed be met at once.

If E_{1m} and E_{2m} represent the maximum pressures across loads L_1 and L_2, then E_m will be the maximum pressure between wires

1 and 2. The current in load L_1 is assumed to lag θ degrees behind E_{1m}, and ϕ_1 degrees behind E_m. If I_o represents the current in middle wire 0, then I_2 will represent maximum current in load L_2. This current lags θ_2 degrees behind E_{2m}, and ϕ_2

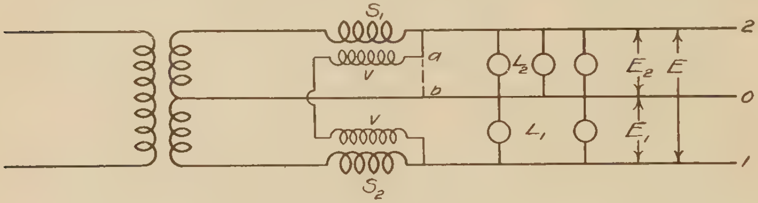


FIG. 164.

degrees behind E_m. According to the notation assumed, the instantaneous values of the electrical quantities involved are,

$$e = E_m \sin \omega t$$
$$e_1 = E_{1m} \sin (\omega t - \alpha_1)$$
$$e_2 = E_{2m} \sin (\omega t + \alpha_2)$$
$$i_1 = I_1 \sin (\omega t - \phi_1)$$
$$i_2 = I_2 \sin (\omega t - \phi_2)$$

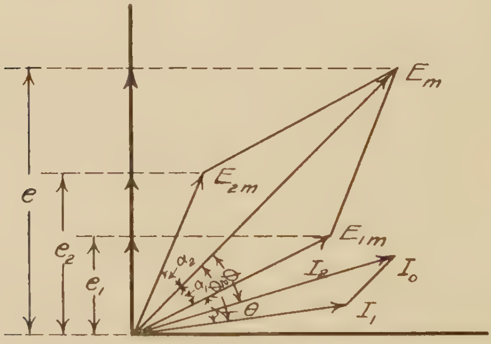


FIG. 165.

The instantaneous power being delivered to loads L_1 and L_2 is

$$w = e_1 i_1 + e_2 i_2$$

Since the meter is direct reading, the proportionality factor is taken care of by the calibration of meter and is omitted. The instantaneous torque on the disk when meter is properly adjusted is $\quad \tau = \frac{1}{2} e (i_1 + i_2)$

but $\quad e = e_1 + e_2$

Hence, $\quad \tau = \frac{1}{2} (e_1 + e_2)(i_1 + i_2)$

If τ is equal to w, the registration will be correct. When this is not the case, the error in registration will depend upon the difference between w and t.

Now $\quad w-\tau=e_1i_1+e_2i_2-\frac{1}{2}(e_1+e_2)(i_1+i_2)$
$\quad\quad =\frac{1}{2}[(e_1i_1+e_2i_2)-(e_1i_2+e_2i_1)]$

But $\quad e_1=E_m \sin(\omega t-\alpha_1)$
and $i_1=I_1 \sin(\omega t-\phi_1)$

Then $\quad e_1i_1=E_{1m}I_1 \sin(\omega t-\alpha_1)\sin(\omega t-\phi_1)$

Expanding $\sin(\omega t-\alpha_1)$ and $\sin(\omega t-\phi_1)$ we get,

$e_1i_1=E_{1m}I_1[(\sin\ \omega t\cos\ \alpha_1-\cos\ \omega t\sin\ \alpha_1)(\sin\ \omega t\cos\ \phi_1-\cos\ \omega t\sin\ \phi_1)]$

and average of e_1i_1 equals average of right-hand member of equation.

Performing the multiplication indicated, and remembering that the average of $\sin^2\omega t$ and $\cos^2\omega t$ is $1/2$, and the average of $\sin\omega t\cos\omega t$ is zero, the expression reduces to

$$\text{av. } e_1i_1=\tfrac{1}{2}E_{1m}\ I_1\cos(\phi_1-\alpha_1)$$

By a similar process of analysis the average of

$$e_2i_2=E_{2m}I_2\cos(\phi_2+\alpha_2)$$

Average of

$$e_2i_1=\tfrac{1}{2}E_{2m}I_1\cos(\phi_2+\alpha_2)$$

and average of

$$e_1i_2=\tfrac{1}{2}E_{1m}I_2\cos(\phi_2-\alpha_1)$$

Substituting these values, the expression for the difference between power and torque becomes

average of $\quad w-\tau=\frac{1}{4}[E_{1m}I_1\cos(\phi_1-\alpha_1)+E_{2m}I_2\cos(\phi_2+\alpha_2)]$
$\quad\quad -\frac{1}{4}[E_{1m}I_2\cos(\phi_1-\alpha_2)+E_{2m}I_1\cos(\phi_2+\alpha_1)]$

The accuracy of the meter evidently depends upon the value of this expression. If the conditions are such that the expression reduces to zero, the registration is correct; if the expression is negative, the average torque is higher than the average load and the meter registration is too high, and when the expression is positive, the registration is too low.

Conditions that are most likely to be met with in practice are $I_1=I_2$, $E_1=E_2$, $\alpha_1=\alpha_2$ and $\phi_1=\phi_2$. When this is the case it is evident that the average of $w-\tau=0$ and the meter registers correctly.

194. Load Unbalanced.—When the load is unbalanced, in

general the expression will not reduce to zero, even if the unbalancing is not sufficient to make E_1 differ materially from E_2.

Let $E_1=E_2$, $\alpha_1=\alpha_2$ and $\phi_1=\phi_2$. Then

$$\begin{aligned} w-\tau &= \tfrac{1}{4}E_{1m}[I_1\cos(\phi_1-\alpha_1)-(I_1-I_2)\cos(\phi_1+\alpha_1)-I_2\cos(\phi_1-\alpha_1)] \\ &= \tfrac{1}{4}E_{1m}\{(I_1-I_2)[\cos(\phi_1-\alpha_1)-\cos(\phi_1+\alpha_1)]\} \\ &= \tfrac{1}{2}E_{1m}(I_1-I_2)\sin\phi_1\sin\alpha_1. \end{aligned}$$

So long as I_2 is less than I_1 and ϕ_1 is less than π_2 the meter will register too high. In practice α_1 will never be equal to 90°.

If the power-factors of L_1 and L_2 are each unity; then $\phi_1=0$, and the meter registers correctly. In general, the meter will register incorrectly on an unbalanced load when voltage coil is connected across the outside wires.

195. Voltage Coil Connected between One Outside Wire and Neutral.—Such a connection is indicated by the dotted line *ab*, Fig. 164, using the same notation as in Art. 193, the power is given by

$$w=e_1i_1+e_2i_2$$

but torque $$\tau=e_1i_1+e_1i_2$$

and $$w-\tau=e_2i_2-e_1i_2$$

It has been shown that

$$\text{av. } e_2i_2=\tfrac{1}{2}E_{2m}I_2\cos(\phi_2+\alpha_2)$$

and $$\text{av. } e_1i_2=\tfrac{1}{2}E_{1m}I_2\cos(\phi_2+\alpha_1)$$

Hence average $(w-\tau)=\frac{1}{2}I_2[(E_{2m}\cos(\phi_2+\alpha_2)-E_1\cos(\phi_2-\alpha)]$

The meter will register accurately only when the average of $(w-\tau)=0$.

If the load is balanced so that $E_2=E_1$, $\phi_2=\phi_1$ and $\alpha_2=\alpha_1$ the average value of $w-\tau=-\frac{1}{2}E_{2m}I_2\,2\sin\phi_2\sin\alpha_2=-E_{2m}I_2\sin\phi_2\sin\alpha_2$.

This expression is zero only when $\alpha_2=0$, for only under such a condition will ϕ be zero. It is perfectly clear, then, that a three-wire induction meter will not register correctly when the voltage coil is connected between outside wires even though the load on three-wire system be balanced. It will register correctly only when the load is balanced and the power-factor is unity. When the voltage coil is connected between neutral and one outside main, the meter in general will be fast if voltage impressed upon voltage coil lags behind; and slow when the pressure coil voltage leads the voltage between outside mains. This answers the

question why it is preferable to connect the voltage coil between outside mains, a practice followed by most manufacturers.

Upon three-wire circuits that are subject to unbalanced loads, two single-phase two-wire meters are preferable. When these are used they are connected as shown in Fig. 166.

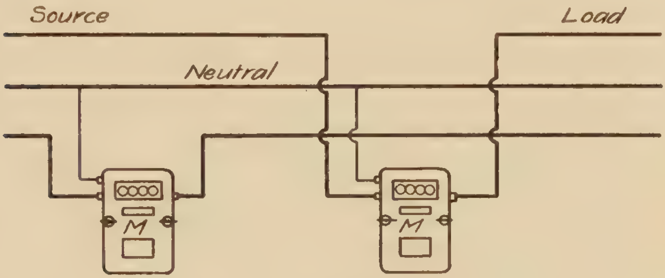


Fig. 166.

196. Polyphase Watt-hour Meters.—In metering energy on polyphase circuits, polyphase induction meters are usually employed, although the practice of some companies favors the use of two or of three single-phase meters, on the ground that when this is done the failure of one meter will not cause a complete loss of the record. The use of two single-phase meters has the advantage that from their registrations the average power-factor of the load can be computed.

On the other hand, the use of more than one meter is subject to objection not only on account of the additional expense, but also on account of the difficulty of explaining to customers the characteristics of polyphase systems.

197. Watt-hour Meters for Two-phase and Three-wire Three-phase Circuits.—One make of polyphase watt-hour meter for a two-phase or a three-wire three-phase system is shown in Fig. 167. The illustration shows clearly that the instrument is a combination of two single-phase metering elements, the armature cylinders of which are mounted on the same shaft or spindle. Only one registering mechanism is thus necessary. The total driving torque is the sum of the torques exerted by the two actuating elements, and the registration is proportional to the energy passing through both.

It was pointed out that for correct registration on inductive load, the flux due to the load current must be in quadrature with

the flux due to the voltage coil current. Exactly the same conditions must exist in each of the metering elements of polyphase meters.

The manner in which two-phase and three-wire three-phase meters are connected to circuits is shown in Figs. 168, 168a, 169, and 170.

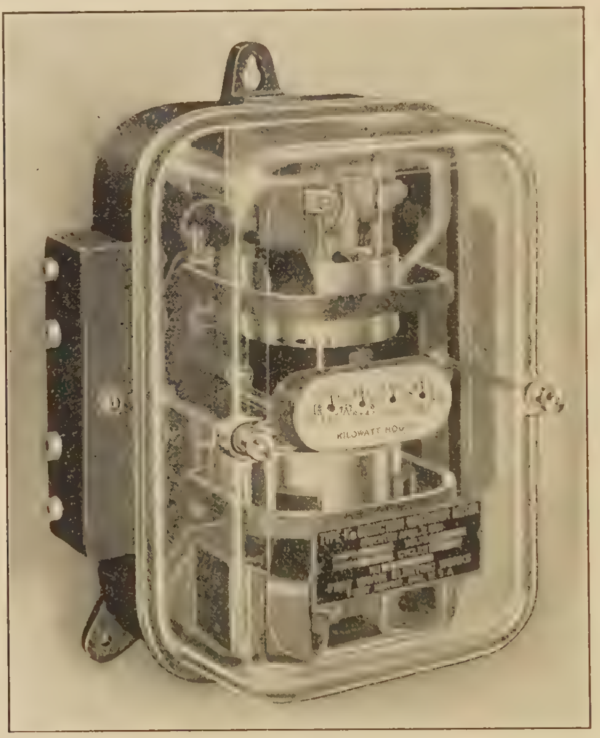


Fig. 167.

When used on four-wire two-phase circuits one operating element is connected in each phase exactly as though it were a single-phase meter and it is evident that under these conditions a meter theoretically correct will register accurately on balanced or unbalanced circuits.

It is not so evident, however, that two single-phase meter elements combined into one instrument will register all of the energy supplied by a three-phase circuit. It is true, nevertheless, as can readily be shown.

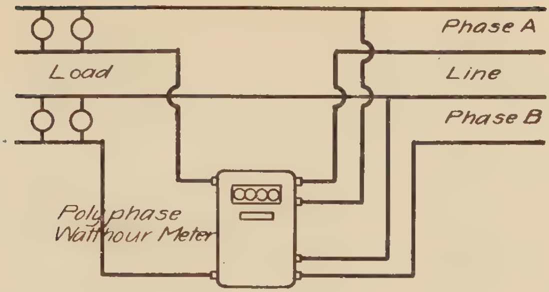


FIG. 168.

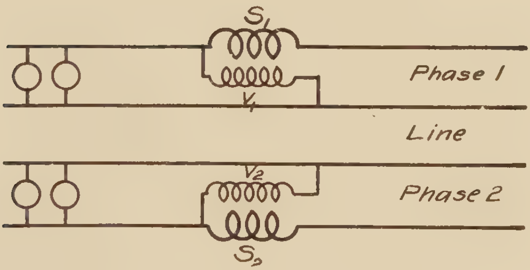


FIG. 168a.

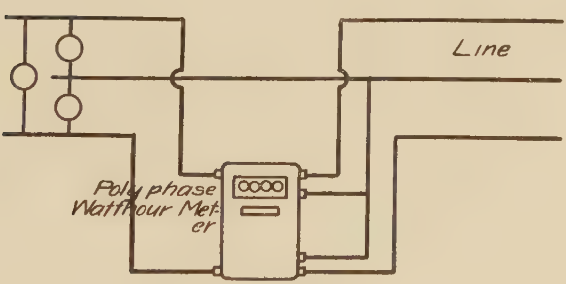


FIG. 169.

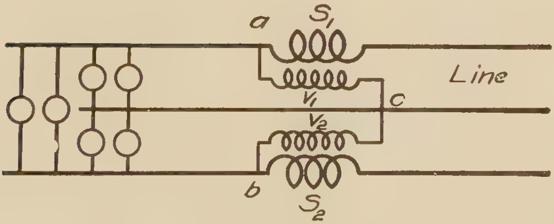


FIG. 170.

198. Relation of Power to Torque in a Y-connected System.—The two general methods of connecting three-phase receiving circuits are shown in Figs. 171 and 172. In the Y-connected system if e_o, e_1, e_2 and i_o, i_1, and i_2 are the instantaneous voltages

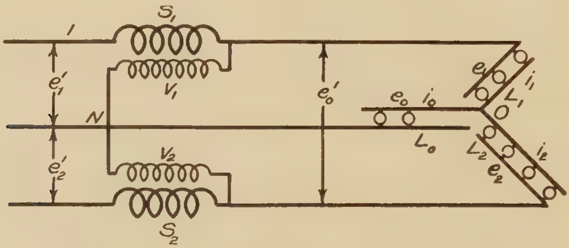


FIG. 171.

and currents applied to loads L_o, L_1, and L_2, the instantaneous power is

$$w = e_o i_o + e_1 i_1 + e_2 i_2$$

but the torque exerted by the two meter elements is

$\tau = e'_1 i_1 + e'_2 i_2$ when the meter is properly adjusted

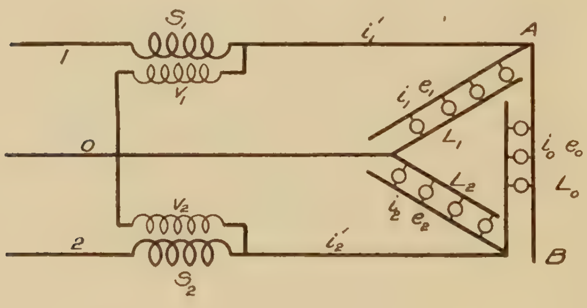


FIG. 172.

The difference between w and τ is

$$w - \tau = (e_o i_o + e_1 i_1 + e_2 i_2) - (e'_1 i_1 + e'_2 i_2)$$

but $\quad e'_1 = e_o + e_1$

and $\quad e'_2 = e_o + e_2$

therefore,
$$\begin{aligned} w - \tau &= (e_o i_o + e_1 i_1 + e_2 i_2) - [i_1(e_o + e_1) + i_2(e_o + e_2)] \\ &= (e_o i_o + e_1 i_1 + e_2 i_2) - (e_o i_1 + e_1 i_1 + e_o i_2 + e_2 i_2) \\ &= e_o(i_o - i_1 - i_2) \end{aligned}$$

Since the middle wire at each instant may be considered as the return for wires 1 and 2, then $i_o - i_1 - i_2 = 0$ under all conditions.

Hence $w - \tau = e_o(i_o - i_1 - i_2) = 0$

and the watt-hour meter registers correctly the total energy in a Y-connected system no matter whether the load be balanced or unbalanced.

199. Relation between Power and Torque in a Δ (delta)-connected System.—Using the same notation in Fig. 172 as in Fig. 171 the instantaneous power supplied to loads L_o, L_1, L_2, is

$w = e_o i_o + e_1 i_1 + e_2 i_2$, and the torque $\tau = e_1 i'_1 + e_2 i'_2$

but $\quad i'_1 = i_1 - i_o$

and $\quad i'_2 = i_2 - i_o$

hence
$$\begin{aligned} w - \tau &= e_o i_o + e_1 i_1 + e_2 i_2 - (e_1 i_1 - e_1 i_0 + e_2 i_2 - e_2 i_o) \\ &= e_o i_o + e_1 i_o + e_2 i_o \\ &= i_o(e_o + e_1 + e_2) \end{aligned}$$

but $$e_o + e_1 + e_2 = 0$$

therefore $$w - \tau = 0.$$

Again the torque at each instant is just equal to the power, and hence, average torque must be equal to the average power and the meter registers correctly on both balanced and unbalanced loads.

It should be noted that no conditions have been imposed upon the character of the electromotive forces or currents and therefore, the demonstrations are true no matter what the form of voltage or current wave or what the power-factor may be. When the meter is theoretically correct and properly connected to the circuit it will register correctly under all conditions of load. That is, any inaccuracy will not be due to method of use, but will be due to faulty characteristics of meter.

Furthermore, it must be perfectly clear that two separate single-phase meters may be used in place of the polyphase meter and that the sum of their registration will give the true energy. One single-phase meter alone will not register the correct energy unless the separate phases are accurately balanced. As this is seldom the case it is best to use either two single-phase or one polyphase meter.

200. Polyphase Meters for Four-wire Three-phase Systems.—In Fig. 173 is shown a Y-connected four-wire three-phase receiv-

ing system. In Article 198 it was shown that a three-wire polyphase meter registered correctly when $e_o(i_o - i_1 - i_2) = 0$ and that under the conditions there assumed $i_o - i_1 - i_2$ always is zero. When, however, another wire is added, as shown in Fig. 173, i_o may no longer be equal to $i_1 + i_2$. When $i_o \gtrless i_1 + i_2$, w is no

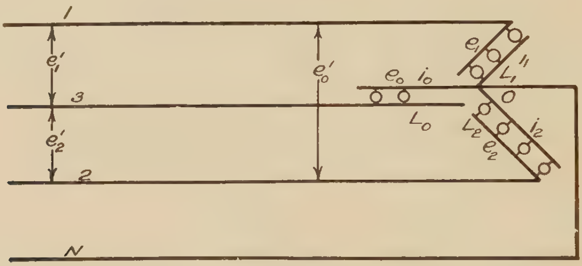


Fig. 173.

longer equal to τ and the meter registration is in error. Thus it is seen that a three-wire polyphase watt-hour meter cannot be used on four-wire circuits.

A combined Y and Δ four-wire receiving system is shown in Fig. 174. Representing the instantaneous currents and pressures

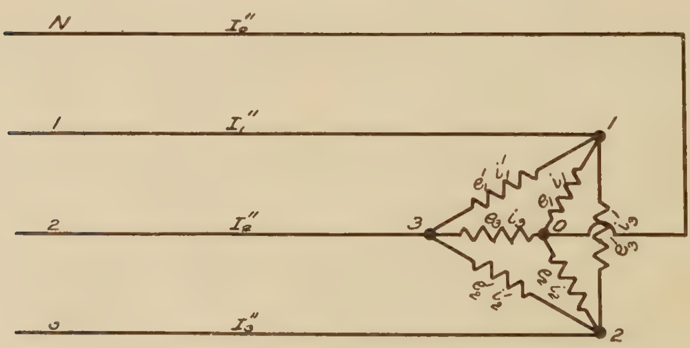


Fig. 174.

by i and e with proper subscripts and primes, the instantaneous value of power consumed in the system is

$$w = e_1 i_1 + e_2 i_2 + e_3 i_3 + e'_1 i'_1 + e'_2 i'_2 + e'_3 i'_3$$

but $e'_1 = e_1 - e_3$; $e'_2 = e_3 - e_2$; $e'_3 = e_2 - e_1$

Substituting these values of e'_1, e'_2, and e'_3 we get

$$w=e_1i_1+e_2i_2+e_3i_3+i'_1(e_1-e_3)+i'_2(e_3-e_2)+i'_3(e_2-e_1) \text{ or}$$
$$w=e_1(i_1+i'_1-i'_3)+e_2(i_2+i'_3-i'_2)+e_3(i_3+i'_2-i'_1)$$

but $\quad i_1+i'_1-i'_3=i''_1;\ i_2+i'_3-i'_2=i''_2;\ i_3+i'_2-i'_1=i''_3$

hence, $w=e_1i''_1+e_2i''_2+e_3i''_3$

e_1, e_2, and e_3 are the voltages between neutral N and mains 1, 2, and 3 respectively; and i''_1, i''_2, and i''_3 are the currents in the corresponding mains. To measure the energy, either three single-phase watt-hour meters or a specially designed polyphase meter is necessary.

A four-wire three-phase meter differs from a three-wire meter mainly in the winding of the current coils. The four-wire meter contains four series coils, two of which, one on each element, are connected in series and carry the current in one line wire. The other two series coils, one on each element, are separate, and each carries the current in one of the other line wires. A diagram of such a connection is shown in Fig. 175.

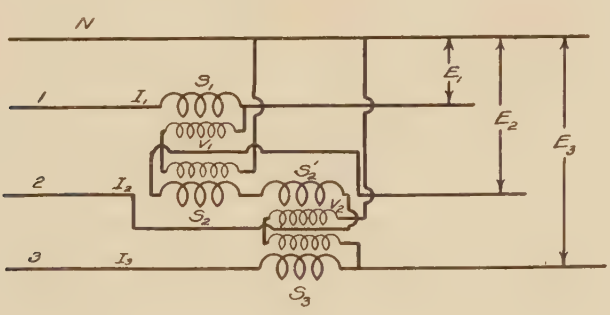


FIG. 175.

Representing the voltages between mains 1, 2, 3, and neutral by E_1, E_2, and E_3 respectively, then according to Article 200 the total power at any instant is equal to

$$w=e_1i_1+e_2i_2+e_3i_3.$$

The method of connections employed show that the driving torque in terms of electrical quantities must be

$$\tau=e_1i_1+e_1i_2+e_3i_2+e_3i_3.$$

Hence, $\quad w-\tau=e_1i_1+e_2i_2+e_3i_3-e_1i_1-e_1i_2-e_3i_2-e_3i_3$
$$=e_2i_2-e_1i_2-e_3i_2.$$

The current coils S_2 and S'_2 are reversed so that at unity power-factor I_2 is 60° out of phase with E_1 and E_3. This has the effect of changing the sign of $-i_2(e_1+e_3)$ and we may write

$$w-\tau=i_2(e_1+e_2+e_3).$$

$$\text{but } e_1+e_2+e_3=0$$

hence, the total energy will be registered by a four-wire meter when connected as shown in Fig. 175, however unbalanced the circuits may be.

201. Balance of Metering Elements.—In order that polyphase meters may register correctly when the circuits are unequally loaded, they must be adjusted so that the driving torques of the actuating elements are equal when the same amount of power is passed through each. If these two torques are not equal, the meter will run too fast when one side is carrying most of the load and too slow when the other side is loaded more heavily.

Since the driving torque is proportional to the product of the maximum values of current and voltage coil fluxes, it is evident that changing the number of turns on the voltage coil will change the torque. This is the method of balancing used by some makers. Another method based on the same fundamental principle consists in changing the reluctance of the path of the voltage flux. This is accomplished by the use of a short-circuited turn, called "balancing loop," upon the voltage coil core. Changing the position of this loop changes the reluctance of that part of the magnetic circuit, and causes more or less of the flux to pass through the disk and interact with the flux of the current coils. This again increases or decreases the torque of that element.

202. Interference of Elements.—One source of error to which polyphase meters are liable is due to the electromagnetic interaction between the elements. This source of error was investigated by the Electrical Testing Laboratories and it was found that different makes differed considerably in this respect. In some makes the effect of interference of the elements was so small that careful tests failed to detect any error due to this cause. In other makes, the interference was such that relatively serious errors might under certain conditions be produced. These facts were brought to the attention of the manufacturers whose meters were defective in this respect, with the result that

these defects have been remedied, and polyphase meters now on the market are practically free from errors due to this cause.

203. Effect of Power-factor on Operation.—In Articles 198 and 199 it was shown that a polyphase meter connected as shown in Figs. 171 and 172 will correctly register the total energy transmitted no matter whether the load be balanced or unbalanced.

The instantaneous torque on each element is

$$\tau_1 = e_1 i_1$$

and $\tau_2 = e_2 i_2$, where e_1, e_2, i_1, and i_2 are the instantaneous line voltages and currents, respectively. In a Δ-connected system e_1 and e_2 are the instantaneous voltages at load

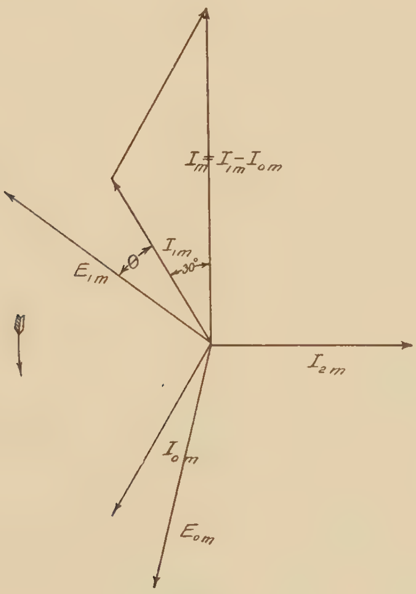


FIG. 176.

terminals, and i_1 and i_2 are the differences between currents in mains 1 and 0 and mains 2 and 0, Fig. 172. Thus, in Fig. 176, I_{1m}, I_{2m}, and I_{om} are maximum values of currents in branches *AO*, *BO*, and *AB* of Fig. 172. I_{1m} is shown as lagging θ degrees behind E_{1m}. The vector difference between I_{1m} and I_{om} is I_m which evidently lags 30 degrees behind I_{1m} and $(\theta + 30)$ degrees behind E_{1m}. The average torque on one element

is then $T_1 = EI \cos(\theta + 30)$ degrees, and by a similar process of reasoning it can be shown that the average torque on the other element is

$$T_2 = EI \cos(\theta - 30) \text{ degrees,}$$

where E is the effective pressure between mains and I is the effective value of current in mains.

When $\theta = 0$, $T_1 = T_2 = EI \cos 30$ degrees

When $\theta = 30$ degrees, $T_1 = EI \cos 60$ degrees

and $T_2 = EI$ or T_2 is twice T_1

when $\theta = 60$ degrees, $T_1 = EI \cos 90 \text{ degrees} = 0$

and $T_2 = EI \cos 30 \text{ degrees} = \frac{1}{2}\sqrt{3}EI$

That is, the total driving torque is exerted on one element only.

When $\theta = 90$ degrees, $T_1 = EI \cos 120 \text{ degrees} = -\frac{EI}{2}$

and $T_2 = EI \cos 60 \text{ degrees} = +\frac{EI}{2}$

In this case the two torques are equal and opposite. When θ is $>$ 60 degrees and $<$ 90 degrees, T_1 is negative while T_2 is positive. The effect of T_1 is thus to drive the meter in a direction opposite to that of T_2.

This clearly shows the importance of properly connecting a polyphase meter to a circuit.

204. Effect of Improper Connections.—A three-wire three-phase meter will in general have six free terminals; four for the series coils and one each for the two voltage coils. From Figs. 171 and 172 it is evident that the series coils may be connected in any two of the line wires, but that when so connected the free ends of the voltage coils must be connected to the third wire, preferably to the same point. It has already been pointed out that when properly connected, the torque on the two elements is in opposite directions on loads whose power-factor is less than .5. It is thus erroneous to assume that the meter will register correctly if the meter disk rotates in the proper direction when either voltage coil is disconnected Hence, disconnecting the voltage coils in succession and noting the direction of rotation cannot be used as a check upon the correctness of the connections unless the power-factor is known.

One wrong connection for a three-phase three-wire meter is shown in Fig. 177. The series coils S_1 and S_2 are properly connected but the voltage coil of V_1 is connected to main 2 instead of main 0 as it should be. When so connected, the instantaneous torques on the two elements are

$$\tau_1 = e_0 i'_1$$

and
$$\tau_2 = e_2 i'_2$$

The average torques on balanced circuits will be

$$T_1 = \text{average } e_0 i'_1$$

and
$$T_2 = \text{average } e_2 i'_2$$

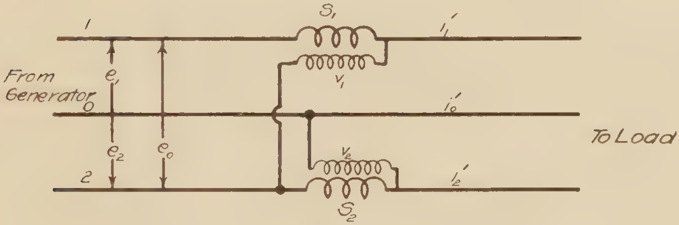


FIG. 177.

Now E_{om} is $(150+\theta)$ degrees out of phase with I_m as shown in Fig. 176. Hence

$$T_1 = EI'_1 \cos (150+\theta) \text{degrees}$$

$= EI \cos (150+\theta)$ degrees where E and I are effective voltage and current respectively. It has been shown, Article 203, that

$$T_2 = EI \cos (30-\theta) \text{ degrees}$$

When $\theta = 0$, $T_1 = -\frac{1}{2}\sqrt{3}EI$, and $T_2 = \frac{1}{2}\sqrt{3}EI$

The two torques are thus equal but in opposite directions. Reversing the connection of the pressure coil of V_1, its torque is reversed and the total driving torque is

$$T = T_1 + T_2 = \tfrac{1}{2}\sqrt{3}\ EI + \tfrac{1}{2}\sqrt{3}\ EI = \sqrt{3}EI$$

and the meter registers correctly. When $\theta = 30$ degrees, $T_1 = EI$ and $T_2 = EI$ and $T = 2EI$. The load, however, is $\sqrt{3}EI \cos 30$ degrees $= 1.5EI$, and the meter registers $33\frac{1}{3}$ per cent too high. This shows that although the meter registers correctly on load of unity power-factor, it will not register correctly on loads whose

power-factor is less than unity. The registration will also be incorrect when the three-phase system is unbalanced. For correct registration, the four-wire three-phase meter may be connected to the circuit in practically one way only. This is due to the fact that there is only one neutral wire, and the voltage coils must both be connected to this neutral conductor. It is very necessary then to know the exact order in which the mains are to be connected, and which is the neutral conductor. Since the voltage between the neutral and any line wire is less than between any two mains, which of the four wires is the neutral conductor

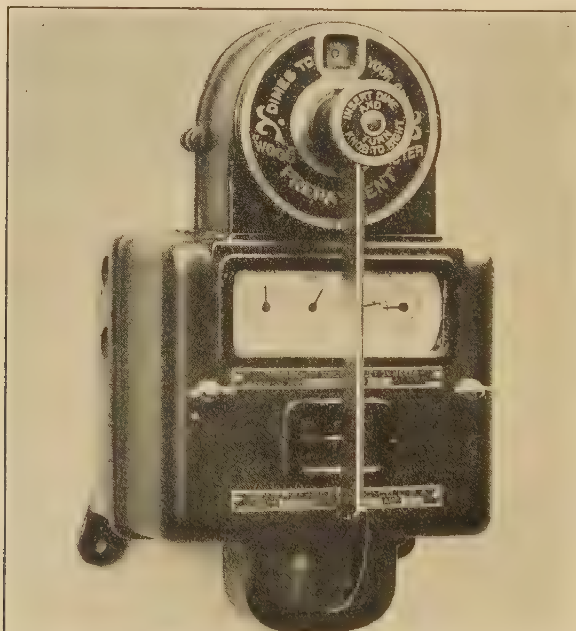


FIG. 178.

can easily be determined by means of a voltmeter. When this is determined, the diagram of connections furnished by the maker must be carefully followed.

205. Prepayment Watt-hour Meters.—In many instances it is advisable to collect pay for energy in advance of its use. For instance, the use of prepayment watt-hour meters simplifies the station bookkeeping, and relieves the proprietor of all responsi-

bility as regards electrical bills when meters are installed in apartment buildings whose tenants frequently change. For such and other service of like nature, prepayment meters have been developed.

The principles of operation of the meter proper are the same as those already discussed. That is, on the electrical side the instrument is either a direct or alternating-current watt-hour meter to which has been added a device which by the insertion of a coin and the turning of a knob automatically closes a switch and keeps the circuit closed until the energy paid for has been used, when the circuit is automatically opened. The external appearance of a General Electric prepayment meter is shown in Fig. 178.

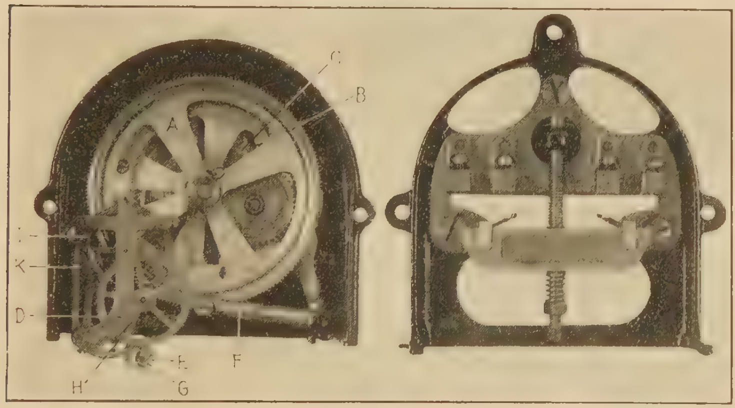


FIG. 179.

206. Prepayment Device.—One form of prepayment device is shown in Fig. 179. This consists of a drum the front of which is formed by the crediting dial and the back by the double gear wheel B. The wheel B contains both spur and annular gears, the second are not shown as they are within the drum. Within the drum is also an actuating spiral spring C, and two gear wheels. One of the gear wheels is mounted on the knob stem to which it is locked by the coin. The other intermediate gear plainly shown, is mounted on a stud which is fastened to the front dial plate. This intermediate gear meshes with the pinion on the knob stem and the annular gear of wheel B. The spur gear of the wheel B meshes with a pinion of the escapement mechanism D. This

mechanism is released by the operation of the registering mechanism of the meter proper, one gear of which meshes with pinion E.

207. Operation.—The prepayment mechanism is operated as follows: The coin that is inserted in the slot in the crediting knob stem acts as a key and locks the stem of the knob to the pinion on its end. On turning the knob one-half turn to the right, the pinion is carried with it, causing the intermediate gear to roll round on the annular gear of the wheel B and to carry with it the crediting dial. This action winds up the springs and at the same time, by the action of a cam, the switch lever F is moved upward, closing the circuit through the meter. This operation of crediting may be repeated until the coin register at the bottom of the meter shows that the full number of coins for which the meter is designed has been inserted. When current is taken the intermediate gear and dial are driven by the main spring in the opposite direction. The pinion E, which is driven by a gear of the meter registering train, carries a cam G. This cam as it revolves oscillates the bell-crank H, which in turn moves a finger backward and forward across the rim of the release gear I in mesh with the damper fan.

Pinion E makes one complete rotation during one oscillation of the finger; during the first half of the rotation, the finger displaces a catch from a pin set in the rim of the gear I, and during the second half of the rotation it is withdrawn from the pin. When the finger K, has returned to its outer position, the gears of the escapement mechanism are free to rotate under the action of disk B, which is driven by the main spring. The gear D makes one rotation every time the catch is released. A pin on this gear pushes the catch back into the path of the stop-pin on gear I and so arrests its motion after it has made the requisite number of rotations to permit the turning back of the crediting dial one place. At every rotation of pinion E this operation is repeated until the purchased energy has been used, when a cam on the dial-plate automatically opens the meter circuit.

It is very evident that since the prepayment device is controlled by the registering mechanism of the watt-hour meter, it can be used on watt-hour meters of any type. In fact, a slight modification of the device and the meter with which it is to be used, permits the installation of the prepayment mechanism at points distant from the meter. When the device is to be installed separately, the escapement mechanism is controlled by an electro-

magnet connected directly into the line. The excitation of the electromagnet is governed by suitable gears and the commutating device in the registering mechanism of the watt-hour meter.

There are several other designs of prepayment meters manufactured, especially in England. In this country the use of the prepayment meter is quite limited, consequently no other forms will be explained. The general appearance of the Fort Wayne prepayment meter with cover removed is shown in Fig. 180.

208. Bases of Energy Rates.—The cost of supplying electrical energy depends not alone upon the amount supplied but also upon the time and rate of supplying the same. Any equitable

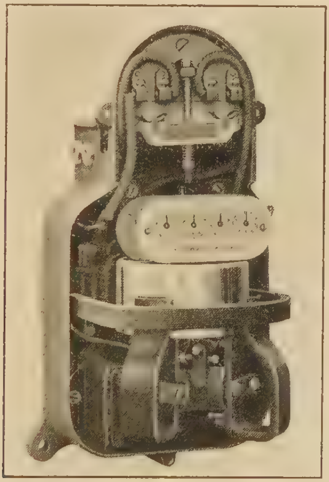


FIG. 180.

method of charging should take these things into consideration. The common method of charging, which allows discounts in proportion to the quantity of energy used, is just in some respects, but it does not take into consideration the time and rate of supply elements as mentioned above.

In order to take into consideration these two elements, two types of instruments have been devised. One type, known as the two-rate meter, permits the collection of two different rates; a relatively high rate for energy used during the peak of the load, and a relatively low rate during the rest of the day. This method of charging tends to discourage the extravagant use of electrical

energy during heavy load, and increase its use during periods of light loads, thus increasing the load factor. The second type of instrument is described in Chapter XV.

209. Two-rate Meters.—The essential difference between a two-rate or double tariff meter and a regular watt-hour meter is the addition of an extra registering train and clock. The clock controls a suitable switching-over clutch mechanism for throwing on or off the separate registering mechanisms. One registering mechanism indicates the energy consumed during the time of peak load, and the other during the other hours of the day. The clock automatically, and at the proper time, connects either the high or low rate train; a high rate being charged for energy consumed during the hours of the peak load. The use of a two-rate meter has not met with great favor in this country. From a mechanical standpoint the two-rate meter is practical and has been successfully carried out in practice.

CHAPTER XIV

INTEGRATING METERS, AMPERE-HOUR METERS

210. Introduction.—In many industries using electric current, it is advisable and often necessary to know the quantity of electricity that has been used within a given time. This is especially true in industries whose operation depends upon electrolytic processes, and in charging storage batteries.

The unit of quantity of electricity is the coulomb, and a coulomb has been defined as the quantity of electricity given by one ampere in 1 second. In 1 hour a constant current of one ampere will give 3600 coulombs. For practical purposes, the coulomb is too small a unit, and hence 3600 coulombs, called the ampere-hour, are used as the unit. Commercial instruments whose registrations are proportional to the quantity of electricity passing are called ampere-hour meters, and are of two types: electromagnetic and electrolytic.

211. Electromagnetic Type Ampere-hour Meter.—In the discussion on watt-hour meters it was shown that the registration is proportional to EIt. Hence, it is evident that if E remain constant the registration will be proportional to It. If t is in hours the scale may be graduated in ampere-hours. Thus, a watt-hour meter with constant voltage coil excitation may be made to register in ampere-hours.

Since alternating currents cannot be used for electrolytic processes, an alternating current-ampere-hour meter could be of little practical use. For direct currents the constant field excitation is best obtained by the use of permanent magnets.

The essential features of the Sangamo ampere-hour meter are shown in Fig. 181. The similarity between the ampere-hour meter and the mercury watt-hour meter is plainly evident. In place of the electromagnet of the voltage field the ampere-hour meter is provided with a permanent magnet. The operation of the ampere-hour meter is in every respect exactly like that of the watt-hour meter. The current flowing from one terminal through the disk to the other terminal reacts with the permanent magnet field. This reaction produces a torque upon the disk,

causing it to rotate. Since the field is constant the torque, and hence, speed, is directly proportional to current strength. The counter torque is obtained by rotating an aluminum disk, which is mounted on the shaft between the poles of permanent magnets exactly as on watt-hour meters. The dial is graduated in ampere-hours instead of watt-hours.

212. Accuracy Characteristics.—Two curves showing the percentage of accuracy of a 10-ampere meter with and without shunt are shown in Fig. 182. It will be observed that on currents below five and above 15 amperes the accuracy falls off quite rapidly.

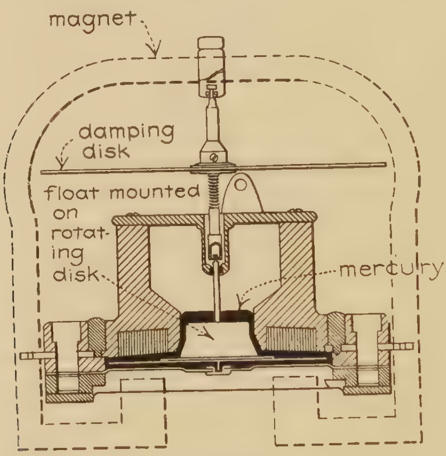


Fig. 181.

This falling off is much more rapid at low values of current than at the higher values. The cause of the deviation of the percentage of accuracy curve from a straight line at low values of current is undoubtedly due to the effect of friction, as no compensating device is used. The deviation at the higher values is due to the demagnetizing effect of current through the disk on the permanent magnet. An external view of Sangamo ampere-hour meter is shown in Fig. 183.

213. Electrolytic Ampere-hour Meters.—The principles according to which electrolytic or ampere-hour meters operate were discovered by Faraday, and are, therefore, known as Faraday's laws. These were discussed in Article 17. The passage of electricity through an electrolyte decomposes the chemical compound, and the mass of the metal deposited is a measure of the

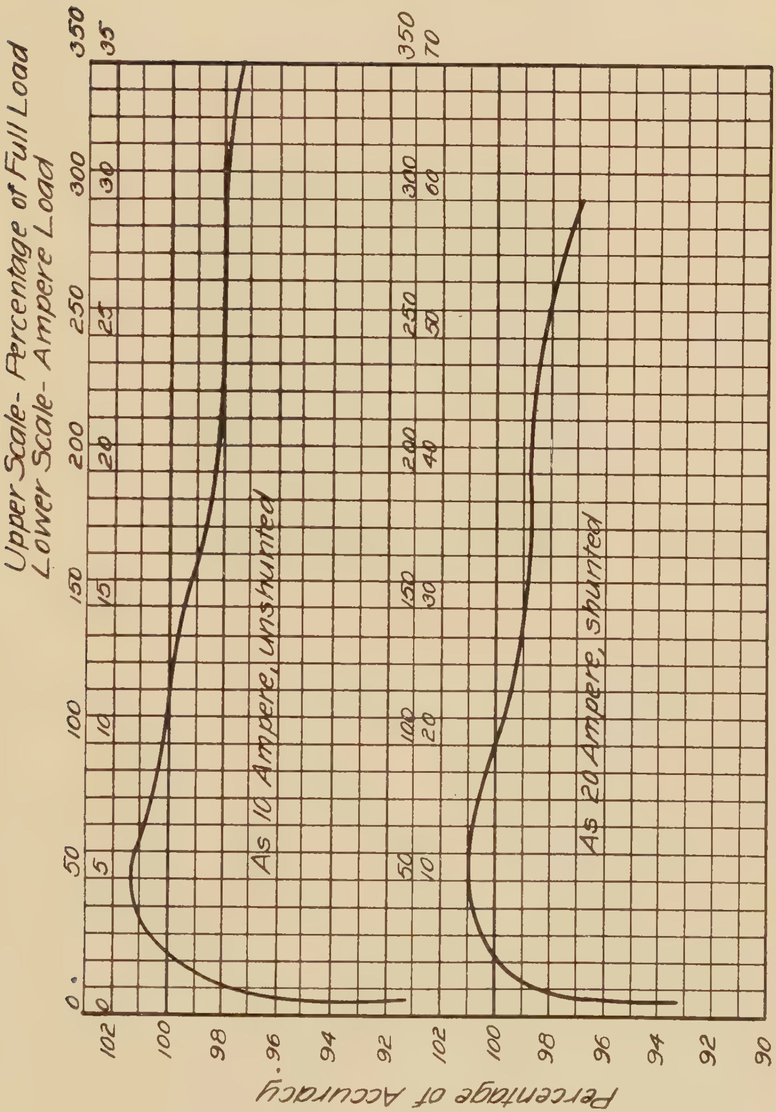

Upper Scale - Percentage of Full Load
Lower Scale - Ampere Load
As 10 Ampere, unshunted
As 20 Ampere, shunted
Percentage of Accuracy
0 50 100 150 200 250 300 350
0 5 10 15 20 25 30 35
50 100 150 200 250 300 350
10 20 30 40 50 60 70
102 100 98 96 94 92
102 100 98 96 94 92 90

Fig. 182.

ampere-hours. As long as the voltage remains constant, the energy transmitted is proportional to the ampere-hours; hence, on constant voltage circuits such instruments will indicate a quantity proportional to ampere-hours. The name wattmeter commonly applied to these instruments is plainly a misnomer.

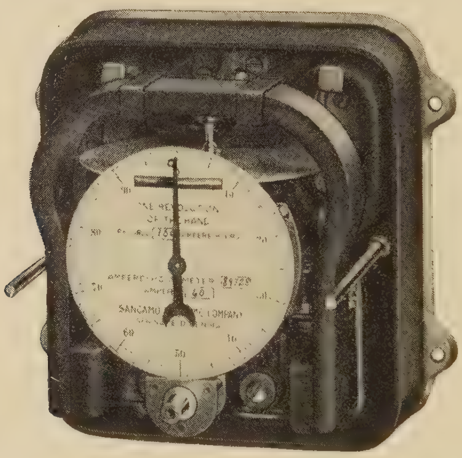


Fig. 183.

214. Edison Electrolytic Ampere-hour Meter.—The Edison electrolytic or chemical meter employed two zinc plates in a solution of zinc sulphate, the bottle containing the plates being placed in the meter at the beginning of the month and replaced by a similar bottle at the beginning of every month. One of the zinc plates (the anode) was carefully weighed before and after this term of service, and from the loss, the current, and hence the monthly bill, was calculated. To pass the whole current through the bottle was impracticable, on account of the large size of plates that would be needed; the bottle was, therefore, put in a shunt circuit, and only about 1/1000 part of the main current passed through it. This shunting caused a large part of the theoretical accuracy and reliability of the electrolytic meter to disappear; the voltage-causing electrolysis being very low at light load, and polarization in the cell, or abnormal resistance due to oxidation of the plates, would make the meter indications too low.

This meter was used to a considerable extent, and may, perhaps, still be found in some installations.

215. The Bastian Ampere-hour Meter.—An electrolytic meter that is still on the market, and perhaps the simplest in construction, operates by decomposing water. The whole current passes through acidulated water, decomposing it into its constituent gases which are allowed to escape. The drop in the elevation of the liquid is a measure of the ampere-hours supplied to the consumer.

In the older type, two platinum electrodes are suspended in the liquid at the bottom of a long glass tube open at the top. The bore of the tube is as uniform as possible throughout its length. The suspending leads are enclosed in two vulcanite tubes screwed into a vulcanite frame which forms a protection for the platinum electrodes enclosed by it. A scale graduated in watt-hours or kilowatt-hours at a definite voltage is fixed in front of the tube in such a manner that the level of the electrolyte can be readily read. The glass vessel is enclosed in a cast-iron or sheet-iron case in front of which is a long window. The electrolyte is a dilute solution of sulphuric acid and water. Since only the water is decomposed, it is necessary to refill the tube with water alone. To prevent evaporation, paraffin oil is poured on top. There are several objections to this meter. Among the most important are the following:

1. Large and variable pressure drop
2. Necessity for refilling with water
3. If left too long in circuit all record is lost
4. Inaccuracy.

The pressure drop across the electrolyte is never less than 1.5 volts, and in a five-ampere meter, under full-load may be three volts. This drop is variable, depending to some extent upon the height of the column of the liquid in the tube.

Since different consumers use different quantities of energy, the refilling of the meters becomes quite complicated. If left too long all record is lost, and disputes are liable to arise between the consumers and the supply company.

The advantages claimed for this meter are:

1. Extreme simplicity
2. Small chance of getting out of order, when properly cared for
3. Low first cost.

In a later form of meter the platinum electrodes have been replaced by nickel, and the electrolyte is an alkaline solution which has no action upon the electrodes. The substitution of nickel for platinum permits the use of larger electrodes at a reasonable cost, and the resulting pressure drop is less. Fig. 184 shows the complete instrument.

FIG. 184.

CHAPTER XV

DEMAND INDICATORS

216. Introduction.—In Article 208 it was mentioned that two types of instruments have been devised for the purpose of making energy rates more equitable. The two-rate meter has been briefly mentioned. With the other type of instrument, known as the maximum demand indicator, the charge is at a rate depending upon the ratio of consumption to the maximum demand. These instruments may be classed under three heads: thermal, induction, and mechanical.

217. Thermal Type.—The main features of the Wright Maximum Ampere Demand Indicator whose operation depends upon the heating effect of the electrical current are shown in Fig. 180. The principle of operation is that of the recording thermometer. That is, the meter does not directly indicate the maximum ampere consumption, but indirectly by heating a column of air whose expansion is proportional to the square of the current passing. As shown in Fig. 186 the instrument consists of a U-shaped tube with a bulb at each end, partly filled with sulphuric acid and hermetically sealed. A resistance of platinoid is wound around bulb A and connected in series with the current or a shunted part of the current. The heat generated by the current flowing through the resistance expands the air in the left bulb, and this expansion forces the liquid into the right hand part of the U. As the liquid rises above a certain height, it flows over into the index tube. The amount

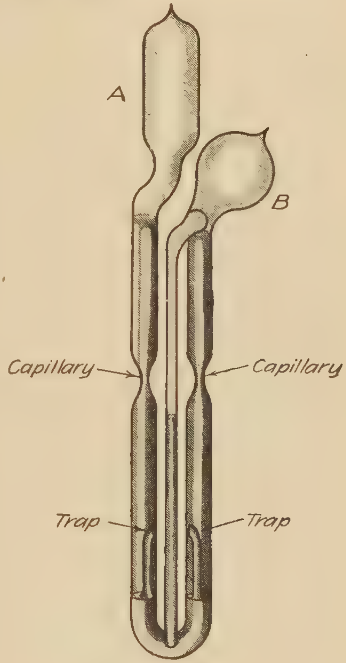


FIG. 186.

of liquid flowing over into this tube is proportional to the expansion of the air in bulb A above that in bulb B. The expansion of the air is proportional to the square of the current flowing through the resistance coil. If the tube has a uniform bore, the height of the liquid in the indicator tube will be a measure of the square of the maximum current. The heat generated by a current in a resistance is equal to

$$H = I^2Rt$$

where I is current, R resistance, and t is the time. It will thus require some time for the air in tube A to reach a maximum temperature, or a temperature sufficiently high to indicate maximum current. The manufacturers state that if the load continues for about 40 minutes the full 100 per cent is indicated. Momentary overloads are not recorded. An overload continuing for some time will, however, fill the index tube and make an accurate record impossible.

Since the indication is proportional to the square of the current, the scale cannot be uniform if the bore of the indicator tube is uniform. An examination of the scale will show that the divisions increase from the bottom up.

After a reading is taken, the instrument is reset by tilting and allowing the liquid to flow back into tube B. To prevent the passage of air from one side of the U to the other, small inverted glass funnels called "traps" are rigidly fastened to the bottom of the U. When the indicator is being reset, the traps remain covered by the liquid, preventing the passage of air into the wrong side of the tube.

Indicators whose maximum capacity is 25 amperes may be used interchangeably on direct- or alternating-current circuits without shunts. Indicators of larger than 25 amperes and less than 200 amperes are provided with shunts and may be used on either direct or alternating currents. Current transformers must be used in all cases on alternating-current circuits where the voltage exceeds 1150 volts, and also where the current capacity is over 200 amperes.

The fact that the indications of the thermal type of indicator depend upon the square of current only, makes it evident that it will not give correct indications of maximum power on circuits whose power-factor is other than unity, or on circuits of variable voltage.

218. Induction Type.—The registration of the induction type maximum demand indicator is determined by the maximum power consumed for a definite time and not upon the maximum current, as in the case of the thermal type. Since the deflection of a wattmeter is determined by the power, it is evident that an ordinary wattmeter could theoretically be used to indicate the maximum watt demand if the meter were provided with another pointer which would remain at the point of maximum deflection. Since it is not desirable to keep records of momentary fluctuations, such a scheme is not used. The recording wattmeter records not only the maximum and minimum demand but also the duration of such a demand.

Since the torque on the movable element of the watt-hour meter varies with the load it too may, with some modifications, be used to indicate the maximum watt demand. The polyphase maximum watt demand indicator of the General Electric Company, shown in Fig. 187, is essentially a polyphase induction watt-hour meter with both actuating elements acting upon the top disk.

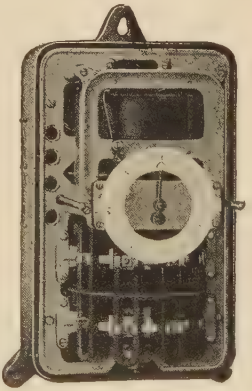


Fig. 187.

To secure the proper time lag, the retarding system consists of several powerful permanent magnets arranged around the periphery of the lower disk. In addition to the retarding effect of the permanent magnets, the motion of the movable element is opposed by three spiral springs connected in series. There are enough convolutions in these springs to permit the disks to make three complete rotations. The regular registering mechanism is replaced by a circular dial with a scale over which move two pointers. One of these pointers is driven by the movable element of the instrument by means of a reducing gear so that it makes only one complete revolution while the movable element makes three. The second pointer is moved by the first in a forward direction only, and is left at the extreme position reached by it, being held in place by a ratchet. The second pointer indicates, therefore, the maximum energy that has passed through the indicator since it was last set. The second pointer is reset by a thumb nut. Since the driving

torque of an induction watt-meter is proportional to the power, and the reaction of the spiral spring is equal to the torque, it follows that the scale of such an indicator is uniform.

219. Time Lag.—As already mentioned, momentary fluctuations of load would not be a just basis for rate making, as they might not indicate a legitimate demand. Hence, some time should elapse before the indicator pointer reaches its extreme position. This time interval is controlled by the amount of magnetic damping of the disk and counter-torque of spiral spring. The following demonstration, which may be omitted by students unacquainted with the calculus, shows this:

Assuming the change in the load to be instantaneous, the driving torque is proportional to the change in load, and is constant so long as the load is constant. The counter-torque is due to the damping magnets and to the reaction of the spiral controlling spring. The torque due to damping magnets is proportional to the speed of the disk, and the torque due to spring is directly proportional to the deflection. This may be expressed by

$$\text{Torque} = K_o\omega + K_1\theta$$

where ω is the angular speed, and θ is the deflection.

But $\omega = \frac{d\theta}{dt} =$ rate at which deflection is changing.

Then $\text{Torque} = T = K_o\frac{d\theta}{dt} + K_1\theta$

and $Tdt = K_o d\theta + K_1\theta dt$

whence $dt = \frac{K_o d\theta}{T - K_1\theta}$

Integrating between $t=0$, and $t=t_1$, which limits for time correspond to $\theta=0$, and $\theta=\theta_1$ we get

$$t_1 = K_o \int_o^{\theta_1} \frac{d\theta}{T - K_1\theta}$$

$$= \frac{-K_o}{K_1} \log \frac{T - K_1\theta_1}{T}$$

This shows that t_1 is a logarithmic function of θ

Solving the above for $K_1\theta_1$, we get

$$K_1\theta_1 = T\left(1 - e^{\frac{-K_1}{K_o}t_1}\right)$$

The time lag is defined as the interval of time taken to record 90 per cent of any change in load, which amounts to the same thing as to say that it is the interval of time required for the pointer to deflect 90 per cent of the maximum deflection due to a given change in load. According to the equation for $K_1\theta_1$, when the pointer has deflected $\frac{1}{n}th$ of the maximum deflection

$$K_1 \frac{1}{n}\theta_1 = \frac{1}{n}T.$$

Then
$$\frac{1}{n}T = T\left(1 - \frac{1}{e^{\frac{K_1}{K_o}t_2}}\right)$$

where t_2 is the time required for a deflection of $\frac{1}{n}\theta_1$

$$\frac{1}{n} = \left(1 - \frac{1}{e^{\frac{K_1}{K_o}t_2}}\right)$$

$$1 - \frac{1}{n} = \frac{1}{e^{\frac{K_1}{K_o}t_2}}$$

and
$$e^{\frac{K_1}{K_o}t_2} = \frac{n}{n-1}$$

and
$$\frac{K_1}{K_o}t_2 \log e = \log\frac{n}{n-1}$$

and
$$t_2 = \frac{K_o}{K_1}\,\frac{\log\frac{n}{n-1}}{\log e}$$

If the deflection is to be 90 per cent of the maximum deflection

$$\frac{1}{n} = .9$$

and
$$n = \frac{10}{9}$$

Then
$$t_2 = \frac{K_o}{K_1}\,\frac{\log 10}{\log e} = 2.3\,\frac{K_o}{K_1}$$

This shows that the time lag t_2 depends mainly upon the ratio of K_o to K_1. Now K_o is determined by the strength of the field of the permanent magnets, and their distance from the shaft.

K_1 is determined by the physical properties of the controlling spring. K_o is large when the permanent magnet field is strong, and K_1 is small when a spring of many convolutions is used. t_2 then is large when many permanent magnets and weak control springs are used. It is also clear that changing either one of these factors will change the time lag. Curve of, Fig. 188,

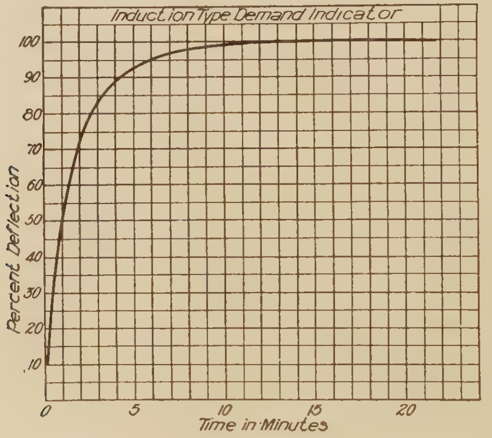


FIG. 188.

shows the relation between the per cent of deflection and time. If K_o and K_1 were known, this curve could be plotted from the equation

$$K_1\theta_1 = T\left(1 - e^{\frac{-K_1}{K_o}t_1}\right)$$

This instrument has the advantage that the maximum demand can be read at once from the dial, and that it is easy to maintain. The disadvantage is that no record of demand is obtained other than that taken down by the meter reader, and that the time at which the maximum demand occurred is not indicated.

The polyphase watt demand indicator may be used on single-phase as well as polyphase circuits. When it is to be used on single-phase circuits, the current coils are connected in series and the voltage coils in parallel.

220. Mechanical Type.—For the want of a better term, we shall call the third form of demand meter the mechanical type, since it is merely an automatic printing attachment to a watt-hour

meter. The attachment is operated by clockwork in connection with the registering mechanism of the meter, while the printing device leaves a record of the rate of the use of energy on a moving tape.

221. Operation.—The mechanical features of one make of the "Printometer" as this type of demand register is called is shown in Fig. 189.

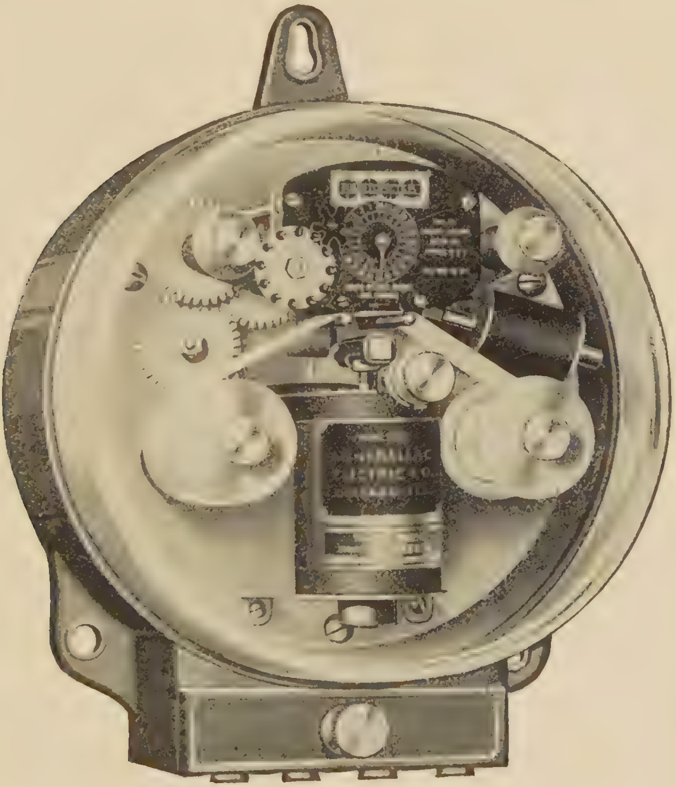


FIG. 189.

The printometer can be attached to any type of watt-hour meter, and when so connected the type or recording wheels of the instrument are electrically interlocked with the registering mechanism of the meter. The movement of the recording mechanism of the printometer is thus in synchronism with the movement of the meter register, and hence the record is an indication of the rate at which energy is being used. The printometer

record is made on a paper tape by means of a copying ribbon which, at regular intervals, is pressed against the type wheels by a rubber platen. The rubber platen is actuated by a solenoid whose circuit is closed at regular intervals by a contact-making clock.

In addition to the energy-recording mechanism, the instrument contains an hour wheel containing numbers from 1 to 24. This hour wheel is also automatically advanced so that by every imprint of the energy there is left a record of the time. This hour wheel is connected to the printing platen through a star wheel and pins, which are plainly shown in Fig. 189. By changing the position of the pins, the wheel can be advanced in such a way as to give readings every hour, half hour, 20, 15, 10, or 5 minutes. Fig. 190 shows a record of hourly readings.

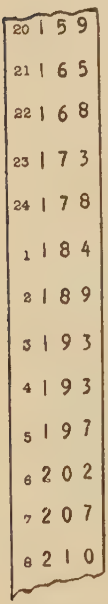


Fig. 190.

The circuit of the solenoid that advances the type wheels is closed through a commutator, which is mounted on one of the spindles of the meter register. This is shown in Fig. 191. On the end of this commutator is a slip ring which is connected to a number of bars across the face. The number of bars depends upon the constant to be used with the attachment. There are three contact-brushes, one bearing upon the slip ring, and the other two at opposite points on the commutator in such relative positions that they alternately close the circuit of the type-wheel actuating solenoid. To prevent the destruction of the commutator by sparking, the circuit is broken by the forward movement of the plunger of the solenoid. As this moves forward it turns a contact wheel which is made of alternating segments of conducting and non-conducting material joined by a metal slip ring. Three brushes make contact with this wheel; one with the slip ring, and the other two with the segments. The distance between the two brushes is such that when one rests on a conducting segment the other rests on a non-conducting segment. The operation of the commutator and contact wheel in closing and breaking the circuit is much the same as that of two three-way switches. The closing of the circuit by the commutator on the registering mechanism of the meter excites the solenoid and drives the plunger forward. The turning of the contact wheel by

the forward movement of the plunger changes the position of the brush on the conducting segment to a non-conducting segment, thus breaking the circuit which again will be closed after the commutator on meter register has rotated the proper distance. The solenoid circuit is thus closed by the slow moving commutator and opened by the quickly moving contact wheel.

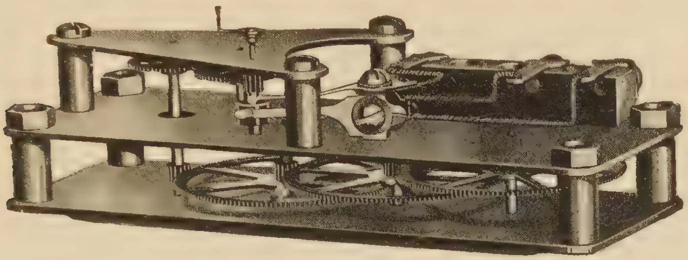


Fig. 191.

Another form of maximum demand indicator is shown in Fig. 192. This form of indicator is merely a combination of a maximum demand indicating device with the registering mechanism of a watt-hour meter. To the registering mechanism is added a pointer which is held in any position by friction. This pointer

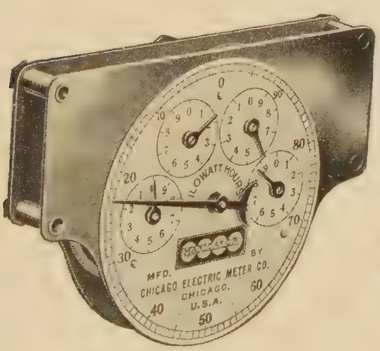


Fig. 192.

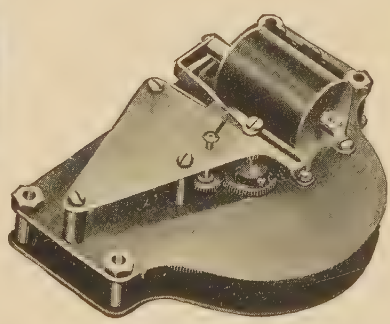


Fig. 193.

is driven forward by means of another pointer which is mechanically connected with the registering mechanism and at the end of every 30-minute interval is automatically set back to zero leaving the indicator proper at its extreme position. The "set back" element is actuated by means of a solenoid mounted on the back of the registering mechanism, as shown in Fig. 193. The circuit

of the solenoid is closed every 30 minutes by means of a contact element mounted on the spindle of an induction motor. The motor is geared so that this spindle makes one revolution every 30 minutes. On closing the solenoid circuit, the plunger is raised and the driving pointer is returned to zero. During the 30-minute interval the plunger descends, driving the entire registering mechanism, the meter acting only as a regulator in controlling the speed. By means of such an arrangement, the consumption of energy for the operation of the instrument is reduced to a minimum.

222. General.—The maximum demand instrument is used chiefly in connection with the sale of energy in large quantities, especially when this energy is supplied by polyphase systems at high voltage. The use of maximum demand instruments for small consumers is on the decrease. The thermal type of demand indicator is not well adapted for polyphase circuits, as the maximum demand in this case is based on the kilowatts rather than kilovolt-amperes. Under these circumstances, the use of the maximum watt demand indicator is preferable.

CHAPTER XVI

INSTRUMENT TESTING

223. Introduction.—Every person who has the care and use of electrical instruments of any kind should observe certain precautions in handling them. This is especially true with reference to portable instruments which are easily damaged by careless or ignorant use.

224. General Precautions.—Too much care cannot be exercised in connecting apparatus for experimental or test purposes. The student should in every case do his thinking in advance and not depend upon correcting mistakes after some trouble has developed. Avoid trouble by arranging everything properly before beginning the test. The supply mains should always be connected through a double pole in a two-wire system or a three-pole switch and each circuit should be protected by fuses. The main switch should be left open until all other connections have been made. When everything is properly arranged, a diagram should be made of the connections and examined in order to be sure that they are not endangered by overloads or short circuits. When everything has been examined, the main switch may be quickly closed and opened to see if there is any indication of short circuits; if everything is in correct working order the main switch may be closed, and the work proceed; a sharp lookout for trouble during the early stages of the work must, however, be maintained. The best general direction to a student is—do not guess—be sure you are right and then go ahead. While making adjustments, gently tap the dials of all indicating instruments so as to free them from frictional errors.

225. Kinds of Tests.—The kind of test to be used in any case depends upon the degree of accuracy desired and facilities for conducting the test. In practice, there are two kinds of tests; these may be called Standardization and Checking tests. The

standardization test consists in comparing the readings of the instrument tested with the fundamental units; the checking or comparison test consists in comparing the readings of the tested instrument with the readings of a similar instrument that has been previously standardized. The second test is the one usually used in central stations, especially the smaller ones.

226. Apparatus for Instrument Testing.—Primary standard instruments, that is, instruments for comparing meters with fundamental units are not suitable for general use, but should be maintained in a few well-equipped laboratories. Large central stations may also be justified in keeping an equipment of these instruments. The instruments used to check working instruments may properly be called secondary standards. From time to time the secondary standards should be checked by some reliable standardizing laboratory, the Bureau of Standards at Washington, D. C., being one of the best. The number and kind of secondary standards to be provided will depend considerably upon the number and kind of working instruments. The most essential secondary instruments are portable indicating ammeters, electrodynamometers, voltmeters, wattmeters, variable resistances or rheostats, and one or more of the portable standard integrating watt-hour meters. If some of the simpler standardizing tests are to be made, it will be well to include standard cells, galvanometer, potentiometer, and standard shunt resistances.

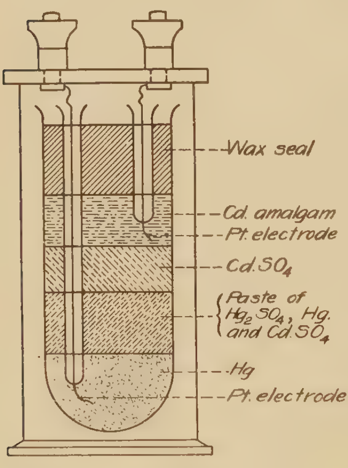


FIG. 194.

227. The Standard Cell.—The International Electrical Congress of 1908 officially adopted the Weston cell as the standard of electromotive force. This cell is constructed of mercury surrounded by mercurous and cadmium sulphate paste for the positive pole; cadmium amalgam for the negative pole; and a solution of cadmium sulphate for the electrolyte. The cells must be built up according to definite specifications, and when so constructed

pressures of different cells agree to a few thousandths of 1 per cent when tested under like conditions. A cross-section of such a cell is shown in Fig. 194.

On January 1, 1911, the Bureau of Standards adopted a new value for the electromotive force of the Weston normal cell, namely:

$$E = 1.01830 \text{ international volts at } 20°\text{C}.$$

The effect of temperature on the Weston cell is slight, so that for commercial measurements no corrections need be made. For more accurate measurements the electromotive force may be calculated from the following formula:

$$E_t = 1.01830 - 0.0000406(t° - 20°) - 0.00000095(t° - 20°)^2$$

where the temperature $t°$ is in centigrade degrees. This new unit of e.m.f. is larger than the old, the change being equal to about 0.08 of 1 per cent in the value of the international volt. This change affects to a slight extent all measurements of the electric current, electromotive force and power, and in some cases necessitates slight changes in measuring instruments.

228. Galvanometer.—It is beyond the scope of this text to give an extended discussion of all of the characteristics of a galvanometer, since it is usually classed as a laboratory instrument. The principle of operation of the galvanometer is the same as that of the permanent magnet, movable-coil ammeter. In fact, the ammeter was developed from the galvanometer. The movable coil of the galvanometer contains many turns and is suspended between the poles of a strong permanent magnet. The controlling force is due to the twisting of the suspension fiber, and the deflection is read by means of a telescope and scale. One make of galvanometer suitable for use with a potentiometer is shown in Fig. 195. The essential difference between a galvanometer and a milliammeter is the high sensibility of the former. This higher sensibility permits measurements of a much higher degree of accuracy.

229. Potentiometers.—The potentiometer is a combination of accurately adjusted resistances used for the comparison of unknown electromotive forces with the electromotive force of a standard cell. For simplicity potentiometers may be considered under two heads; namely, the slide-wire type and deflection type.

230. Slide-wire Type.—The fundamental principle upon which the operation of either form is based is Ohm's law, or in other words, the fact that the voltage drop along a conductor is directly proportional to the resistance so long as the current remains constant. The application of this principle to the comparison or measurement of electromotive forces will be best understood by reference to Fig. 196. This figure shows a storage battery connected in series with a high resistance AB. A part of this resistance is shunted by a circuit CD, the contact D being movable. In the shunt circuit is connected a sensitive galvanometer G and the unknown source of e.m.f. E. The connections of E are reversed with reference to those of the storage battery.

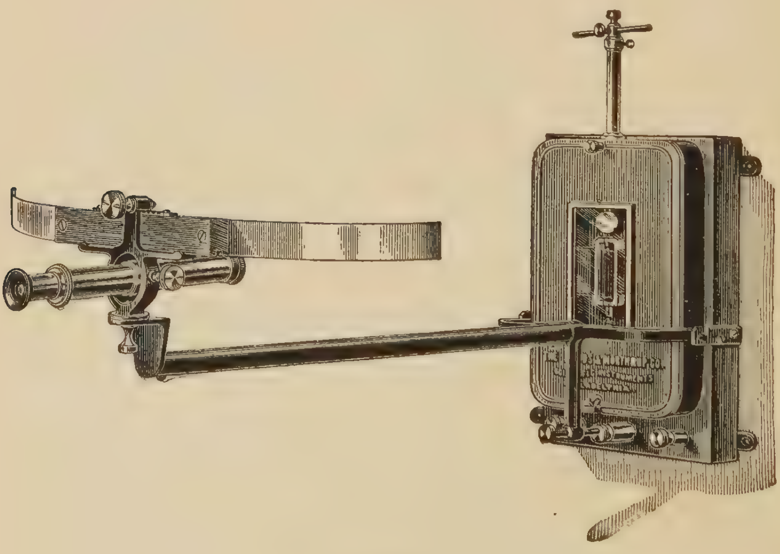


FIG. 195.

231. Operation.—When a measurement is to be made, the contact, D, is moved either to the right or left, as the case demands, until the galvanometer shows no deflection. When this condition is reached there is no current flowing through the shunt circuit, and if I is the current in the battery circuit, whose resistance from C to D is R, the voltage drop between CD equals IR. Since

no current flows through the shunt circuit, this voltage must equal the unknown voltage E, or

$$E=IR.$$

If now E be replaced by a standard cell and the position of D be again adjusted until the galvanometer shows no deflection, the voltage drop is equal to the e.m.f. of the standard cell. Let this new resistance between C and D be R_1,

then $$E_s=IR_1$$

and $$\frac{E}{E_s}=\frac{IR}{IR_1}=\frac{R}{R_1}$$

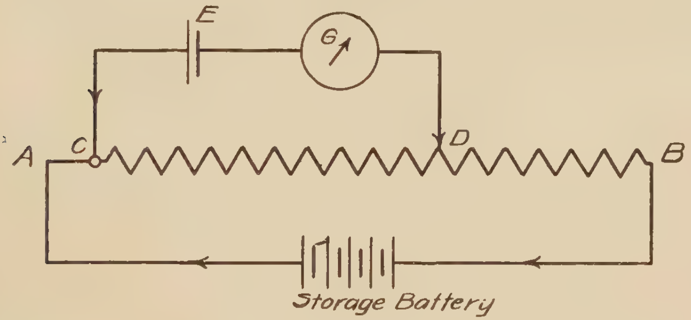


FIG. 196.

Thus, if the instrument is adjusted so that R and R_1 can be read off from a dial, E can be calculated, for

$$E=E_s\frac{R}{R_1}$$

232. Leeds & Northrup Potentiometer.—One make of the slide-wire type of potentiometer is shown in Fig. 197. A diagram of the internal connections of this instrument is shown in Fig. 198. The similarity between this and the diagram of Fig. 196 is plainly evident. As shown in the diagram, the essential part consists of three resistances D to A, A to C, and C to B, connected in series. The contact points of resistance, D to A, are marked to correspond to the electromotive force of the standard cell when corrected for temperature according to formula of Article 227.

The resistance, A to C, consists of fifteen 5-ohm coils adjusted to a high degree of accuracy. CB is a 5.5-ohm slide-wire wound around a marble cylinder.

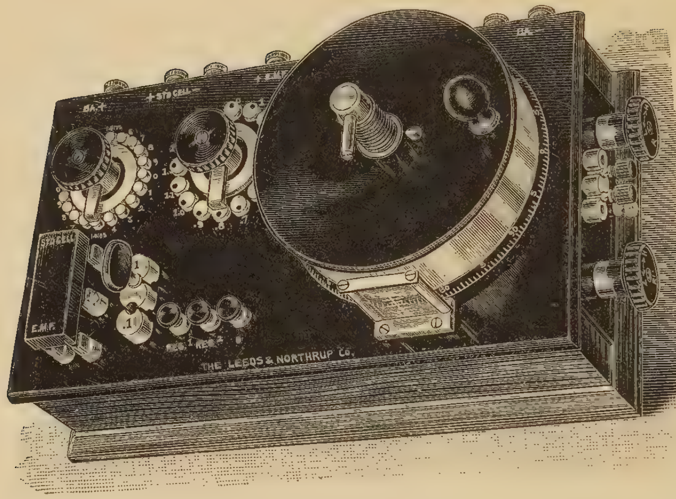


FIG. 197.

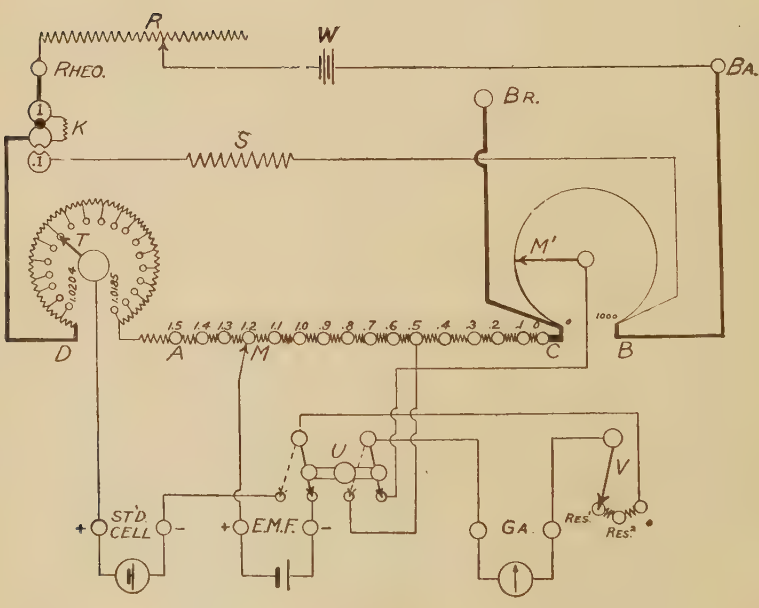


FIG. 198.

233. Operation.—When an unknown electromotive force is to be measured, the connections are made as indicated in the diagram, Fig. 198. The contact T is set at a point corresponding to the e.m.f. of the standard cell when corrected for temperature; the switches U and V are moved to the left, and key V is closed. If the galvanometer shows a deflection, rheostat R is adjusted until the galvanometer shows no deflection when V is moved to R_o and closed, and balanced is again obtained by adjusting R. When this adjustment has been made, the current through AC is exactly 0.02 ampere. The voltage drop across any one of the 5-ohm coils is consequently one-tenth of a volt, and that across CB is .11 volt. It will be observed that the unknown e.m.f. is connected between M and M' in series with the galvanometer. Since both M and M' are movable, the maximum voltage drop

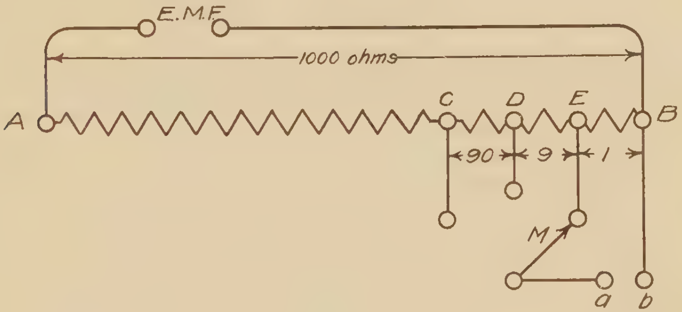


FIG. 199.

from A to B is 1.61 volts. To measure the unknown e.m.f. M' is placed at zero, U is moved to the right and M is moved from lower to higher values until a change of one contact reverses the direction of the deflection of the galvanometer. M is then moved back one point so the deflection is in the same direction as before, and M' is adjusted until the galvanometer shows that no current is flowing when V is on R_o. The value of the unknown e.m.f. may then be read from the positions of M and M'. The value of the e.m.f. is marked on the dial in place of the resistance.

To measure pressures higher than 1.5 volts the unknown pressure must be connected across a high resistance commonly called volt box, while the potentiometer terminals are shunted across a definite fraction of the high resistance. This method of connection is shown in Fig. 199. The potentiometer is connected to

a and b, and by moving switch M the potentiometer reading may be 0.1, 0.01, or 0.001 of the unknown e.m.f. Current measurements can also be made on the potentiometer by measuring the drop in volts across a standard shunt. The current is calculated by Ohm's law. The connections for current measurements are shown in Fig. 200, in which S represents the standard shunt whose resistance is known. The potentiometer terminals are represented by a and b. For precision measurements of direct current and electromotive force, the potentiometer method is far superior to any other, and is today the standard method for such measurements. For commercial work, however, which does not require such a high degree of accuracy the method possesses several disadvantages. The two chief disadvantages are the time required to make the measurements, and the cost of the apparatus.

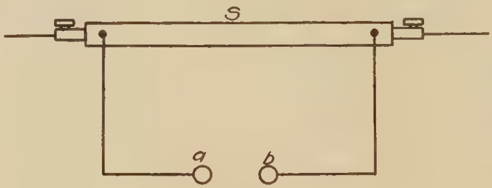


Fig. 200.

234. Deflection Type Potentiometer.—For checking portable instruments Mr. H. B. Brooks of the Bureau of Standards has designed a potentiometer which combines to some extent the accuracy of the slide-wire potentiometer and the speed of operation of deflection instruments.

It was pointed out that the final adjustments of the Leeds and Northrup potentiometer are made by moving contact M'. This adjustment is time consuming and for most commercial measurements unnecessary, were it possible to read or estimate the unbalanced electromotive force. Brooks' deflection potentiometer differs from the slide-wire form mainly in that it permits the reading of the unbalanced e.m.f. and the modifications necessary to bring this about.

Fig. 201 shows the complete deflection-type potentiometer as designed by Mr. Brooks and built by Leeds & Northrup Co. It is seen that the galvanometer is built into the apparatus, so that the instrument is self-contained with the exception of the standard

cell and auxiliary storage cell. If the instrument is to be portable, the storage cell could be replaced by two dry cells, which, with the standard cell, could be enclosed in the case, making the entire apparatus self-contained.

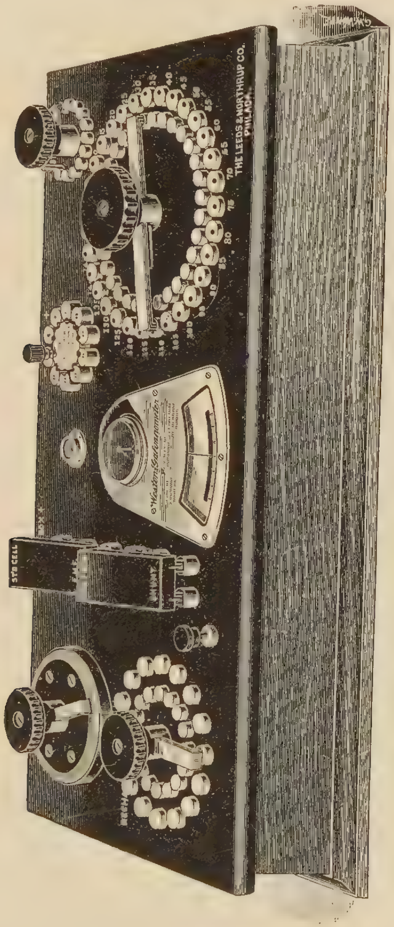


Fig. 201.

235. Theory and Operation.—The essential principles of the instrument will be understood from Fig. 202, which is a diagram of the connections of a simple potentiometer for measurement of voltage and current.

When voltages higher than the voltage of the standard cell are to be measured, it is necessary to use a volt box or some high resist-

ance as already pointed out. Such a connection is shown at (*a*), Fig. 202. R is such a resistance. A suitable fraction of this resistance R/p is connected to the circuit r_1r_2. When r_1 has been adjusted so that the galvanometer shows no deflection, we have:

$$\frac{1}{p}E = \frac{r_1}{r_1+r_2}e_1$$

or

$$E = r_1 \frac{e_1}{r_1+r_2} p$$

where p is the ratio of the whole resistance R to the portion R/p; or p is the multiplying factor of the volt box. The current through galvanometer is then

$$i_g = \frac{e_1 \dfrac{r_1}{r_1+r_2} - \dfrac{E}{p}}{r_g + \dfrac{r_1r_2}{r_1+r_2} + R\dfrac{p-1}{p^2}}$$

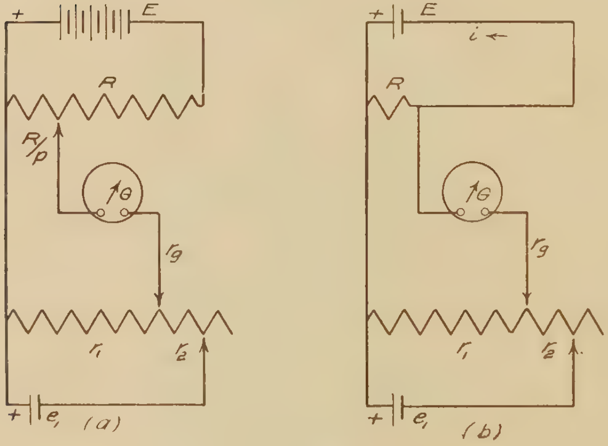


Fig. 202.

The first term in the numerator of this expression is the voltage drop in the portion r_1 of the potentiometer wire when the galvanometer circuit is open; it is therefore numerically equal to the setting of the potentiometer. The second term in the numerator is the voltage drop which would exist around the portion R/p if the galvanometer circuit were open. The denominator is the total resistance in the galvanometer circuit. The expression

shows that the current through the galvanometer is equal to the unbalanced portion of the e.m.f. divided by the total resistance of the galvanometer circuit.

Similarly, by the aid of (*b*), Fig. 202, it can be shown that when potentiometer is used for current measurements the current through the galvanometer is given by

$$i_g = \frac{e_1 \dfrac{r_1}{r_1 + r_2} - E}{r_g + \dfrac{r_1 r_2}{r_1 + r_2}}$$

This and the preceding expression show, the possibility of reading any desired part of the pressure on a properly calibrated galvanometer scale provided the total resistance of the galvanometer circuit is kept constant.

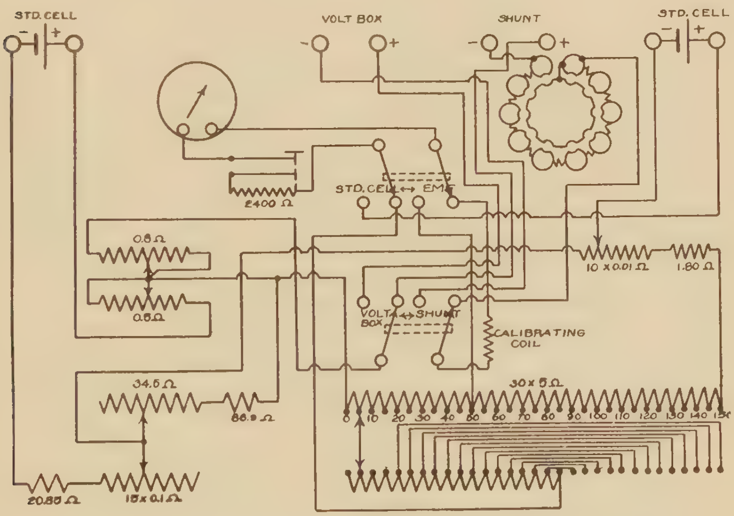


FIG. 203.

When used for current measurements, the current through R is in general not equal to the line current whose value is desired, being greater or less than the line current by the amount of the galvanometer current. It has been found that a simple expedient will correct for this difference and make the reading of the potentiometer (when divided by R) give accurately the value of the line current.

The manner in which these principles are worked out in detail is shown in Fig. 203. Anyone wishing a more complete explanation of the principles of the deflection potentiometer is referred to Bulletin of The Bureau of Standards, Vol. 8, No. 2, from which the foregoing explanation is abstraced. A view of the potentiometer with its accessories is shown in Fig. 204, which also shows a wattmeter connected for test. The voltage supply line is at the left, a slide resistance being used to set at the desired value. The current is supplied by a storage battery, not shown in the figure. The current is controlled by the carbon rheostat at the right. The volt box, auxiliary storage cell, and standard cell are back of the potentiometer.

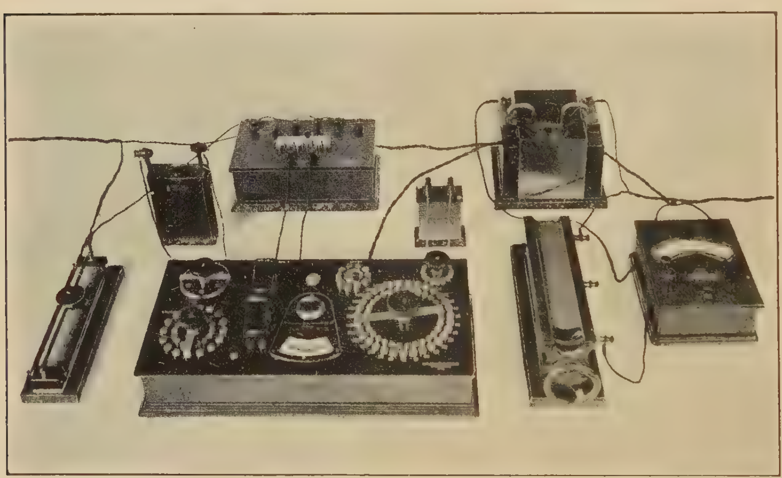


Fig. 204.

236. Standard Resistances or Shunts.—For accurate measurement of current a set of manganin standard resistances is required. These are usually made for oil immersion and give from 0.01 to 1.5 volts' drop at maximum current, those intended for high currents giving the lower full-load drop. To keep down the size of the shunt, the accuracy for very heavy currents is usually less than for more moderate ones. The resistances are made accurate to a small fraction of 1 per cent so the results obtained by their use leave little to be desired for commercial purposes. The usual values of these shunts are 1.000, 0.1, 0.01, 0.001, and 0.0001 ohm.

The resistances are made in two forms. Fig. 205 shows a one-ohm standard of the Reichsanstalt form. Fig. 206 is a 0.00002-ohm shunt of different form.

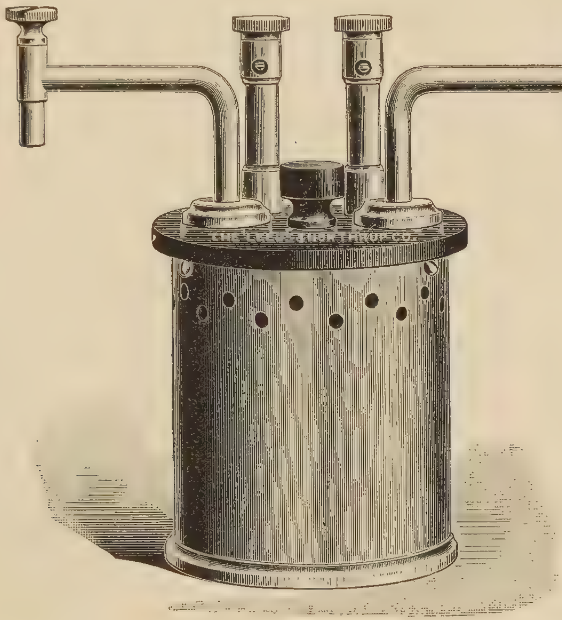


FIG. 205.

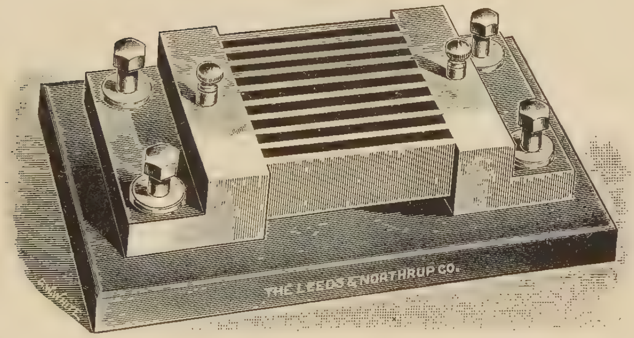


FIG. 206.

With the slide-wire form of potentiometer it has been common practice to use current shunts whose values are decimal multiples or submultiples of an ohm. These, however, are not the most convenient for all cases, as sometimes calculations will have to

be made during the test to determine the setting of the potentiometer to correspond to the ammeter reading in order to expedite the work. To facilitate and expedite the readings when a deflection potentiometer is used, it is necessary to choose shunts whose values will not make these calculations necessary.

Assuming that the fundamental range of the potentiometer is, say, 150 "dial units," for rapid and convenient work, one scale division on an instrument under test should correspond to one dial unit. Under such conditions the resistance of shunt R is given by

$$R = \frac{\text{e.m.f. corresponding to one dial unit}}{\text{amperes per division of ammeter}}$$

If one dial unit equals 0.01 volt, current shunts required for ammeters of 1, 2, 5, 10, and 20 amperes per scale are 0.01, 0.005, 0.002, 0.001, and 0.0005 ohm, respectively. A single shunt will usually do for several ranges; thus the 0.01 ohm shunt is suitable for 100 ampere-100 division, 120 ampere-120 division, and 150 ampere-150 division instruments. The same shunts are equally convenient in testing wattmeters of corresponding current range.

237. Variable Resistance Rheostat.—In many tests it is necessary to connect to the circuit a resistance which can be varied

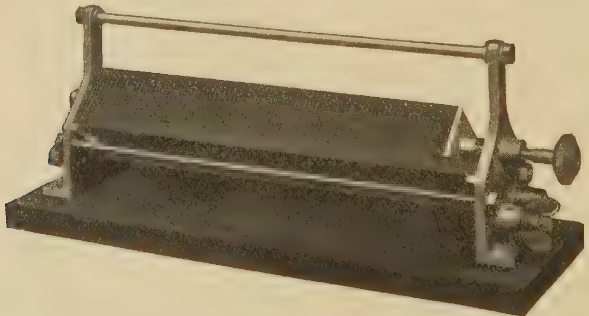


Fig. 207.

gradually so as to maintain the current constant. One of the most convenient forms of rheostat is shown in Fig. 207. The construction and operation of the rheostat is easily understood from the figure. The resistance consists of many carbon plates resting on slides made of insulating material. The resist-

ance is varied by compressing, by means of a hand screw, the plates more or less firmly.

The chief advantages of such a rheostat are the nicety with which the current can be controlled, the large current-carrying capacity and non-inductive property. This last property makes it suitable for alternating as well as direct currents.

238. Lamp Bank.—A convenient resistance for many tests can be made from carbon filament incandescent lamps. A suggestive diagram of a lamp bank is shown in Fig. 208. The resistance of the lamp bank can be varied through wide limits by closing different switches. For instance, if switches *a* and *b* alone are closed, the lamps in circuit 1 will be lighted; by closing *a*, *b*, and *c*, circuits 1 and 2 will be in parallel, etc.

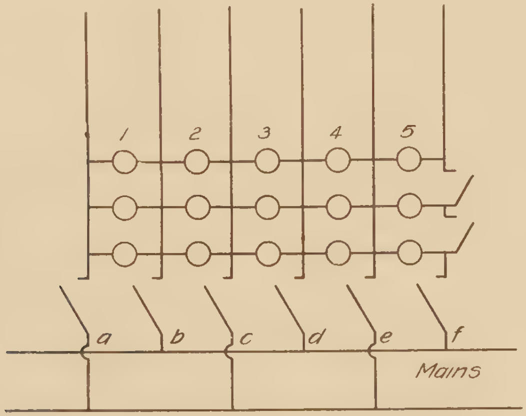


Fig. 208.

Connecting circuits in parallel, decreases the resistance and increases the current. To increase the resistance, either unscrew enough lamps in one circuit until the current is the required value, or close switches *a* and *d*, or *a* and *f*, which will connect the circuits 1, 2, and 3, or 1, 2, 3, 4, and 5 in series. The maximum resistance that can be obtained by the device shown is the resistance of five lamps in series; the smallest resistance is the resistance of all lamps in parallel. An ingenious student can readily modify the arrangement suggested to meet his particular needs. Such a lamp bank can be used for many purposes in a central station.

239. Water Rheostat.—In case of an emergency or when very heavy currents are to be measured, the water rheostat is about the only controlling device that can be used. The water rheostat consists of two or more metal plates, usually iron, in a vessel of salt water. The vessel is usually a wooden water-tight box or barrel. The resistance of such a rheostat is varied by immersing more or less of the iron plates, by moving them nearer together or farther apart, or by changing the amount of salt in the solution.

Other forms of apparatus will be discussed in connection with the tests where used.

CHAPTER XVII

TESTING AMMETERS

240. Introduction.—As pointed out in the previous discussion, all ammeters have a very low resistance and are to be connected in series with the circuit in which the current is to be measured. On account of the extremely low resistance great care must be exercised in connecting them to be sure that excessive current will not flow through the instrument when the circuit is closed. In no case should an ammeter be connected to a circuit without a resistance or rheostat in series with it—the resistance being sufficiently large to reduce the current to a proper value. If the conditions do not enable the student to know beforehand the approximate value of the current, the resistance may be cautiously cut out after final connections have been made.

241. Comparison of Ammeters.—The readings of two or more ammeters may be compared by connecting them in series to a

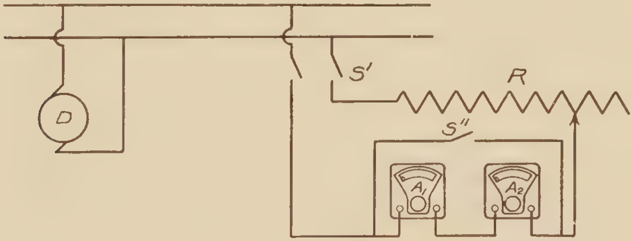


FIG. 209.

suitable source of electromotive force. In series with the ammeters should be connected the lamp bank or other suitable resistance. The resistance is varied step-wise beginning with a small current, and both instruments should be read at the same time. If one of the ammeters has previously been standardized, a comparison of its corrected readings with the readings of the other ammeter will enable the student to determine the correction for the second ammeter. The connections for this test are shown in Fig. 209. In this figure, D is the source of electromotive force

which is preferably a storage battery; A_1 and A_2 are the ammeters to be compared, and R the variable resistance. The lamp bank will serve very nicely for this purpose, although the carbon rheostat is preferable when a low voltage storage battery is available. S' is a double-pole switch which should be left open until all other connections have been made, and S'' is a short-circuiting switch for the ammeters. This switch should be kept closed except when the readings are being taken. When direct-current ammeters are compared or tested, care must be taken to see that they are connected so as to deflect in the proper direction.

242. Calibration Curve.—A curve showing the relation between the correct values of the quantity measured and the indications of the instrument should be drawn from the data obtained

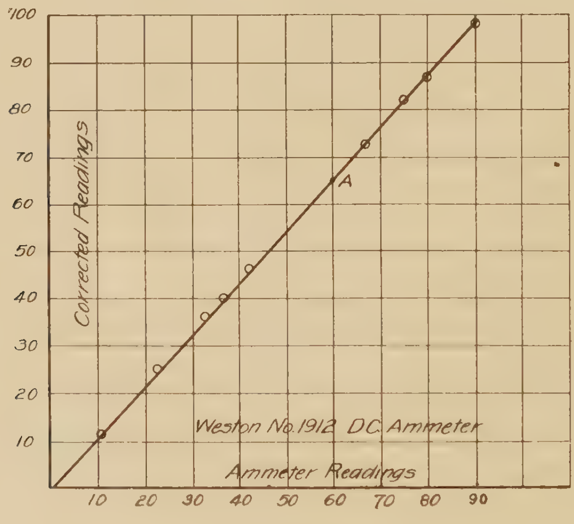


FIG. 210.

when an instrument is tested. Thus in the case just considered if the readings of one ammeter are known to be correct, these values may be plotted vertically or as ordinates, while the readings of the inaccurate ammeter may be plotted horizontally or as abscissas. A curve or line drawn through the points thus determined will show at a glance the variations in the readings; or if a reading on the inaccurate ammeter is known, the correct value of the current can be at once found from the curve. This is brought out more clearly in Fig. 210, which is drawn from data

obtained by comparing a Weston D. C. milliammeter with a Kelvin balance. The data are as follows:

EXAMPLE

Test No. 1.—Comparison of milliammeter with Kelvin Balance.
Apparatus.—Weston Milliammeter.
Kelvin centiampere balance.
Rheostat.
Temperature 22° C.

TABLE III

Ammeter readings	Balance readings	Correction
110	122.48	+12.48
202	222.25	+20.25
230	250	+20.0
330	362.8	+32.8
365	402	+37.0
425	463.9	+38.9
668	722.2	+54.2
752	821.8	+69.8
800	870	+70.0
900	983	+83.0

In Fig. 210, the ammeter readings have been plotted horizontally and the correct, or balance readings, vertically. Through the points thus determined the straight line has been drawn. This curve shows that the per cent error of the ammeter is practically constant. Furthermore, if the milliammeter has been used in practice and a reading of 600 milliamperes obtained, it is easy to determine the correct value of the current from the curve. Thus, the ammeter reading of 600 corresponds to the vertical line marked 60 at the bottom; running vertically on this line we find it intersects the curve at the point A which corresponds to 650 on the vertical scale; hence, the correct reading is 650 milliamperes. Any other reading can be interpreted in the same way.

It is often just as advantageous to draw a curve showing the relation between the correction to be applied and reading. Such a curve is shown in Fig. 211, which is also drawn from the data of Table III. The irregularities of curve, Fig. 211, are perhaps in

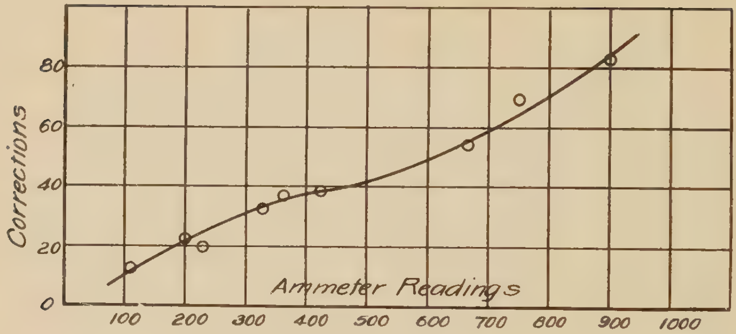


FIG. 211.

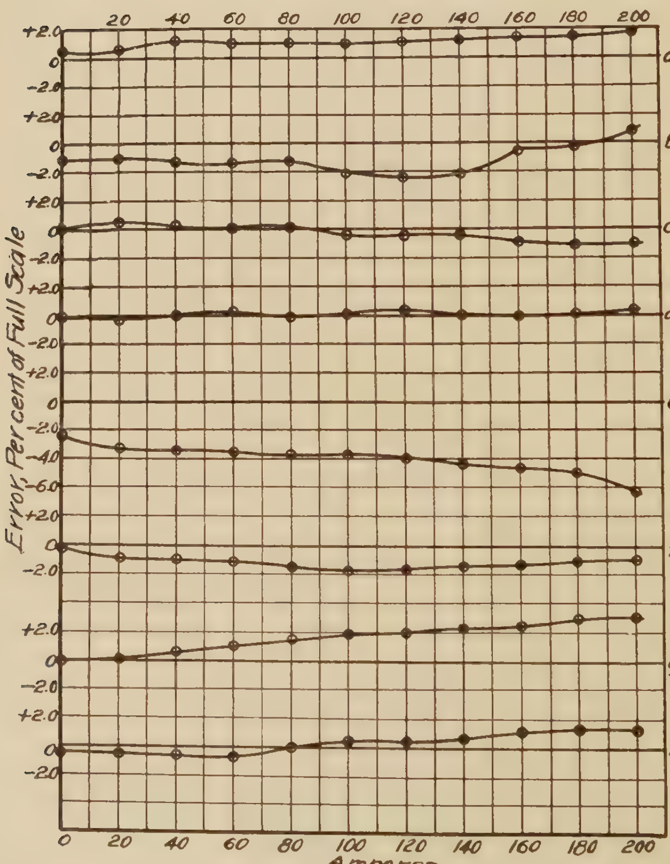


FIG. 212.

this case due to friction and lack of sensibility as the milliammeter tested had received some rough usage.

In Fig. 212 are shown the correction curves of eight American make switchboard ammeters whose movements are shown in Fig. 25. Since all of these instruments were new when tested, the irregularities seem to indicate that the scales do not fit, and also that the sensibilities were not closely adjusted. Where the deviation is pronounced, the discrepancy may be due to non-uniformity of the magnetic field. A straight line calibration curve which shows a constant percentage error is due to a change in the strength of controlling spring or in the strength of the magnetic field.

244. Calibration of D. C. Ammeters by Means of Standard Resistance and Voltmeter.—Since currents can be accurately determined by measuring the voltage drop across a standard

FIG. 213.

resistance, the same principle can be used for calibrating ammeters. The only apparatus necessary is a standardized millivoltmeter and shunt and rheostat or other adjustable receiving circuit. A good instrument for this purpose is shown in Fig. 213. This is a voltmeter with three ranges, 0–1.5, 0–3, and 0–150 volts. The lower range is the most convenient for current measurements. The scale is divided in such a way that it can be read accurately to 0.2 of a scale division, and the accuracy of the instrument is such that measurements of potential may be made within 0.1 of 1 per cent. When direct-current ammeters are to be calibrated they are connected in series with a source of potential, the resistance R, and shunt as shown in Fig. 214. The voltmeter

is connected across the terminals of the shunt as shown. The readings of the ammeters and voltmeter are taken simultaneously, the current is changed and new readings taken, etc. By such a step-wise process the ammeters can be calibrated over the whole scale. The correct values of current are calculated in accordance with Ohm's law. If R_s is the resistance of the shunt and E is the millivoltmeter reading, the current for any one reading is

$$I=\frac{E}{R_s}.$$

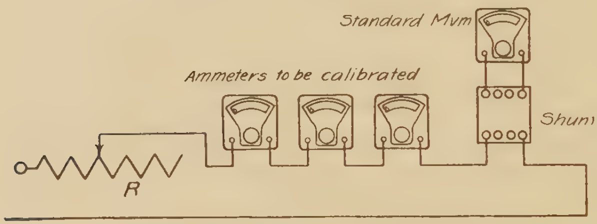


Fig. 214.

When the current is changed E will change, but in each case the current is obtained in the same way. Thus, if the millivoltmeter reading is 0.456 volts and the shunt resistance is .01 ohm the current is

$$I=\frac{0.456}{0.01}=45.6 \text{ amperes.}$$

The calculated values should be plotted vertically and the ammeter readings horizontally, as shown in Fig. 210.

When many ammeters of different ranges are to be calibrated, more than one standard shunt will be necessary. Suppose that a full scale deflection of the millivoltmeter is obtained with a current of 500 amperes; with 50 amperes the deflection will be only 10 per cent of the full scale; below this, the percentage error of the readings will impair the result. Thus, if ammeters below 50 amperes are to be calibrated, another shunt of 10 times the former resistance should be used; it will give a full scale deflection for 50 amperes and may be used down to five amperes. For smaller currents still other shunts should be used.

Millivoltmeters are made for any range from 15 millivolts to 1500 millivolts for full scale deflection, and, therefore, it is necessary to know the relative values of the shunt and millivoltmeter

resistances if appreciable errors are to be avoided. For instance, if the resistance of millivoltmeter is R_v and of shunt R_s, the current through instrument is

$$I_v = \frac{R_s}{R_s + R_v} I$$

where I is current through ammeter. The per cent error in the calibration will then be

$$\frac{100R_s}{R_s + R_v}.$$

When $R_s = \frac{1}{99} R_v$ the error is 1 per cent. If appreciable errors are not to result, $\frac{R_s}{R_v}$ must be small.

245. Deflection Potentiometer Method.—When the deflection potentiometer is used, it replaces the millivoltmeter, and if the proper shunts are available the test can be performed without any calculations. The observer at the ammeter sets successively

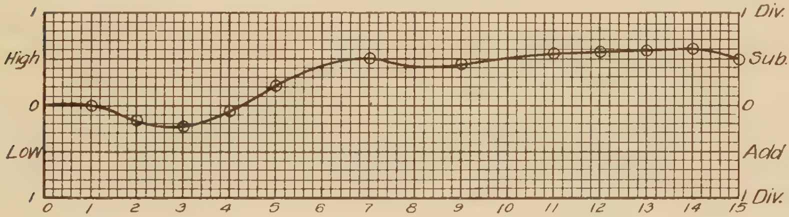


FIG. 215.

on 10, 20, 30 etc. . . . . divisions of the scale, and the observer at the potentiometer sets the main dial to the same numbers, and depresses the key. The small deflection of the galvanometer gives the correction to be applied to the reading of the instrument under test, one division of the galvanometer corresponding to 0.1 division of the ammeter. The correction curve can be plotted at the time the readings are being taken by putting the pencil on the proper vertical line, Fig. 215. If the galvanometer reads two divisions to the right, the ammeter is in error by 0.2 ampere, and the pencil mark is made two divisions below the zero line on the chart; if the galvanometer reading is one division to the left, the mark is made one division above the zero line of the chart.

The scale points may be checked several times if desired, and a smooth curve drawn through the pencil marks. Thus a correction curve may be quickly drawn without recording a single figure, and without any computation.

For rapid work, where ammeters of different ranges are to be tested the shunts may be arranged as shown in Fig. 216. This method may be used with a millivoltmeter as well as potentiometer. As indicated in the figure the shunts are soldered together, and their free ends are connected to a millivoltmeter. The line current enters at A, and may leave at B, C, or D. The section AB is of low resistance and large current carrying capacity; BC is of lower current capacity and higher resistance, etc.

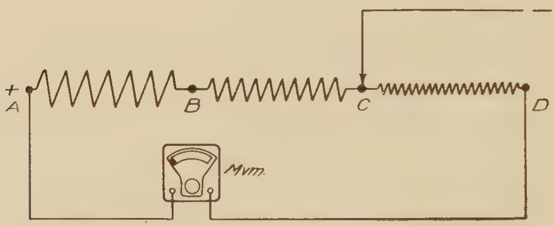


FIG. 216.

It will be evident that the method outlined above may be used for calibrating not only ammeters, but shunts and millivoltmeters as well. Thus, if the ammeter and millivoltmeter have been standardized, the value of the resistance of the shunt can readily be obtained. According to Ohm's law $R=\frac{E}{I}$, and, when E and I are known, R can be determined. Similarly $E=RI$, and a standardized ammeter and shunt may be used for calibrating the millivoltmeter. In case the ammeter cannot be depended upon, an ammeter shunt can be calibrated by connecting it in series with a standard shunt, and connecting the millivoltmeter first across the standard shunt, and then across the shunt to be calibrated. The ratio of the millivoltmeter readings is the ratio of the resistance of standard to the resistance of shunt being calibrated. This can be shown as follows:

The current through both resistances is the same and equal to I.

Let R = resistance of standard

and R' = resistance of shunt being checked

Then $E = IR$

and $E' = IR'$

Whence $\frac{E}{E'} = \frac{IR}{IR'} = \frac{R}{R'}$

and $R' = \frac{E'R}{E}$

where E is the millivoltmeter reading across the standard and E' the reading across the shunt.

246. Difference between D. C. and A. C. Ammeters and Voltmeters.—The essential differences between direct-current and alternating-current instruments have been pointed out in detail. A brief summary of the main points, with reference to calibration, is necessary. In direct-current ammeters and voltmeters, the force actuating the pointer may be any function of the current, although in most cases it is proportional to the first or second power of the current; the straight-line relation being the most convenient on account of the consequent uniformity of scale.

In alternating-current ammeters and voltmeters the instantaneous value of the actuating force is proportional to the square of the current or electromotive force at that instant. The average force upon which the steady deflection depends is proportional to the average square of the current or electromotive force. The indications of alternating-current ammeters and voltmeters are really a measure of the average value of the square of the alternating quantity.

Alternating-current indicating instruments, whose indications are independent of frequency and wave form, when calibrated by using direct current, indicate correctly effective values when used in measuring alternating-current or electromotive force. This relation may be shown as follows:

Let $I =$ direct current causing a given deflection.

and let $I_A =$ alternating current causing the same deflection. Then the deflection with direct current is equal to KI^2, and with alternating current it must be K times the average of i^2.

or $$KI^2 = K \text{ average } i^2$$

whence $$I^2 = \text{average } i^2$$

and $$I = \sqrt{\text{average } i^2} = I_A$$

Since the instrument scale is graduated in values of I, it indicates $\sqrt{\text{average } i^2}$, or effective values when used on alternating-current circuits. Thus, alternating-current ammeters whose indications are independent of wave form and frequency when calibrated on direct current, indicate correct effective values of alternating current. The instruments which may be calibrated on direct current are the hot-wire, electrodynamometer, and electrostatic types.

It is evident from previous discussion of the hot-wire indicating instruments that the indications are proportional to I^2 when direct current is flowing, and to the average value of i^2 when alternating current is being measured. The indications of the instrument are correct effective values when the instrument is used on alternating-current circuits.

The electrodynamometer when standardized on direct current, indicates effective values when used for measuring alternating currents. When the two coils are connected in series, the torque exerted upon the moving system, for a given relative position of the coils, is proportional to the square of the current as already pointed out, and the torque is independent of the direction of the current. Most makes of this type of instrument give readings of equal accuracy on either direct or alternating currents of ordinary commercial frequencies and wave form. Owing to eddy currents in surrounding metal, and non-uniformity of current distribution in conductors, some makes are subject to slight errors on even commercial frequencies, and on circuits of high frequencies the same causes will produce errors in the readings of all electrodynamometer type instruments.

In addition to the foregoing, the electrodynamometer ammeter has certain limitations. If the current is carried into and out of the moving coil by the usual spiral springs only small currents may be used without injury to the springs. In one of the earliest forms the current is taken into and out of the moving coil by mercury cups; in the Kelvin balance the axis about which the moving system turns is horizontal, and ligaments of fine wire are used as supports and conductors. Both of these instruments are slow and inconvenient to use and require that the current to be measured be quite steady. The readings of the Kelvin balance change appreciably with heating, when kept in circuit for any length of time. The balances also have frequency errors which increase with the ampere capacity of the balance. Several European

makers arrange the fixed and movable coils in parallel so that the latter carry only a small part of the current to be measured. In order to avoid errors in the division of the current, due to inductance, the ratios of the inductance to the resistance of each coil are made small and as nearly equal as possible by adding non-inductive resistances to each coil. These instruments are suitable for checking alternating-current ammeters which cannot be accurately calibrated with direct-current.

Both the fixed and movable coils of the electrodynamometer voltmeter are made of fine wire and connected in series with a non-inductive high resistance. The high-resistance multiplier reduces to a low value the time constant (ratio of inductance to resistance) of the electrodynamometer coils, and, hence, well made voltmeters of this type, calibrated on direct current, show practically negligible errors on commercial alternating-current circuits. These instruments, if properly made, are suitable for checking other working instruments.

Since alternating-current voltmeters require considerably larger currents than the moving coil direct-current types, some provision should be made for ventilation. Unless this provision is made, the relatively large current develops considerable heat, which accumulates, raising the temperature of the springs and other parts of the instruments and affecting the readings. For very accurate work the series resistance should be mounted separately from the instrument and ventilated.

247. Calibration of A. C. Ammeters.—From the foregoing discussion, it is evident that some types of alternating-current ammeters can be calibrated in exactly the same manner as D. C. ammeters. The soft iron and induction type ammeters should, however, be calibrated on alternating current of the same frequency as that on which they are to be used. The most satisfactory method of calibrating these instruments is by the use of an intermediate D. C. – A. C. standard, such as a hot-wire ammeter or an electrodynamometer. A convenient method of connections for such tests is shown in Fig. 217. The diagram shows a standard D. C. ammeter connected to a source of D. C. electromotive force. By means of a double throw switch the instrument *C*, which may be an electrodynamometer or hot-wire ammeter, can be connected in series first with the D. C. ammeter, and then with the A. C. ammeter to be calibrated. The instrument *C* thus acts as an intermediate standard, and if it is a hot-wire ammeter, its

zero reading should be maintained. If the pointer does not return to zero, it may be set to zero by means of the adjusting screw.

248. A. C. - D. C. Comparator.—A most ingenious method of eliminating the intermediate standard has been devised by Dr. E. F. Northrup of Princeton University. His method is based upon the principle that the heating of the effective value of an alternating current is the same as that of the direct current of the same nominal value. The essential principles of the A. C.-D. C. Comparator, as the instrument is called, will be clearly understood from Fig. 218. In the illustration, AB and CD are two small wires of equal length, diameter, and resistance,

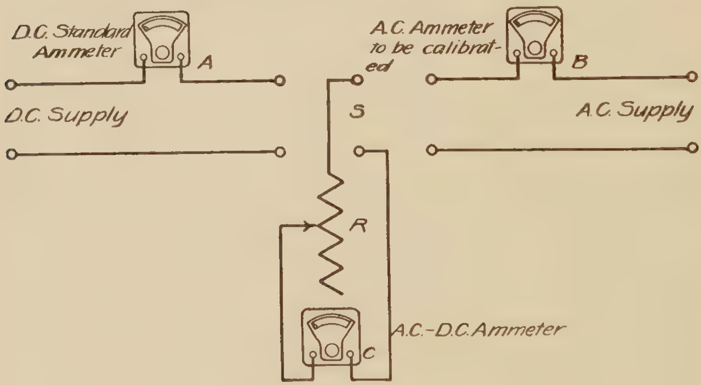


FIG. 217.

stretched, as shown, about 5/32 in. apart and pulled back at their middle point by a cross piece resting on each wire. A spring attached to this cross piece keeps a constant tension on the two wires. Upon the cross piece is mounted a small mirror m. When the two wires are heated they will lengthen, but if both are elongated the same amount, the spring will simply pull the wires farther back and the mirror will remain parallel to its first position. It will be evident, however, that if one wire elongates more than the other, the mirror will tilt, and a very slight tilting can be observed by a telescope and scale in front of the mirror. As both wires will be influenced to the same extent by external temperature changes, the indications of the instrument will not be influenced by them. Equal currents flowing through both wires will cause equal elongations and no deflection; but a slight difference in the currents in the two will cause unequal elonga-

tions due to heating, and a deflection. In the diagram of Fig. 218, R is a non-inductive resistance through which flows the alternating current to be measured; f' and h' are potential points on it, the resistance between which is accurately known. The connecting resistances ef and gh are made equal to $e'f'$ and $g'h'$ respectively. SS are the two parts of a switch by means of which the terminals of the wire CD may be connected either to e' and g', or e and g. R is a standard regulating rheostat and M a storage battery.

To make a measurement of current, throw the switch S so as to connect the two wires in parallel, then by means of the small rheostat, make adjustment until the comparator makes no deflection when the current that is to be measured flows through R.

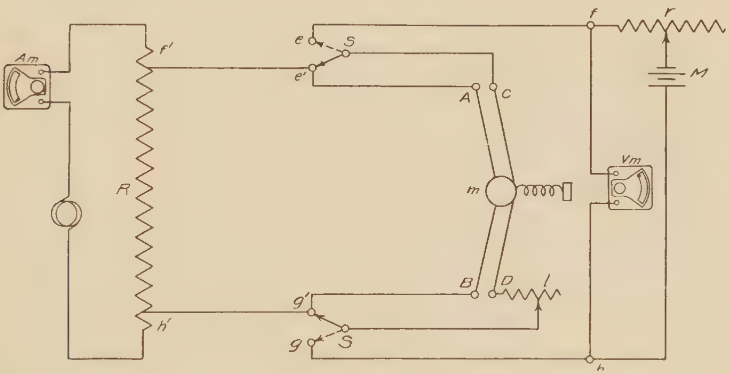


Fig. 218.

Now throw the switch so as to connect CD to the direct-current side and regulate r until there is again no deflection. When this is obtained, the heating value of the direct current is equal to that of the alternating current. The e.m.f. indicated by the voltmeter is equal to the fall of potential across $f'h'$; calling this E, the current through the ammeter is $\frac{E}{R}$ plus the current which flows through the wire AB. The current through AB varies with the value of E, but as it is previously determined for all values of E for which the instrument is intended, and is furnished in the form of a curve, the current through the ammeter is known. The manufacturers guarantee that with this instrument, measurements can be made throughout its range to an accuracy of one-fifth of 1 per cent.

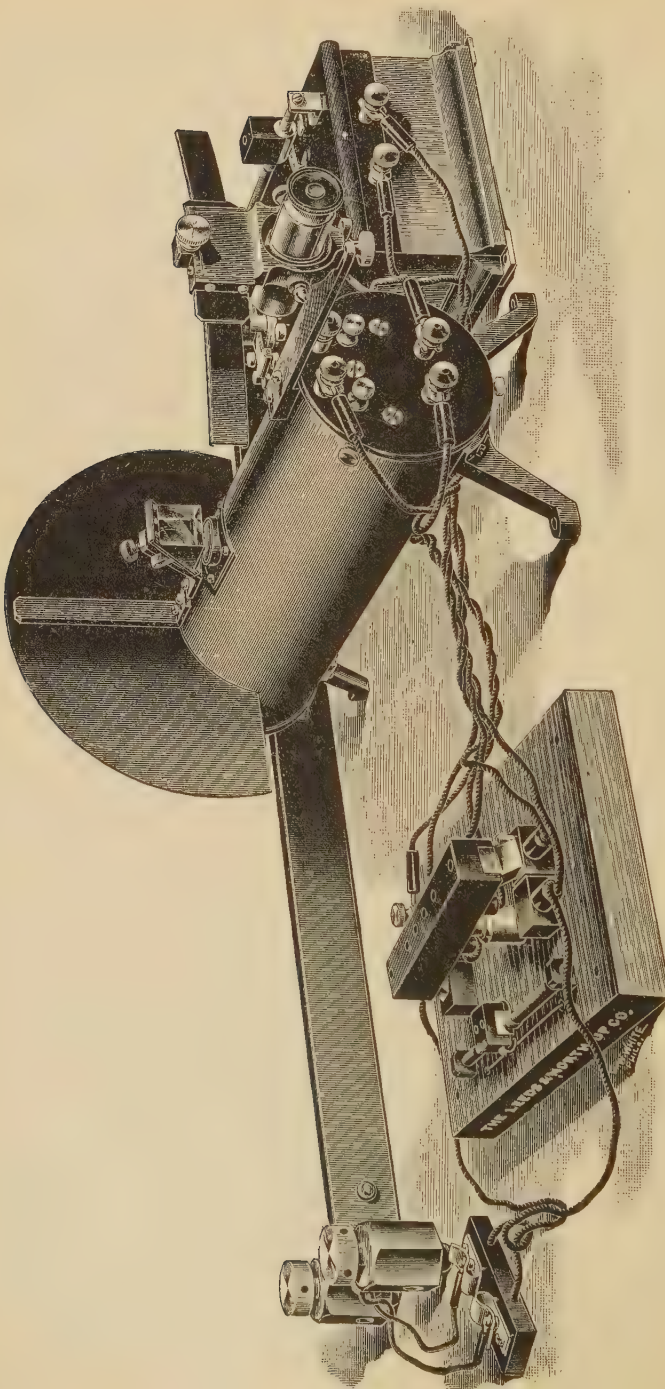


Fig. 219.

Voltmeters as well as ammeters can be readily calibrated. By means of proper shunts and other accessories, alternating-current instruments can be calibrated rapidly and accurately from one-twentieth of an ampere up and from five volts up. The complete apparatus is shown in Fig. 219.

For measuring high frequency alternating currents of low value, the Duddell thermo-ammeter described in a previous assignment is very convenient. Its chief advantage lies in the fact that it can be calibrated on direct current and when so calibrated will indicate correct effective values of alternating current of any frequency or wave form. Furthermore, it has very little self-induction or capacity and may be used as an ammeter or voltmeter, according to whether it is constructed with a high or low resistance heater.

CHAPTER XVIII

TESTING VOLTMETERS, WATTMETERS, POWER-FACTOR, AND FREQUENCY METERS

249. Introduction.—Some of the most common and convenient methods of checking ammeters are, with slight modifications, applicable to voltmeter calibration as well, the main difference being in the manner of connecting the instrument to circuit, and the necessity of some means of adjusting the pressure instead of current.

250. Comparison of D. C. Voltmeters.—Fig. 220 shows the connections for comparing a voltmeter V_1 with the standard voltmeter V_2. The standard voltmeter discussed in Article 244 is well suited for direct-current low range voltmeter tests. As is clear from the diagram, the two instruments are

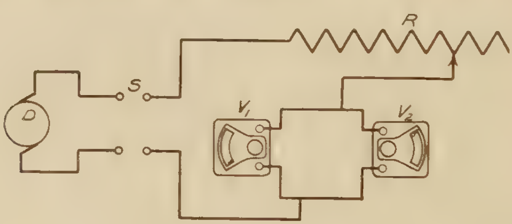


FIG. 220.

connected in parallel with each other, and in series with a high resistance. Different readings are obtained by varying the series resistance. As pointed out in the discussion on indicating voltmeters, the deflection or indication of the voltmeter is proportional to the current through the instrument. This current, under constant pressure, is determined by the resistance in the circuit. If R_v is the resistance of voltmeter, R the series resistance, and E the electromotive force, the deflection may be expressed by

$$d = K\frac{E}{R+R_v}$$

From this it is evident that the deflection will depend upon R, and that increasing R decreases the indication of the instrument. The value of R must be high in order to get a low reading if E is large. Thus, to reduce the reading one-half, the value of R must be equal to R_v.

If it is not convenient to provide a large enough series resistance for a sufficient number of readings, the connections may be modified as shown in Fig. 221. To get the maximum desired deflection the two voltmeters are connected across a sufficient number of lamps in series. For lower readings, the voltmeter connections are changed so as to include one lamp less. The deflection in each case will be determined by the voltage drop across the number of lamps included between the voltmeter

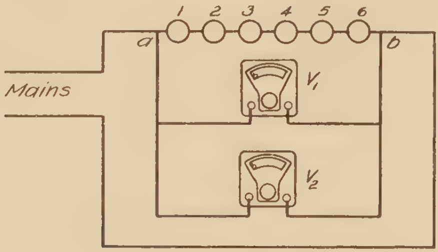


FIG. 221.

terminals, so long as the current through the lamps remains constant. Thus, if r is the resistance of one lamp, I the current, the voltage drop across one lamp is Ir, across two lamps $2Ir$, etc.

Where it is possible to vary the e.m.f. of the source, the voltmeters may be connected as shown in Fig. 220, with R omitted. Different readings are then obtained by changing the excitation of the generator, if that is used, or by changing the number of cells in series, if a storage battery is used.

Both instruments must be left in the circuit when readings are taken if the connections of Fig. 220 are used. Both instruments should also be read at the same time. When multipliers and leads are used, they should be checked with the instrument for which they are intended.

251. Potentiometer Method.—The most accurate method of checking voltmeters is by means of the potentiometer. The deflection potentiometer is especially well adapted to this class of work. If the procedure outlined in Article 245 is followed,

the checking may be done accurately and rapidly. The correction curve, Fig. 215, may be plotted as the work of checking proceeds. The connections for such a test are the same as shown for the voltage coil of wattmeter, Fig. 204, the current line being disconnected.

252. Testing A. C. Voltmeters.—Alternating-current voltmeters, whose indications are not affected by frequency and wave form, may be compared in the manner just explained. For calibrating induction voltmeters, either an intermediate standard or an A. C.–D. C. comparator is most convenient.

The connections for an intermediate standard are shown in Fig. 222, where V is either a hot-wire voltmeter or some other

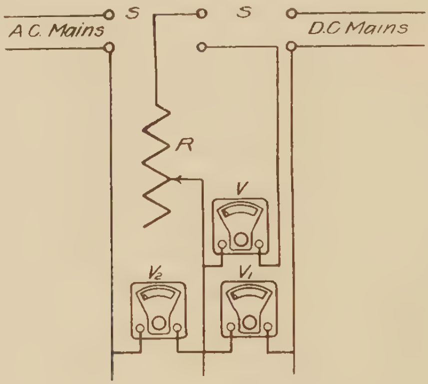


FIG. 222.

instrument unaffected by frequency and wave form; V_1 is a D. C. standard and V_2 the A. C. voltmeter to be tested. The connections of the diagram are practically the same as those of Fig. 220. That is, the voltmeters are connected in series with a high resistance R, and the source of e.m.f. Different readings are obtained by changing R. The diagram of Fig. 223 shows a system of connections similar to those of Fig. 221. In addition to the two-pole double throw switch, a single-pole double throw switch is most convenient to use. In place of this, however, two single-pole single throw switches will answer. In case this system of connections is used, R may be most conveniently made up of lamps in series, and different readings obtained by changing the connection a.

When the double-pole switch S is closed to the right, the

intermediate standard and D. C. voltmeter are connected in parallel to the D. C. circuit; and when the switch is closed to the left, the intermediate standard and A. C. voltmeter are connected in parallel to the A. C. circuit. The single-pole switch must be changed every time the main switch S is changed.

253. Use of A. C.–D. C. Comparator.—As pointed out in the preceding discussion, the Leeds and Northrup A. C. D. C. comparator is also well adapted for accurate calibration of A. C. voltmeters. A diagram of connections for voltage measurements is shown in Fig. 224. The voltmeter V_m to be tested is connected, as shown, in series with a variable resistance R. R_v is a high non-inductive resistance in series with the alternating-current circuit. S is a double-throw switch. When this switch is to the left or on "check," the two comparator wires AB and CD are

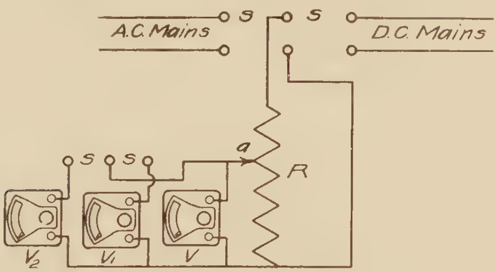


FIG. 223.

in series, and are both traversed by the same current. The value of this current is adjusted by R_v. Under these conditions the comparator is adjusted so that it will show no deflection. When the test is to be made, the switch is thrown to the right or to "test." Under these conditions the same current will flow through wire AB as before, for the resistance of r' is equal to that of CD, which it replaces. By adjusting r, the direct current through CD is made equal to the current through AB, and the deflection is brought back to zero. This current is read by a standard D. C. ammeter, or milli-voltmeter and shunt. If this current is I, the difference of potential across the A. C. voltmeter is IR_v, as is plainly evident. To check other points adjust R.

254. Calibration Curves.—Calibration curves should be drawn for voltmeters in the same way as for ammeters. The following table will show how to arrange the data obtained from a voltmeter test.

EXAMPLE

Test No. 2.—Test of Voltmeter.

Apparatus.—Weston D. C. Voltmeter No. 19229.
Weston Laboratory Standard Voltmeter No. 316
Lamp bank.

Temperature 22° C.

TABLE IV

Standard	Instrument tested	Correction	Remarks
			See Curve, Fig. 225
75	80	−5.0	.....
81	83	−2.0	....
88	88	0.0	....
98	97	+1.0	....
112	111	+1.0	....
118	116.5	+1.5	....
134	132	+2.0	....
141	138.5	+2.5	....
150	147	+3.0	....

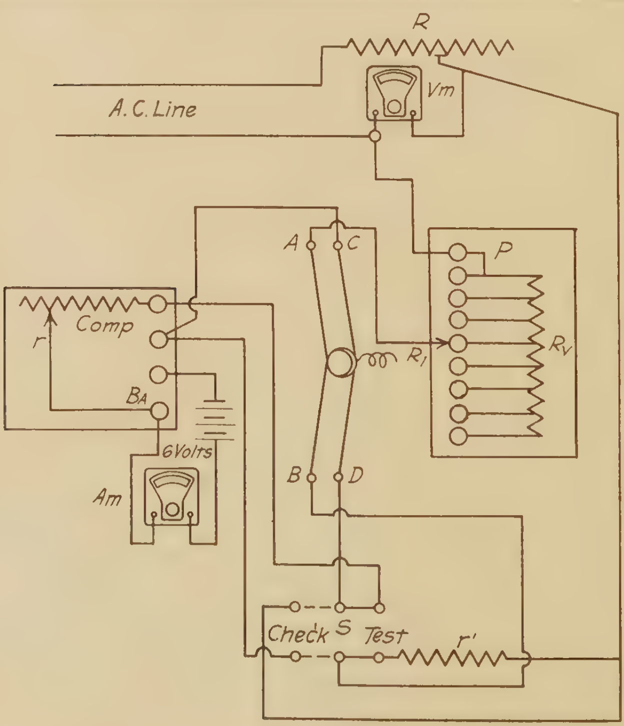


Fig. 224.

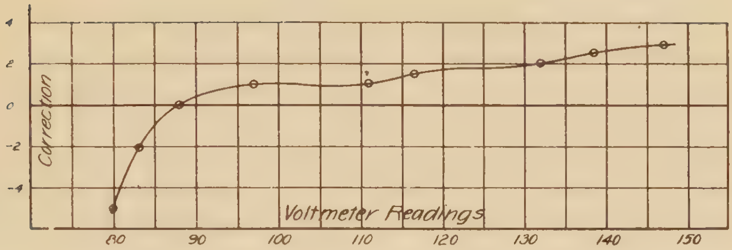


Fig. 225.

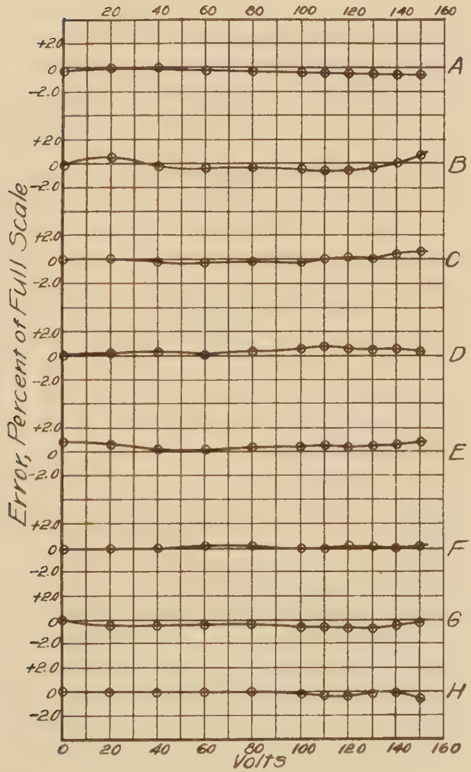


Fig. 226.

Calibration curves for eight switchboard, moving coil permanent magnet type of voltmeters are given in Fig. 226. Curve *A* shows that the controlling spring was too strong, or else the magnetic field too weak for the scale used. The errors as a whole are small.

256. Test of Electrodynamometer Type Wattmeter.—The indications of wattmeters are proportional to the product of amperes and volts supplied. On direct currents, the product of amperes and volts gives the correct power, but on alternating currents this product must be multiplied by the power-factor of the circuit. This, however, is not a serious objection, for when a wattmeter is calibrated with direct current it indicates correct power when used with alternating current, provided the inductance of the pressure coil is negligible. This fact was demonstrated in Chapter X.

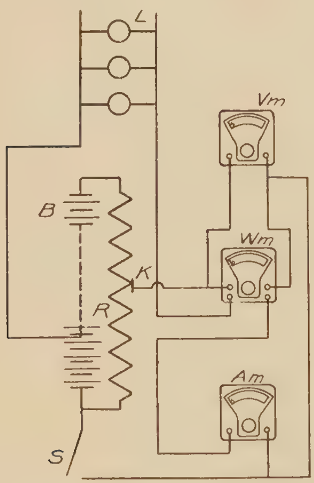


Fig. 227.

A wattmeter should indicate correct power supplied to a circuit when either the current, or pressure, or both, vary in value within their respective limits stated on the instrument. In connecting a wattmeter to the testing circuit, provision must be made for varying both the current and electromotive force, and for measuring these accurately. This is best done by connecting the instruments as indicated in Fig. 227. In this diagram, *B* represents a storage battery or other source of electromotive force; the battery is shunted by a high resistance, *R*, to one terminal of which is connected one terminal of the potential coil of the wattmeter *Wm*, while the other terminal of the wattmeter is connected to a movable contact *K*. The current coil of the wattmeter is connected in series with the load *L* and ammeter *Am*. *L* may be the adjustable lamp bank already described. A standardized voltmeter *Vm* is connected in parallel with the potential coil of the wattmeter. When compensated wattmeters are tested, the independent connection is used for potential connection; this cuts out the

compensating coil as is evident from Fig. 86. When the connections are made as shown, three separate tests can be made as follows:

1. With constant voltage and variable current
2. With variable voltage and constant current
3. With variable voltage and variable current

To make the test with constant voltage, carefully read the instruments, all circuits being opened. Insert the smallest load desired and close the switch S. By inserting a smaller or greater number of lamps, the desired value of current can be obtained. Close K and move it along until the voltmeter indicates the proper voltage.

To obtain reading for different currents, vary the number of lamps. For variable voltage test, adjust the current through ammeter for maximum desired value. Move K so as to get lowest voltage desired. For other readings move K until a sufficient number up to maximum is obtained, the current in the meantime is kept constant. For variable current and voltage, adjust the number of lamps and move K until suitable readings on the voltmeter and ammeter are obtained. For different readings change these adjustments. In each case, the three instruments should be read simultaneously. The results may be tabulated as follows:

EXAMPLE

Test 3.—Calibration of Wattmeter.

Apparatus.—Weston Wattmeter No. 4263.
Standard Voltmeter No. 316.
Standardized Ammeter No. 21131.
Lamp bank and rheostat.

Temperature 22.5° C.

TABLE V

Voltmeter reading	Corrected volts	Ammeter reading	Corrected amperes	True watts	Watt-meter reading	Correction	Remarks
110	111.5	2.50	2.75	306.62	283.90	+22.7	. .
110	111.5	4.50	4.95	551.92	511.00	40.9	. . .
110	111.5	6.50	7.20	802.80	747.00	65.8	. . . .
110	111.5	8.70	9.57	1067.50	1000.00	67.5	. . . .
110	111.5	13.00	14.35	1600.00	1500.00	100.0	. .
110	111.5	17.50	19.25	2137.00	2000.00	137.0	. .
110	111.5	22.00	24.00	2675.00	2500.00	175.0	. .
110	111.5	27.00	29.70	3212.50	3000.00	212.5	. . .
							Curve Fig. 228

Fig. 228 shows the correction to be applied at any wattmeter reading with constant voltage and changing current. Similar curves may be plotted for the other two cases, viz., when current

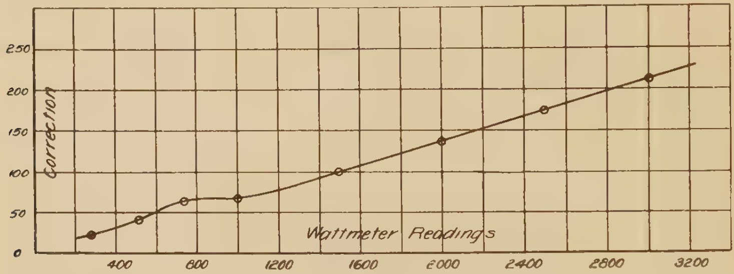


Fig. 228.

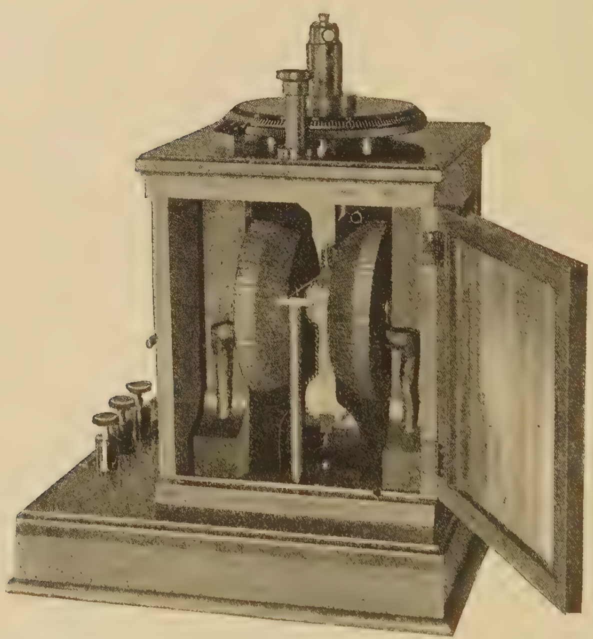


Fig. 228a.

is kept constant and voltage is changed, and when both current and voltage are varied.

The foregoing method of calibration applies only to wattmeters whose indications are independent of frequency and wave

form. Induction wattmeters cannot be calibrated on direct current. Wattmeters of this type are most conveniently calibrated by placing them in circuit with an electrodynamometer wattmeter which has previously been standardized. A good instrument for this purpose is the Watt dynamometer shown in Fig. 228a. Each phase of a polyphase wattmeter must be calibrated separately. A diagram of connections for testing induction wattmeters is shown in Fig. 229, where Ws represents the standardized wattmeter and Wi the induction instrument to be tested. The current through the potential coil is partly determined by the frequency of the alternating current under constant voltage, and hence the frequency at which the instrument was calibrated should always be recorded.

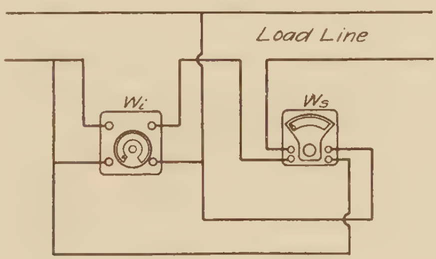


FIG. 229.

258. Testing Single Phase Power-factor Meters.—The power-factor of a circuit has been defined as the ratio of the true power to the apparent power being delivered by an alternating current. If I and E are the effective values of current and pressure respectively, the power is given by the expression $W=KIE=IE \cos\theta$, where K, or $\cos\theta$ is called the power-factor. Thus $K=\frac{W}{I\times E}$. W can be measured by means of a standard wattmeter and I and E by means of a suitable ammeter and voltmeter. The correct power-factor can thus be calculated from the readings of three standardized instruments, wattmeter, ammeter, and voltmeter, connected as in the diagram of Fig. 230. The correction to be applied is then obtained by substracting the indicated power-factor from the calculated power-factor. In order to use this method for calibrating a single-phase power-factor meter, the load L must be inductive and variable. The methods of obtaining loads of variable power-factor are de-

scribed in Chapter XX. At this point it may be noted that a small induction or synchronous motor can be used for the inductive load. The power-factor of the induction motor may be varied by loading it more or less, and the power-factor of the synchronous motor may be varied by varying its field excitation. This is not the most satisfactory method, as the range through which power-factor can be varied is limited.

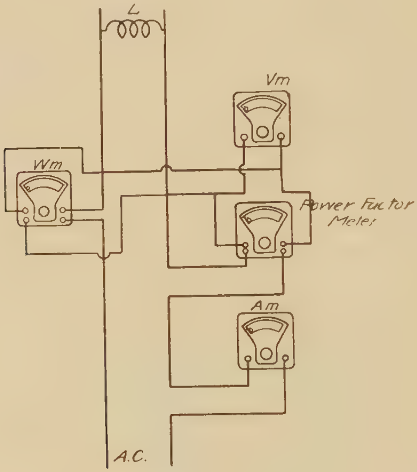


Fig. 230.

259. Testing Polyphase Power-factor Meters.—When the separate circuits of a polyphase system have the same power-factor, the power-factor of the whole system is equal to the power-factor of one of the phases. When the load is unbalanced so that the separate phases have different power-factors, there is no one power-factor that has a definite value or physical significance. The power-factors of the separate phases of a three-phase system are $\cos\theta_1$, $\cos\theta_2$, and $\cos\theta_3$, while by definition the power-factor of the system is

Power-factor $= \dfrac{W_1+W_2+W_3}{E_1I_1+E_2I_2+E_3I_3}$. The polyphase meter, however, gives the mean of $\cos\theta_1+\cos\theta_2+\cos\theta_3$. The power-factor as given by definition is evidently not the same as that indicated by meter. When the load is balanced, the power-factor is

$$\text{power-factor} = \frac{W_1+W_2+W_3}{\sqrt{3}\,E_1I_1} = \frac{W}{\sqrt{3}\,EI}$$

where W represents the power expended in balanced load, E the voltage between mains, and I the current in one main. To test such an instrument on balanced load, it is sufficient to determine the power-factor of one phase and compare that with the instrument indication. For tests of polyphase power-factor meters, the testing apparatus necessary are a standarized ammeter, voltmeter, and wattmeter. These must be connected to each phase successively in the manner shown in Fig. 230. Provisions should be made for transferring the instrument connections from one phase to another without disturbing the flow of energy. This is readily accomplished by the aid of a polyphase meter board shown in Fig. 231. While the test is under way, some characteristics of the meter may also be observed. Disconnect the potential circuit, and observe the rotation of the vane under the influence of the revolving field produced by the series windings alone. Reverse two of the series connections and observe the effect. The revolving field of an induction motor will behave in a similar manner. Reverse the potential connections and observe the effect. Can you explain this?

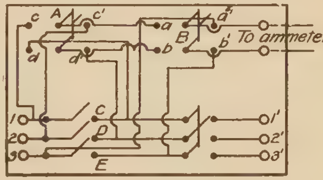


Fig. 231.

Polyphase power-factor indicators will give correct indications only on balanced circuits, although slight unbalancing will not greatly affect the reading. Determine what effect an unbalanced load has on the indication. The Westinghouse Electric and Manufacturing Company suggests the following method for checking their instruments: The moving part should be perfectly balanced; that is, when no current is passing through the coils, the pointer should remain in any position in which it is placed. The instrument should now be correctly connected to the circuit in the usual manner with the exception that the wire to the lower left-hand binding post should remain disconnected. If the meter is correct in calibration, the pointer, with full-load current on the meter, will come to rest at a position coinciding with a red line in the upper left-hand part of the scale. Should the pointer not come to rest at this point, it should be shifted on the shaft until it rests on the red line. Care should be taken not to disturb the balance by moving the pointer.

This procedure simply insures the maintenance of the original calibration.

260. Testing Frequency Meters.—The frequency of an alternating current depends upon the speed and number of poles of the generator. Thus, if the generator has p field poles and makes n revolutions per minute the frequency is

$$f=\frac{p}{2}\times\frac{n}{60} \text{ cycles per second.}$$

To test the accuracy of a frequency meter it is only necessary to measure the speed of the generator, count the field poles and calculate the correct frequency by the above formula. The speed can easily be determined by means of an accurate tachometer or speed counter.

Any inaccuracy in the resonance type of frequency meter can be corrected by filing off, or adding to the solder weight at the top of the reed. It is not advisable for an inexperienced person to attempt this.

In place of a calibration curve it is preferable to arrange a table showing the instrument indication and correct frequency. The induction type of frequency meter may be calibrated in exactly the same way, but adjustment is made by varying the series resistance until the instrument reads correctly.

261. Testing Recording Meters.—Since recording instruments are mainly modified forms of indicating meters, they may be tested in exactly the same way as indicating meters of the same type.

CHAPTER XIX

TESTING WATT-HOUR METERS

262. Introduction.—In order that higher efficiency in the operation of watt-hour meters may be maintained, not only the most reliable meters must be used, but constant vigilance is necessary in keeping these meters accurate. For this purpose, the meter departments of some companies are equipped with the highest grade of primary standards and all necessary appliances for checking the accuracy of the secondary standards which are employed in meter testing. To the secondary standards already mentioned should be added the rotating standard watt-hour meter.

263. Rotating Standard Watt-hour Meter.—The instrument to which has been given the name "rotating standard" is at best only a secondary standard. The principles of operation of the rotating standard meter are the same as those of the service watt-hour meters, the construction, however, is modified to meet certain condtions. The conditions that necessitated changes in construction are portability, wide range of current capacity, and ease of determining the number of revolutions. The first condition is fulfilled by omitting the iron case and enclosing the operating parts of the meter in a carrying case as shown in Fig. 232.

While it is not difficult to make the series winding so that different loads might be safely carried, for accuracy and rapidity of testing, it is necessary to construct the series coils in such a manner that the torque will be the same at different loads. This is accomplished by making the current coils in sections, and mounting the sections so that they can be connected in series or parallel. The number of ampere turns of the series coils are made equal at different full-loads by either a sliding contact, or by changing the external connections. The field windings are usually for full-loads of 1, 5, 10, 20, and 40 amperes, or for 1, 5, 10, 50, and 100 amperes.

The last condition is fulfilled by affixing a pointer to the end of the shaft. This pointer moves over a dial graduated into 100

parts making it possible to read to 1/100 of a revolution, and even closer. The whole number of revolutions is indicated by two smaller pointers, plainly shown in Fig. 232. Fig. 233 shows a side view of the Duncan rotating standard watt-hour meter with

FIG. 232.

case removed. In Fig. 234 is shown a General Electric induction test meter. The figures plainly show the similarity between the regular service meter and the test meter as the rotating standard should be called. The use of the portable standard watt-hour

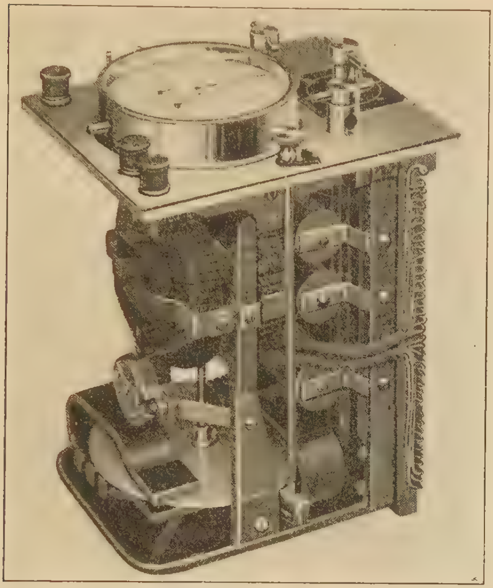


FIG. 233.

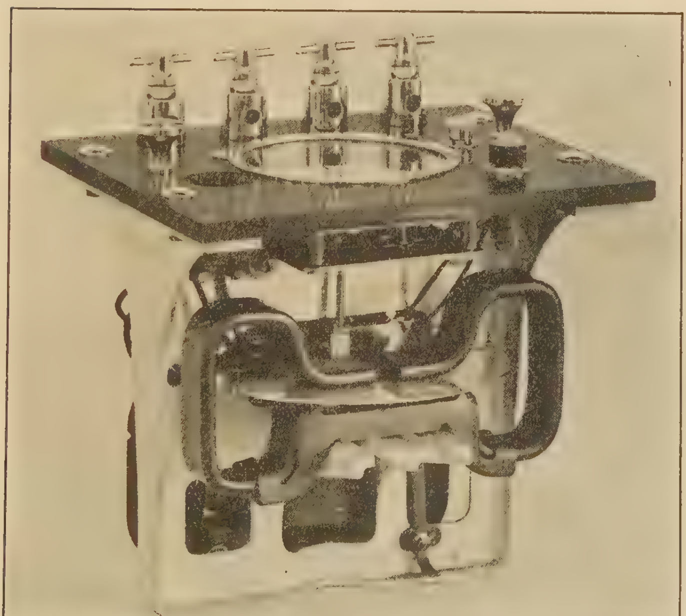


FIG. 234.

meter has several advantages, among which may be mentioned:

1. The use of only one instrument greatly facilitates the testing of meters in service.

2. Both test and tested meters are subjected to the same conditions; and hence, any variation in load during the test is automatically corrected by the test meter.

3. Only one man is necessary for testing, as the time of starting and stopping the test meter may be controlled by a push button.

4. The portable test watt-hour meters are more rugged than indicating instruments, and consequently, they will stand harder usage.

Against these advantages must be placed the fact that the results obtained are liable to be less accurate than those obtained by the use of well calibrated indicating instruments.

264. Kinds of Tests.—The meter committee of the National Electric Light Association recommends that in view of the diversity of conditions, the lack of recognized standard testing methods, meter tests be classified as follows:

1. Shop tests
2. Installation tests
3. Periodic tests
4. Complaint tests
5. Inquiry tests
6. Re-tests
7. Repair tests
8. Special tests.

265. Shop Tests.—Upon the receipt of the meters from manufacturers, also when removed from service and before being placed in the stock room, all meters should be carefully examined and tested. Any defects that may be discovered should be corrected at that time. These tests, preliminary to installation or storing, are called shop tests.

266. Installation Tests.—Even if a meter is found to be correct in the shop, it is not safe to assume that it will be correct after being placed in service, and therefore, a test after installation is necessary. In the case of commutator meters, a test is imperative. One good plan is to inspect the meter immediately after being connected to service, and then after about 4 weeks, test the meter to determine its percentage of accuracy.

The commutator of a new meter is not always in good working

condition when installed and it is advisable to place the meter in service for a time before making a final test. Then again, accidents affecting the accuracy of the meter are liable to occur during the installation of the meter and it is necessary to determine whether the accuracy of the meter has been affected. Induction meters should be tested as soon after their installation as possible.

The best method of making installation tests is by means of a standardized portable watt-hour meter.

267. Periodic Tests.—No matter how good the construction of a meter, nor how accurate its registration, its accuracy will diminish with time. Tests at regular intervals are, therefore, necessary to determine whether the permissible error is not exceeded.

Those periodic tests should be made at intervals to suit the circumstances. Commutator meters being more liable to become inaccurate should be tested more frequently than induction meters whose rugged construction and absence of commutator makes them immune to certain troubles. No definite rule can be formulated for determining the interval of time between periodic tests of different capacities of meters and for different classes of business, yet every company should appreciate the necessity for testing every class of meters before its maximum error exceeds the permissible limits. The interval between tests must be determined by experience, cost of metering, amount of bill, etc.

268. Complaint Tests.—When a consumer complains of his bill, it is frequently customary to test the meter unless a test has been made very recently. Complaint tests are conducted in the usual manner, except that it is customary to test the meter, not merely on light and full-loads, but also on other loads, especially on the normal load or load most generally used.

269. Inquiry Tests.—Inquiry tests are tests ordered by the company itself before the bill is rendered, to determine whether or not the meter has been operating properly.

270. Re-tests.—Re-tests are tests made before current is re-introduced to meter which has been out of service for a long time; or, if the meter has been opened by any one not authorized, or if the meter has been moved or reconnected.

271. Repair Tests.—Meters that have been repaired in service should be tested immediately after the completion of the repairs. Such tests are properly termed repair tests.

272. Special Tests.—Any tests not properly coming under any of the foregoing headings may be classed as special tests.

273. Meter Constants.—Since the number of revolutions of the meter disk is merely proportional to the energy that has passed, some constant must be used to convert the number of revolutions into watt-hours or kilowatt-hours. Two kinds of constants are used in practice, namely—dial constant, and test constant.

274. Dial Constant.—On large capacity meters the difference between any two readings seldom gives the watt-hours directly. To get the energy that has been registered, the difference between the dial statements must be multiplied by a constant which is usually marked on the dial. This is known as the dial constant. It has different values for meters of different capacities.

275. Test Constant.—Manufacturers of meters usually stamp or print upon some part of the meter a constant which is used in testing, but as there is no uniformity in the meaning of this constant, its significance must be explained.

276. Watt-hour Constant.—Since the meter registers in watt-hours, for every revolution of the disk, a definite quantity of energy must have been delivered to the load circuit. This energy, in watt-hours corresponding to one revolution of the disk, is called the watt-hour constant.

278. Watt-minute or Watt-second Constant.—A watt-hour is the energy delivered by one watt in 1 hour, similarly a watt-minute, or watt-second, is the energy or work done by one watt in 1 minute or 1 second, respectively. A watt-minute is thus equal to 1/60 watt-hour, and a watt-second equals 1/3600 watt-hour, and hence, the watt-minute constant is equal to $60 \times$ watt-hour constant, and a watt-second constant is equal to $3600 \times$ watt-hour constant.

$$
\begin{aligned}
\text{Let } K_h &= \text{watt-hour constant} \\
K_m &= \text{watt-minute constant} \\
\text{and } K_s &= \text{watt-second constant} \\
\text{then } K_s &= 3600K_h = 60K_m.
\end{aligned}
$$

The test constant as used by manufacturers is either one of these or a multiple of one of these constants.

279. Use of Constant in Testing.—The accuracy of a meter is expressed by the ratio of its indication to the actual watt-hours in per cent. In algebraic symbols this may be expressed by

$$100 \times \frac{\text{meter watt-hours}}{\text{actual watt-hours}} = \text{percentage of accuracy.}$$

The numerator is, of course, the indication of the watt-hour meter during a given time, while the denominator is the actual number of watt-hours as measured by standard instruments. If the actual watt-hours are determined by means of an indicating wattmeter, the indication will have to be multiplied by the time during which the observations were made. Thus, the actual watts as given by the indicating instruments multiplied by $T/3600$, will give the actual watt-hours. T is the duration of test in seconds, and 3600 is the number of seconds in 1 hour.

It is usually impossible to determine accurately the meter watt-hours for a short time from the dial indications of the meter, and, hence, in practice the number of revolutions of the disk during a definite time is determined by means of a stop watch. The meter watt-hours are then computed as above.

Since the watt-hour constant has been defined as the number of watt-hours corresponding to one revolution of the disk, the total number of watt-hours will be equal to the watt-hour constant times the number of revolutions, or

$$\text{watt-hours} = K_h \times R$$

where R is the number of revolutions counted in time T. We can, therefore, write

$$\text{Meter watts} \times \frac{T}{3600} = K_h \times R$$

or

$$\text{Meter watts} = \frac{K_h \times R \times 3600}{T}$$

This is the standard formula as used, in one form or another, by all meter manufacturers. If all meter manufacturers used this formula the watt-hour constant would be the test constant, but as different manufacturers use different modifications, this simple relation does not hold in every case. The constants as used by the various companies are:

General Electric Company:

$$\text{Test Constant} = \text{watt-hour constant}$$

$$\text{or } K_h = K_g$$

Duncan Electric Manufacturing Company:

$$\text{Test Constant} = \text{watt-minute constant}$$

$$\text{or } K_h = \frac{K_d}{60}$$

Westinghouse Electric and Manufacturing Company:

Test Constant = watt-second constant

$$K_h = \frac{1}{3600} K_w$$

Fort Wayne Electric Works:

Test Constant = 36 times the watt-hour constant, or

$$K_h = \frac{K_f}{36}$$

Where K_h represents the watt-hour constant and K_g, K_d, K_w, K_f, represent the test constants of the General Electric, Duncan, Westinghouse, and Fort Wayne Companies respectively.

If a rotating standard watt-hour meter has been used for determining the actual energy passed through the meter under test, the percentage of accuracy is given by

$$100 \frac{R \times K \text{ (of meter under test)}}{R' \times K' \text{ (of standard)}} = \text{percentage of accuracy}$$

R and K are the number of revolutions and constant of the meter under test, while R' and K' are the corresponding quantities of the rotating standard. It is thus evident that before the above formula can be used, K and K' will have to be reduced to the same basis. That is, a Westinghouse rotating standard cannot be used to test a Fort Wayne meter until the constants have been reduced either to watt-hours or watt-seconds, see Table VI.

280. Methods of Loading.—In practice there are several different methods of loading meters under test. The most common methods are:

1. The consumer's load
2. Portable lamp bank
3. A specially designed and constructed load box
4. Portable storage batteries
5. Step down transformers.

281. The Consumer's Load.—While this method is convenient, in that little accessory apparatus is necessary, the annoyance to the consumer and the liability to misunderstanding make it advisable to avoid this method as much as possible.

282. Portable Lamp Bank Method.—One form of lamp bank which may be used for this purpose has been suggested. These lamps are operated at the lamp voltage and the load is changed by changing the number of lamps in the circuit.

283. Special Load Box Method.—Load boxes may be merely self-contained non-inductive resistances to which may be attached indicating instruments. One form of such a load box is shown

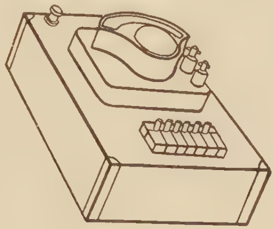


FIG. 235.

in Fig. 235 which is known as the Knopp load box. This is a variable resistance box upon which is mounted an indicating ammeter. The resistance box has several coils which may be

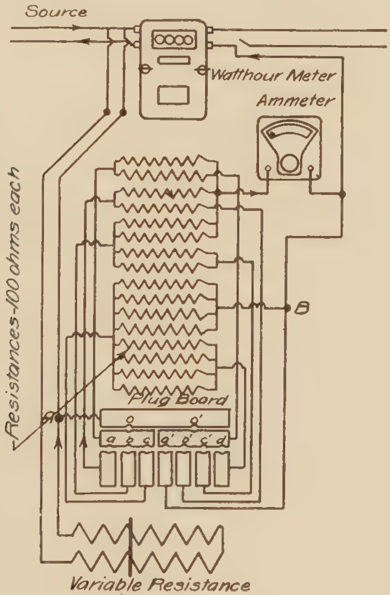


FIG. 236.

connected in different ways for different loads. The exact power consumption under a given voltage may be predetermined and marked on the box. In series with the loading resistance is

connected a second resistance whose value may be varied by sliding contacts. By the use of this second resistance the voltage drop across the load coils may be made equal to the voltage at which the power consumption of the coils was determined. The pressure coil of the watt-hour meter is connected so that the voltage impressed upon the meter is the same as that impressed upon the load coils. The reading of the ammeter is thus sufficient for determining the load. Fig. 236 shows the internal connections of the Knopp load box. When the box is to be used on 110-volt circuits, the plugs O, a, b, and c, are used. On 220-volt circuits additional resistance is connected in series by using plugs O', a', b', and c'. The Knopp load box may be used on either direct-current or alternating-current circuits.

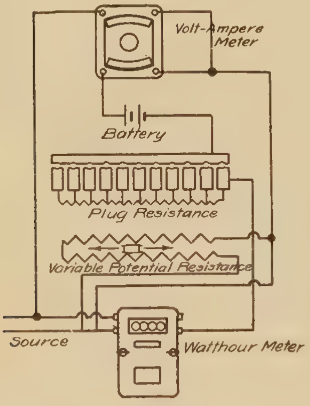


FIG. 237.

284. Portable Storage Battery Method.—For testing direct-current meters on the consumers' premises the portable storage battery has many advantages. The load current is supplied at a low voltage and hence the energy required for making the test is much less than when the load current is taken from the line. Any desired current can be obtained in a simple manner, and when once the adjustments are made, the current will remain practically constant. For ease of manipulation and rapidity of operation, some regulating device must be used with the battery. One good arrangement of resistances is shown in diagram Fig. 237. Other devices can easily be designed.

285. Low Voltage Transformer Method.—There are on the market several different makes of low-voltage transformers combined with resistances for alternating-current meter testing. The general principles of all are the same, hence only one will be described. The appearance of the Rollinson load box is shown in Fig. 238, the internal connections of which are shown in Fig. 239. The diagram clearly shows that the load box is primarily a step down auto-transformer, the secondary of which supplies current to the series coils of the tested and testing meters. The resistances for controlling the voltage impressed on the primary

are R_1, R_2, and the circular dial rheostat. The load current is thus varied by connecting these resistances in series or parallel by adjusting the rheostat, and by connecting in the load a greater or smaller number of secondary turns.

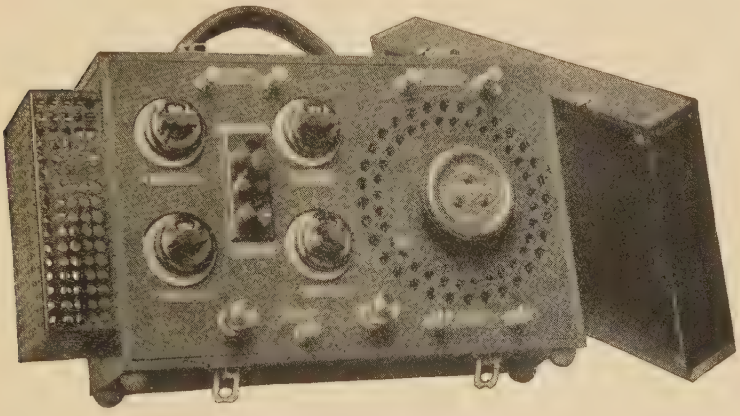


FIG. 238.

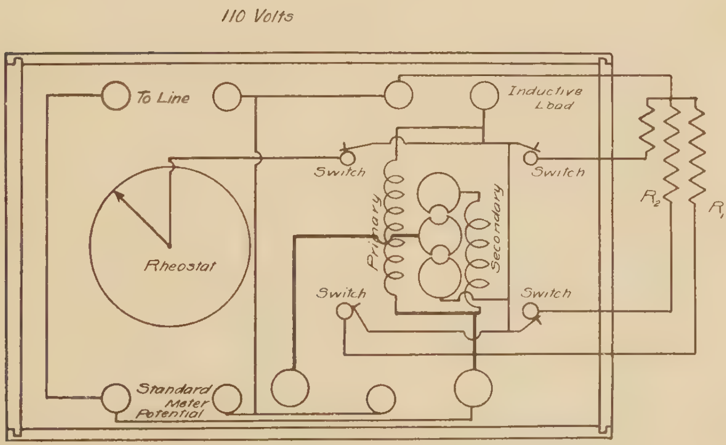


FIG. 239.

The manner of connecting the tested and testing meters to the load box is shown in Fig. 240.

Where step-down transformers are used, it is advisable first to make a thorough test of the influence of the transformer upon

the power-factor. The power-factor of the testing circuit will in most cases be less than unity, and under extreme conditions unduly low power-factors may be obtained. It is thus necessary to test all such load boxes for power-factor under working conditions.

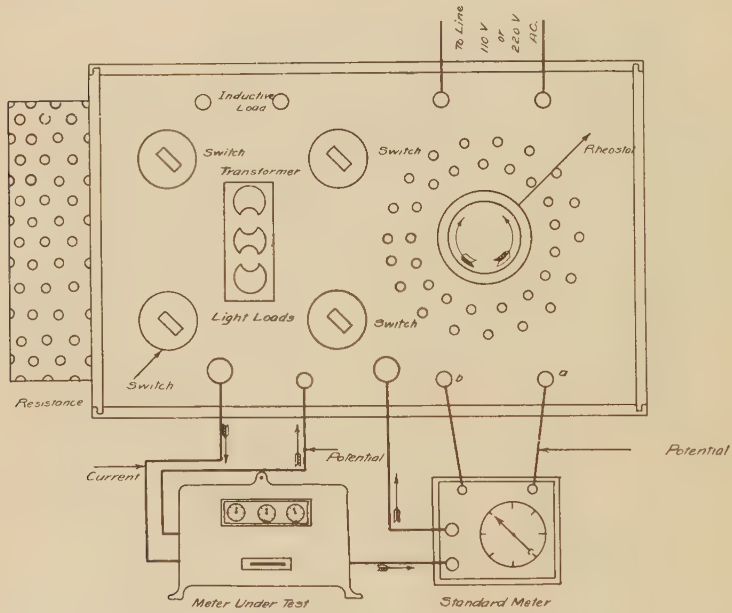


FIG. 240.

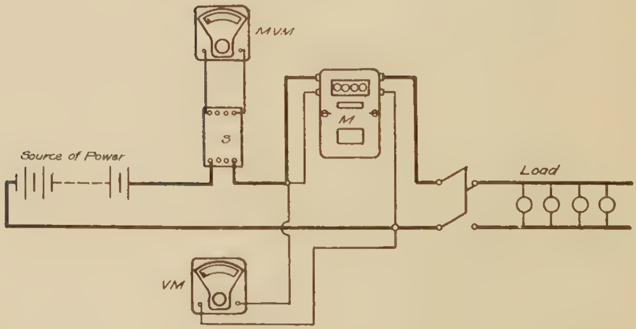


FIG. 241.

286. Determination of Watt-hour Constant, Experimentally.— To determine K_h for a direct-current watt-hour meter experi-

mentally, connect the instruments as shown in Fig. 241, where M represents a direct-current integrating meter, Vm a voltmeter, and MVM a standard millivoltmeter with its shunt S. The source of power is preferably a storage battery. Both the pressure and current are kept constant and the exact time of a definite number of revolutions is determined by means of the stop watch. The current is obtained from the millivoltmeter indications, divided by the resistance of the shunt. K is then calculated from

$$K = \frac{\text{watts} \times T}{R}$$

287. Method of Procedure.—Before taking any readings, enough time must elapse for the pressure coil to reach normal temperature. This is true of both direct-current and alternating-current meters. This time can be shortened considerably if some provision is made for subjecting the pressure coil to double voltage for a short period. This can easily be done on three-wire circuits, where pressure coils are connected between neutral and one main. Before taking any readings, make a white mark on the disk, adjust the voltage to the proper value, and insert all of the load resistance where a variable resistance is used. If the lamp bank is used, insert three or four lamps in series, and gradually increase the current until the armature commences to rotate. Repeat this three or four times and take the average of the currents. All values of the current below this value are not recorded by the meter, since they do not cause rotation of the armature at the normal voltage.

Adjust the resistance of lamps so that the full-load current passes through the meter, count the revolutions in 1 minute or some other suitable interval of time.

Repeat the test with decreasing values of load current. Care must be taken that the speed is not affected by external causes, such as air draughts, touching with the hand, etc. If the test constant, as given by the manufacturers, is not given in watt-hours, it can readily be changed by means of relations already given.

This constant should be determined at different loads and different temperatures. When this is done, it will be observed that the value of the constant depends to some extent upon the temperature and load. A diagram showing this relation for a

commutator type watt-hour meter is shown in Fig. 242. It will be observed that the constant at any given temperature first decreases to a minimum and then increases with increase in load. This increase is even more prominent at overloads. A change

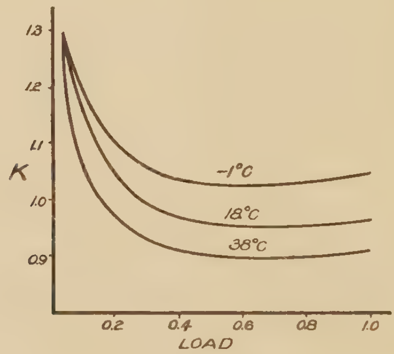


FIG. 242.

in temperature has a similar effect, namely, as the temperature increases the constant, K, decreases.

In Fig. 241 a storage battery is represented as the source of current for the series coils. For testing the commutator type, or

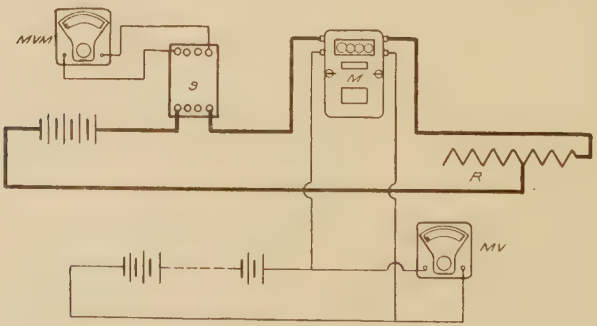


FIG. 243.

direct-current watt-hour meters, it is a good plan to provide a few cells of a storage-battery, preferably of the portable form, for supplying the load current, and many small cells for the excitation of the voltage circuit. When two sources of pressure

are used, the connections of the voltmeter and voltage circuit of watt-hour meter are to be made according to Fig. 243.

In making the foregoing test, it is best to take at least three readings at each load, and to vary the load so that the constant may be determined at 10, 25, 50, 100, and 125 per cent of the load.

From the data obtained, a curve similar to that shown in Fig. 242 should be drawn. If any reading is wrong, it will easily be detected by the corresponding point not coming on the curve.

TABLE VI.—TABLE TESTING CONSTANTS OF STANDARD TYPES OF WATT-HOUR METERS

Capacity of meters in amperes	Sangamo types "F" and "D"				G. E. types "C6," "J2" and "D2"			
	Testing constant				Testing constant			
	100–125 volts		200–250 volts		100–120 volts		200–220 volts	
	Watt-hours	Watt-seconds	Watt-hours	Watt-seconds	Watt-hours	Watt-seconds	Watt-hours	Watt-seconds
2½	......	....	......	....	......	....	......	....
3	......	....	......	....	.125	450	.25	900
5	½	1,800 "F"	1	3,600 "F"	......	....	......	....
5	⅔	2,400 "D"	1⅓	4,800 "D"	.2	720	.4	1,440
10	⅔	2,400	1⅓	4,800	.4	1,440	.75	2,700
15	......	....	......	....	.6	2,160	1.25	4,500
20	1⅓	4,800	2⅔	9,600	......	....	......	....
25	......	....	......	....	1.	36,00	2.	7,200
30	2	7,200	4	14,400	......	....	......	....
40	2⅔	9,600	5⅓	19,200	......	....	......	....
50	......	....	......	....	2.	7,200	4.	14,400
60	4	14,400	8	28,800	......	....	......	....
75	......	....	......	....	3.	10,800	6.	21,600
80	5⅓	19,200	10⅔	38,400	......	....	......	....
100	6⅔	24,000	13⅓	48,000	4.	14,400	7.5	27,000
150	10	36,000	20	72,000	6.	21,600	12.5	45,000
200	13⅓	48,000	26⅔	96,000	......	....	......	....
300	20	72,000	40	144,000	12.5	45,000	25.	90,000

TABLE VI—*Continued*

Capacity of meters in amperes	G. E. types J, JI, JN, FN, DN, and DI				G. E. type "I"			
	Testing constant				Testing constant			
	100–110 volts		200–220 volts		100–130 volts		200–260 volts	
	Watt-hours	Watt-seconds	Watt-hours	Watt-seconds	Watt-hours	Watt-seconds	Watt-hours	Watt-seconds
2½	......	....	......	....	......	....	......	....
3	.2	720	.4	1,440	.2	720	.4	1,440
5	.5	1,800	1.	3,600	.3	1,080	.6	2,160
5	......	....	......	....	......	....	......	....
10	.5	1,800	1.	3,600	.6	2,160	1.25	4,500
15	1.	3,600	2.	7,200	1.	3,600	2.	7,200
20	......	....	......	....	......	....	......	....
25	1.	3,600	2.	7,200	1.5	5,400	3.	10,800
30	......	....	......	....	......	....	......	....
40	......	....	......	....	......	....	......	....
50	2.	7,200	4.	14,400	3.	10,800	6.	21,600
60	......	....	......	....	......	....	......	....
75	......	....	......	....	5.	18,000	10.	36,000
80	......	....	......	....	......	....	......	....
100	......	....	......	....	6.	21,600	12.5	45,000
150	......	....	......	....	10.	36,000	20.	72,000
200	......	....	......	....	12.5	45,000	25.	90,000
300	......	....	......	....	20.	72,000	40	144,000

TABLE VI—*Continued*

Capacity of meters in amperes	Westinghouse types "B" and "C"				Fort Wayne type "K" (345,000 and above)			
	Testing constant				Testing constant			
	100–110 volts		200–220 volts		110 volts–2w		220 volts–2w	
	Watt-hours	Watt-seconds	Watt-hours	Watt-seconds	Watt-hours	Watt-seconds	Watt-hours	Watt-seconds
2½	......	....	......	....	......	....	......	....
3	.....	......	...	......	....	....	......	....
5	.333	1,200	.666	2,400	.25	900	.5	1,800
5	.. ..	....	...	....	......	....	.....	....
10	.666	2,400	1.333	4,800	.5	1,800	1.	3,600
15	......	....	......	....	.75	2,700	1.5	5,400
20	1.333	4,800	2.666	9,600	1.	3,600	2.	7,200
25	......	....	......	....	1.25	4,500	2.5	9,000
30	......	....	......	....	.....	....	......	....
40	2.666	9,600	5.333	19,200	2.	7,200	4.	14,400
50	......	....	......	....	......	....	......	....
60	......	....	......	....	2.5	9,000	5.	18,000
75	......	....	......	....	3.75	13,500	7.5	27,000
80	2,666	9,600	5.333	19,200	......	....	......	....
100	......	....	......	....	5.	18,000	10.	36,000
150	......	....	......	....	7.5	27,000	15.	54,000
200	......	....	......	....	10.	36,000	15.8	57,000
300	....	....	.....	....	.....	....	.....	....

TABLE VI—*Continued*

Capacity of meters in amperes	Fort Wayne type "K" (344,999 and less)				Gutmann meters			
	Testing constant				Testing constant			
	110 volts–2w		220 volts–2w		50 volts		100 volts	
	Watt-hours	Watt-seconds	Watt-hours	Watt-seconds	Watt-hours	Watt-seconds	Watt-hours	Watt-seconds
2½	...	...	...	...	...	...	...	...
3	.25	900	.5	1,800	...	...	...	...
5	.25	900	.5	1,800	...	...	¼	900
10	.5	1,800	1.	3,600	⅓	1,200	⅓	1,200
15	1.	3,600	1.5	5,400	⅓	1,200	⅓	1,200
20	1.	3,600	2.	7,200	...	...	...	...
25	1.	3,600	2.	7,200	½	1,800	1.	3,600
30	2.	7,200	2.5	9,000	...	...	...	...
40	2.	7,200	3.	10,800	...	...	...	...
50	2.	7,200	4.	14,400	1.	3,600	2.	7,200
60	3.	10,800	5.	18,000	...	...	...	...
75	3.	10,800	6.	21,600	2.	7,200	2.	7,200
80	...	...	...	...	...	...	...	...
100	4.	14,400	8.	28,800	2.	7,200	3.	10,800
150	6.	21,600	12.	43,200	3.	10,800	6.	21,600
200	8.	28,800	16.	57,600	6.	21,600	...	...

TABLE VI—*Continued*

Capacity of meters in amperes	Duncan meters				Columbia meters			
	Testing constant				Testing constant			
	100–125 volts		200–250 volts		110 volts		220 volts	
	Watt-hours	Watt-seconds	Watt-hours	Watt-seconds	Watt-hours	Watt-seconds	Watt-hours	Watt-seconds
2½	⅛	450	½	1,800	$\frac{5}{36}$	500	$\frac{5}{18}$	1,000
3	...	...	...	...	...	...	...	...
5	¼	900	½	1.800	$\frac{5}{18}$	1,000	$\frac{5}{9}$	2,000
10	½	1,800	1.	3,600	$\frac{5}{9}$	2,000	$1\frac{1}{9}$	4,000
15	1	3,600	2.	7,200	$\frac{5}{6}$	3,000	$1\frac{2}{3}$	6,000
20	...	...	...	...	...	...	...	...
25	1	3,600	2.	7,200	$1\frac{7}{18}$	5,000	$2\frac{7}{9}$	10,000
30	...	...	...	...	...	...	...	...
40	...	...	...	...	...	...	...	...
50	2	7,200	4.	14,400	$2\frac{7}{9}$	10,000	$5\frac{5}{9}$	20,000
60	...	...	...	...	...	...	...	...
75	3	10,800	6.	21,600	$4\frac{1}{6}$	15,000	$8\frac{1}{3}$	30,000
80	...	...	...	...	...	...	...	...
100	4	14,400	8.	28,800	$5\frac{5}{9}$	20,000	$11\frac{1}{9}$	40,000
150	6	21,600	12.	43,200	$8\frac{1}{3}$	30,000	$16\frac{2}{3}$	60,000
200	8	28,800	16.	57,600	$11\frac{1}{9}$	40,000	$22\frac{2}{9}$	80,000
300	...	...	...	...	$16\frac{2}{3}$	60,000	$33\frac{1}{3}$	120,000

288. Test for Percentage of Accuracy.—In commercial practice, it is not often necessary to determine K_h or the watt-hour constant, as the test constant is given by the maker, as well as the formula, by which the number of revolutions are to be translated into watt-hours. The quantity that is of most interest commercially is the percentage of accuracy which is defined as the ratio of the registered watt-hours, expressed as a percentage, in a given time to the true watt-hours, or kilowatt-hours. That is, the important question is how much too fast or too slow is the meter, rather than the characteristics of the meter.

When an ammeter and voltmeter are to be used in testing a direct-current watt-hour meter, the connections shown in Figs. 241 and 243 may be used. The watts registered are calculated from the constant of the meter, number of revolutions of disk, and duration of test, thus:

$$\text{Meter watts} = \frac{K_h \times R \times 3600}{T}$$

The true watts are obtained from the readings of the indicating instruments and are equal to $I \times E$. The percentage of accuracy is then equal to

$$\text{Percentage of accuracy} = \frac{100 K_h \times R \times 3600}{T \times I \times E}$$

It must be remembered that K_h is the watt-hour constant, and as some manufacturers of meters use other constants, the value

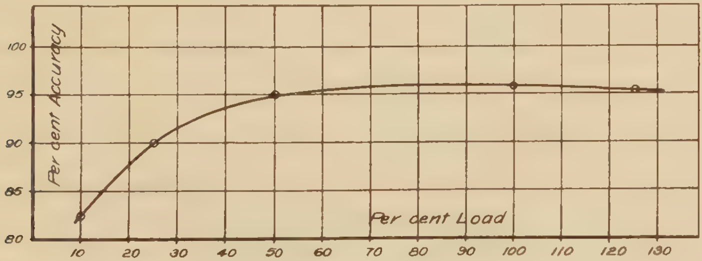


FIG. 244.

of the manufacturer's constant in terms of K_h will first have to be determined before substituting in the above formula. The percentage of accuracy should be determined for 10, 25, 50, 100, and 125 per cent of load, and a curve plotted for the load and percentage of accuracy. The form of such a curve is shown in Fig. 244.

EXAMPLE

Test No. 4.—

Test of Scheefer D. C. Watt-hour Meter.

Capacity 30 amperes.

Apparatus.—

Portable Wattmeter, Weston.

Stop watch.

Lamp bank.

Temperature 20° C.

TABLE VII

Per cent load	No. revolutions	Time seconds	Meter watts	Correct watts	Per cent accuracy	Remarks
10	10	185.3	259	300	82.5	
25	10	68.8	67.5	750	90.2	Poorly com-
50	20	67.3	14,260	1,500	95.1	pensated
100	30	50	2,880	3,000	96	for friction.
125	40	53.6	3,581	3,750	95.5	Curve Fig. 244.

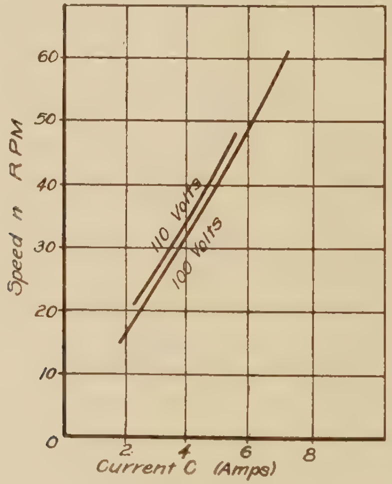


Fig. 245.

It is evident from the form of the curve that the friction of the bearings and commutator is not properly compensated at light loads. The speed of the rotating element in a well-designed meter should be proportional to the load. Whether this relation is fulfilled is well shown by the curve of Fig. 245, where the load current is plotted horizontally and speed vertically.

289. Test of a D. C. Three-wire Meter.—The three-wire direct-current meter differs from the two-wire meter in having its series or current coil divided into two coils that are, or should be, exactly alike. These two coils are connected, one in each outside wire of a three-wire circuit, while the common voltage

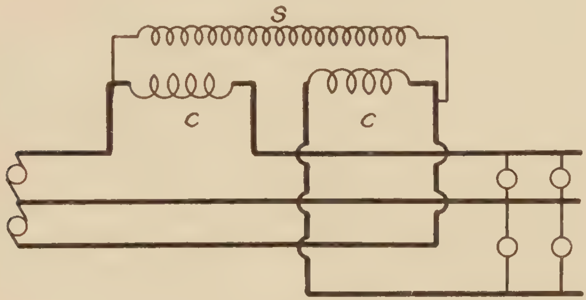


FIG. 246.

coil may be connected between the neutral and an outside wire, or across the outside wires, according to the design of the meter. The limitations of the different connections have already been discussed. The second method of connection is shown in Fig. 246 in which CC are the current coils and S the shunt or pressure coil.

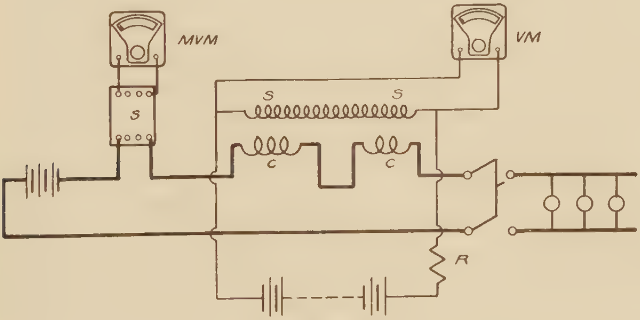


FIG. 247.

The connections for testing such a meter are shown in Fig. 247. The current coils are connected in series and also in series with one wire of a two-wire circuit. If potential cells are available, the best plan is to connect the voltage coil to a separate source of electromotive force, as shown, otherwise it may be connected

in parallel with the current coils. The operation of the test is the same as other tests that have been described in which a millivoltmeter is used to measure the current and a standard voltmeter to measure the voltage. A wattmeter may be used in place of voltmeter and millivoltmeter.

290. Test for Balance.—In addition to the foregoing test for balanced load, it is also necessary to determine the effect of each current coil in causing rotation. In a well-designed meter, the coils should supply equal torques. To determine whether the torques are equal, two simple tests may be made. The simplest method consists in connecting the two-current coils in series, but in such a way that the resulting torques are in opposition. If the two coils exert equal torques, no motion will result. When this is not the case, the armature will rotate either forward or backward, depending upon which coil exerts the greater torque. The resulting speed will undoubtedly be very slow, and for that reason it will be advisable to overload the coils for a short time.

The second method consists in connecting to the circuit only one of the series coils at a time. If the two coils are exactly balanced, the resulting speeds should be exactly equal, so long as voltage and current remain unchanged. The speed with only one current coil should also be equal to one-half the speed when both coils are operating, current and voltage remaining constant.

291. Test of Ampere-hour Meters.—The simplest and most convenient method of testing an ampere-hour meter is by means of a standardized ammeter of proper range and a stop watch. The ammeter is connected in series with the ampere-hour meter through a rheostat or lamp bank. The current is adjusted to the value desired and maintained constant throughout the test. The ammeter reading multiplied by the duration of test in hours gives the actual ampere-hours passed through the meter. The calculated ampere-hours compared with the registration of the meter will indicate the error. The number of ampere-hours registered by the ampere-hour meter may be calculated from the number of rotations of the disk and ampere-hour constant. The chemical ampere-hour meters that are graduated in watt-hours can be tested either with a wattmeter or standard test watt-hour meter in the same way as watt-hour meters.

CHAPTER XX

METHODS OF OBTAINING DIFFERENT POWER-FACTORS

292. Introduction.—Since alternating-current meters must be tested on loads of different power-factors, it will be advisable first to discuss some methods of obtaining these. Several methods may be used for this purpose; the particular one to be used in any case will depend upon conditions present and apparatus available.

293. Reactance Coil Method.—As has already been pointed out, the current flowing through an inductive circuit is expressed by

$$I=\frac{E}{(R^2+L^2\omega^2)^{\frac{1}{2}}}$$

and the difference in phase between the pressure E and current I is obtained from $\tan\theta=\frac{L\omega}{R}$. Evidently, if $L\omega$ is varied, $\tan\theta$,

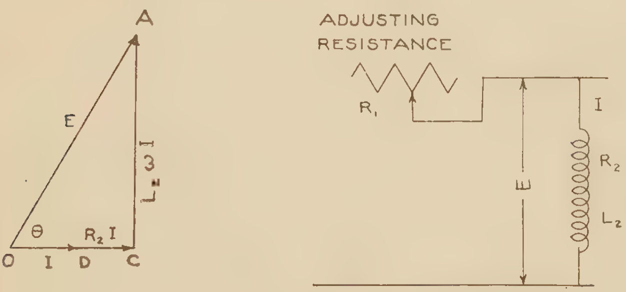


FIG. 248.

and hence, θ can also be varied. This relation will perhaps be better understood by referring to Fig. 248. An alternating current of frequency f, supplies current to a resistance and inductance coil. If I is the current flowing through the inductance coil R_2L_2 and E the electromotive force between its terminals, the relation between these quantities is then shown by the vector diagram. The voltage drop due to the resistance of inductance

coil is R_2I, while that due to self-inductance is $2\pi fL_2I$. R_2I is in phase with the current, and $2\pi fL_2I$ is at right angles or 90° ahead of I. Drawing $OC = R_2I$, and $CA = 2\pi fL_2I$, we get the right-angled triangle OCA, of which OA, the vector sum of OC and CA, is the electromotive force E. The value of θ thus depends upon OC and CA and can be changed by changing either. In practice, the change in power-factor is commonly obtained by changing the inductance. This is accomplished by providing a movable iron core for the inductance coil as shown in Fig. 249. By introducing or withdrawing the laminated iron core, different values of power-factor can readily be obtained. The

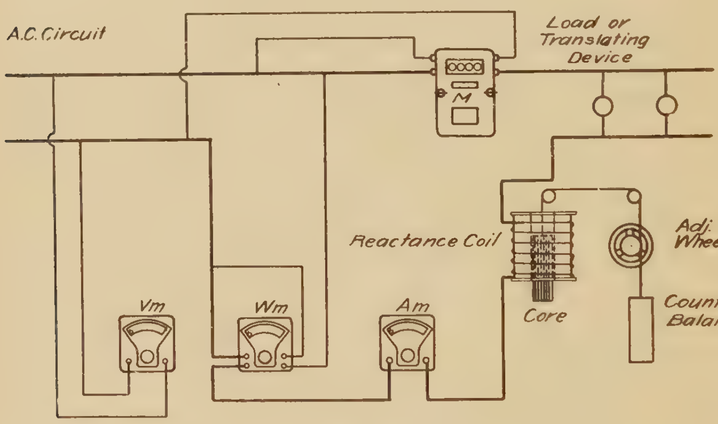


FIG. 249.

exact value of the power-factor is calculated from the readings of the wattmeter, voltmeter, and ammeter. It has already been pointed out that at low power-factors corrections must be made to the wattmeter readings.

294. Two Transformer Method.—Another method of securing an inductive load for single-phase watt-hour meters is by means of two transformers connected one to each phase of a two-phase circuit. The secondaries of the transformers are provided with several taps and are connected in series. The manner of connecting transformers and instruments to the circuits are shown in Fig. 250. As is evident from the diagram, the current coil of the meter to be tested is connected to one-phase of the two-phase circuit, while the pressure coil is connected in series with the secondaries of the transformers. The ammeter, wattmeter,

and voltmeter are connected as usual. The value and position of the pressure applied to the meter will evidently be equal to the vector sum of the secondary voltages of the transformers. By

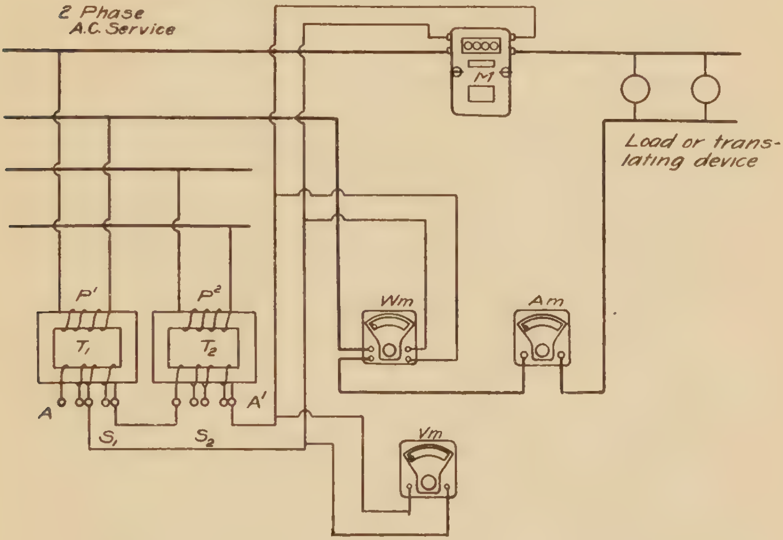


FIG. 250.

varying the relative values of these, different values of power-factor can be obtained. The vector diagram of Fig. 251 shows how this is accomplished. Let $OA = E_1$ represent the secondary voltage of transformer T_1. This voltage will be in phase with I, the current through the meter.

If the secondary voltages of the transformers are equal in magnitude, OB will represent the secondary pressure of the transformer T_2. The resultant voltage, or that impressed upon the pressure coil of the meter, is then equal to OC, which leads the current by the angle θ. By changing the relative values of the secondary voltages, the value of θ can be changed from 0 to 90°. The vector diagram shows the relative values of these pressures for a phase difference θ'. It is clear that not only power-factor, but the impressed pressure may be

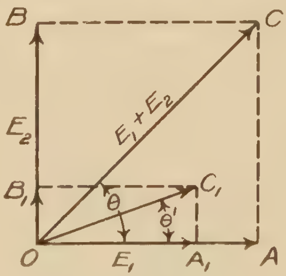


FIG. 251.

varied between wide limits by changing connections A and A' in Fig. 250. If two transformers are specially constructed for this test, the connection of S_1 and S_2 may be made to a circular switch. The power-factor and voltage for a given position of the switch may be calculated once for all and marked on the

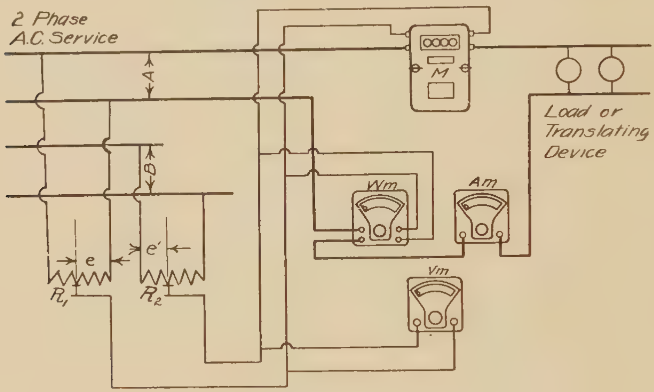


FIG. 252.

switch. If that is done, the position of the switch will indicate the voltage and power-factor, and the voltmeter and ammeter may be omitted, unless needed for other measurements.

295. Two Resistance Method.—Perhaps a simpler method consists in replacing the two transformers by two slide contact resistances as shown in Fig. 252. A vector diagram similar to that in Fig. 251 will show the relation of the quantities involved. The value and position of E is determined by the position of the sliding contacts R_1 and R_2. That is, E is the vector sum of e and e', Fig. 253, which represent the potentials between sliding contacts and middle wire. The

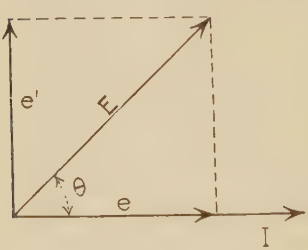


FIG. 253.

resistances R_1 and R_2 must be non-inductive and capable of withstanding the line voltage, and have a current capacity of 1 to 2 amperes. A non-inductive load is connected to the meter as in the transformer method.

By adjusting R_1 and R_2, any desired value and phase position

of E may be obtained. In most cases it may be possible to use lamps in place of resistances. The adjustment, however, will be much more troublesome. The power-factors may be calculated for certain positions of the contacts and the points so determined may be marked with the corresponding power-factor. When this is done, the ammeter and voltmeter may be omitted and the required power-factor reproduced by simply setting the contacts at the same points.

296. Two Generator Method.—A very convenient, and at the same time accurate, method of varying the power-factor for testing purposes may be obtained by providing a special motor

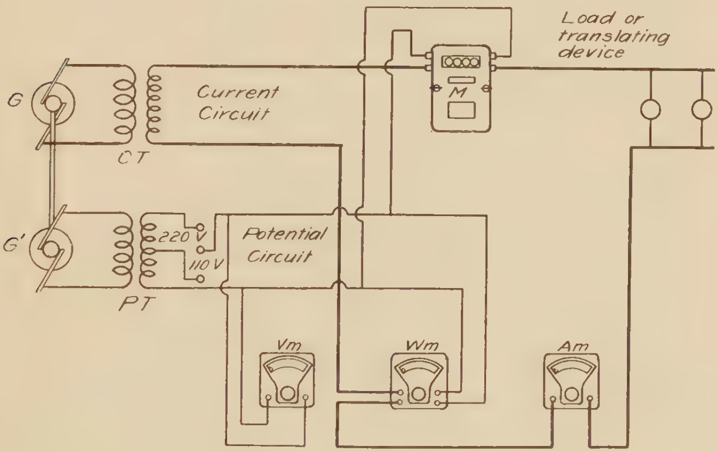


FIG. 254.

generator set consisting of one driving motor and two similar alternating-current generators, all connected to the same shaft. The alternators need not be alike in every respect, but should have the same frequency. One generator may preferably be of low voltage and comparatively large current output for supplying current to meters under test. The other generator may be of low-current capacity but relatively high voltage to serve as a source of pressure. Fig. 254 shows the connections for such a system. If the generators are low voltage, the current and potential transformers may be omitted.

One of the generators must be connected to the shaft in such a way that the position of its armature may be shifted with reference to the other. The frequencies of the two generators

will always be the same on account of the rigid connection, and by shifting the armature of one with reference to the other, any power-factor can be obtained. Furthermore, the power-factor can be calculated from the relative position of the armatures if the position of zero, or unity power-factor is accurately known. Thus, if each generator has four poles for every revolution of the armature, the current or voltage will pass through two cycles. That is, for every 360° of armature motion, the electromotive force passes through 720 electrical degrees, or 1 angular degree equals 2 electrical degrees. If then the armatures are in a position of unity power-factor, and one is rotated 5° on the shaft, there will result a displacement of 10° between the electromotive forces. The power-factor has then been changed from cos 0° to cos 10°. In general, then, if ϕ is the angle through which the armature of one machine has been shifted with reference to the other, p is the number of poles on each generator, and $\cos\theta$ is the power-factor, we can express $\cos\theta$ in terms of ϕ and p thus,

$$\cos\theta = \cos\frac{p}{2}\times\phi$$

Measuring ϕ, the power-factor can be calculated. Any error in ϕ is, however, multiplied $\frac{p}{2}$ times, and, hence, for very accurate work ϕ should be determined by the aid of a vernier. In some cases it may be more convenient to have the machines constructed in such a way that the armatures remain rigidly connected to the shaft, while the field of one is movable. Of course, the same effect is obtained as in the previous case.

297. Ammeter Method of Measuring Power-factors.—For the want of a better name, the author, having devised the two following methods, has decided to call them "Ammeter Methods." The reason for this is that at most only two ammeters, and with the use of a polyphase switchboard, only one ammeter is necessary for an approximate determination of the power-factor.

In plants using quarter-phase— commonly called two-phase— or three-phase generators, the methods will undoubtedly prove useful and accurate enough for commercial purposes.

298. Ammeter Method on Two-phase Circuits.—The connections for testing the watt-hour meter on a two-phase three-wire circuit are shown in Fig. 255. The series coil of the watt-hour meter carries the vector sum of the two-phase currents.

When the voltage coil of the watt-hour meter is connected as indicated in the diagram, the power-factor is the cosine of the angle between the series current and pressure across mains 1 and 2.

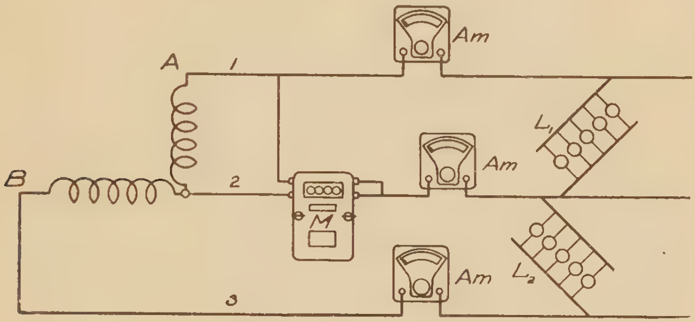


FIG. 255.

Thus in Fig. 256 let OE_1 represent the voltage between mains 1 and 2, and OE_2 the voltage between mains 2 and 3. Since the load is supposed to be non-inductive, OI_1 and OI_2 may be considered as representing the currents in the separate phases. The current in main 2 is the vector sum of OI_1 and OI_2 and is, therefore, represented by OI.[1] The power-factor of the load registered by the watt-hour meter, when connected as indicated in Fig. 255, is cos θ, where θ is the angle between OE_1 and OI. The value of θ is determined from the relative values of currents in lamp banks L_1 and L_2. That is, if I_1 is current in load L_1, and I_2 the current in load L_2, the power-factor is equal to

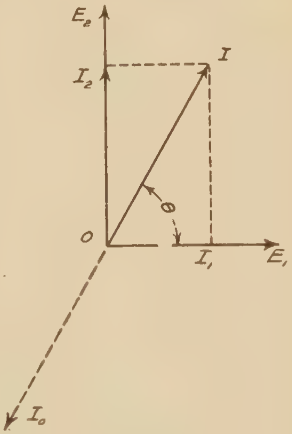


FIG. 256.

$$\cos \theta = \frac{I_1}{(I^2_1 + I^2_2)^{\frac{1}{2}}}$$

and hence, can be determined from the readings of two ammeters, one placed in main 1 and the other in main 3. Since, $(I^2_1 + I^2_2)^{\frac{1}{2}} = I$, the current in the series

[1] To be exact the current in main 2 should be represented by dotted line OI_0. The value of power-factor will be the same in either case.

coil of watt-hour meter, an ammeter placed in main 2 will indicate $(I^2_1+I^2_2)^{\frac{1}{2}}$ of I. The power-factor then reduces to

$$\cos\theta=\frac{I_1}{I}$$

When $I_2=0$, the total load is on L_1; $I_1=I$, and $\cos\theta=1$.

When $I_1=0$, the total load is on L_2, and $\cos\theta=\frac{0}{I_2}=0$.

Hence, by adjusting the load between L_1 and L_2, any power-factor between 0 and 1 can be approximately determined. According to the foregoing discussion, two ammeters are needed;

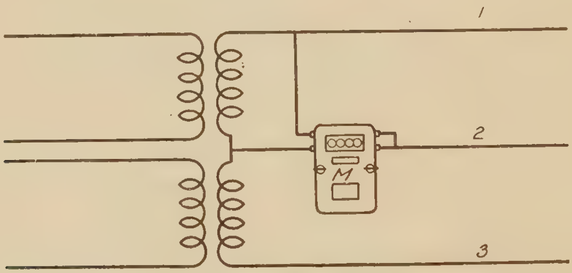


FIG. 257.

later it will be shown how the connections may be made so that one ammeter will suffice.

In case the generator is not wound for three-wire connection, which the foregoing method presupposes, a three-wire circuit can be obtained by connecting two transformers, as shown in Fig. 257. The primary circuits of the transformers are connected to the separate phases of the generators and the secondaries are interconnected as shown.

299. Ammeter Method on Three-phase Circuits.—When three-phase alternators are available, the same general plan may be followed. When the series coil of the meter to be tested is connected to the middle wire, as indicated in Fig. 258, and the pressure coil across E_1, the power-factor is

$$\cos\theta=\frac{1}{2}\frac{I_3-I_2}{(I^2_2+I_2I_3+I^2_3)^{\frac{1}{2}}}=\frac{I_3-I_2}{2I}$$

where I, I_2, and I_3 are currents in middle and outside wires, respectively.

When the potential circuit is connected across E_3, the power-factor is

$$\cos \theta = \frac{1}{2} \frac{2I_3 + I_2}{(I^2_2 + I_2 I_3 + I^2_3)^{\frac{1}{2}}} = \frac{2I_3 + I_2}{2I}$$

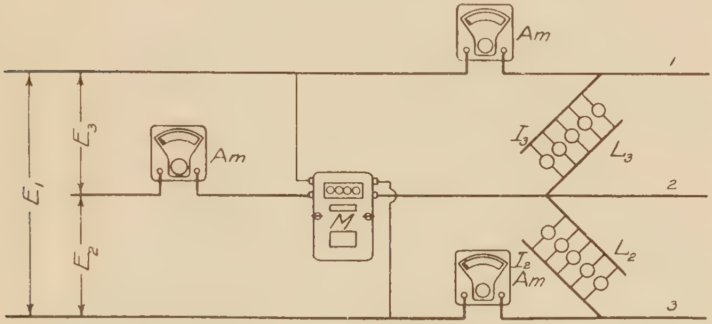


FIG. 258.

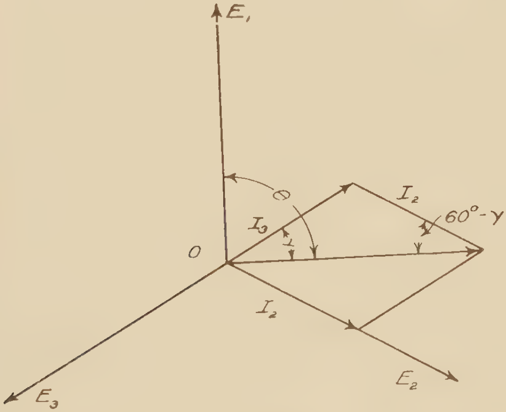


FIG. 259.

These formulas can be derived as follows:

Assuming the time-phase displacements of the e.m.f.'s of the three-phase generator to be 120°, Fig. 259 is a vector diagram of the quantities involved. From this diagram it is plainly evident that both the magnitude and time-phase of I with reference to E_1 depend upon the relative magnitudes of I_2 and I_3.

Let θ be the angle between I and E_1, then the angle between I_3 and E_1 is 60° when load is non-inductive. Hence,

$$\cos \theta = \cos (\gamma + 60)$$
$$= \tfrac{1}{2} \cos \gamma - \tfrac{1}{2}\sqrt{3} \sin \gamma$$

From the triangle whose sides are I, I_2, and I_3, we get

$$I_3 : I_2 : : \sin (60-\gamma) : \sin \gamma.$$

whence
$$\sin \gamma = \tfrac{1}{2} I_2 \frac{\sqrt{3}}{(I^2_3 + I_2 I_3 + I^2_2)^{\frac{1}{2}}}$$

and
$$\cos \gamma = \tfrac{1}{2} \frac{2I_3 + I_2}{(I^2_3 + I_2 I_3 + I^2_2)^{\frac{1}{2}}}$$

Substituting these values in the expression for $\cos \theta$ and reducing, we get,

$$\cos \theta = \tfrac{1}{2} \frac{I_3 - I_2}{(I^2_3 + I_3 I_2 + I^2_2)^{\frac{1}{2}}}$$

But
$$(I^2_3 + I_3 I_2 + I^2_2)^{\frac{1}{2}} = I$$

Hence,
$$\cos \theta = \tfrac{1}{2} \frac{(I_3 - I_2)}{I}$$

When $I_2 = I_3$, or when both phases are equally loaded, $\cos \theta = 0$. When $I_2 = 0$, $I_3 = I$, and $\cos \theta = .5$; and when $I_3 = 0$, $I_2 = -1$, and $\cos \theta = -.5$. Hence, by such a connection the power-factor can be varied between $-.5$ and $+.5$.

By a similar process of reasoning it can be shown that when the pressure coil is connected across E_3, the power-factor is

$$\cos \theta = \tfrac{1}{2} \frac{2I_3 + I_2}{(I^2_3 + I_2 I_3 + I^2_2)^{\frac{1}{2}}} = \frac{2I_3 + I_2}{2I}$$

When $I_2 = 0$, $I_3 = I$, and $\cos \theta = 1$.
When $I_3 = 0$, $\cos \theta = 0.5$. That is, when all the load is on L_3 the power-factor is 1, and when all the load is on L_2 the power-factor is 0.5.

The advantage of this method lies in the ease with which both the connections and calculations can be made. The ordinary ammeter—voltmeter—wattmeter method necessitates three instruments, and it is a well-known fact that at low power-factors the inductance of the pressure coil of the wattmeter introduces errors which are proportional to $\sin \theta$. This error, due to the inductance of the pressure coil, may be as great as that due to the unbalancing of the pressures in the method under consideration. The fact that one ammeter is sufficient, makes this method especially advantageous for small power plants whose supply of instruments is limited.

In order to use only one ammeter, the polyphase switchboard,

shown in Fig. 231 may be used. The three mains are connected to binding posts 1, 2, 3, and the three load terminals to posts 1′2′, and 3′, while the ammeter is connected to leads indicated. This connection is shown in Fig. 260. An examination of the diagram will show that when the three pole, and three single-pole switches are closed, the ammeter will register no current. When, however, both double pole, double-throw switches are closed to the left and single-pole switch *C* is opened, the

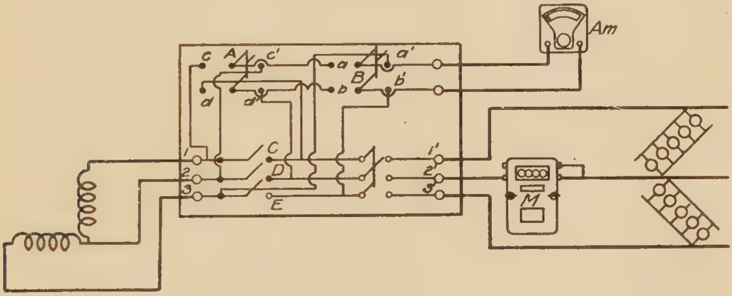


Fig. 260.

ammeter will give the current in one lamp bank. An examination of the diagram will show the operations necessary to obtain in succession the currents in the other lamp bank and in the middle wire.

It has been shown by experiment that for the purpose of watt-hour meter testing the method is accurate enough for commercial purposes, and in fact the results obtained by the ammeter-method are just as accurate as the three-instrument method unless accurately calibrated instruments of proper range are available. The adjustable load must be non-inductive.

CHAPTER XXI

SPECIAL TESTS OF A. C. WATT-HOUR METERS

300. Test for Quarter-phasing.—As pointed out in Article 180, the displacement between the magnetic field due to the current coil and that due to the voltage coil must be 90° for accurate registration on inductive load. One of the first tests to be made on a single phase meter is to determine whether this relation exists.

Since no torque should be exerted upon the rotating disk when the current lags or leads by an angle of 90 degrees, in order to

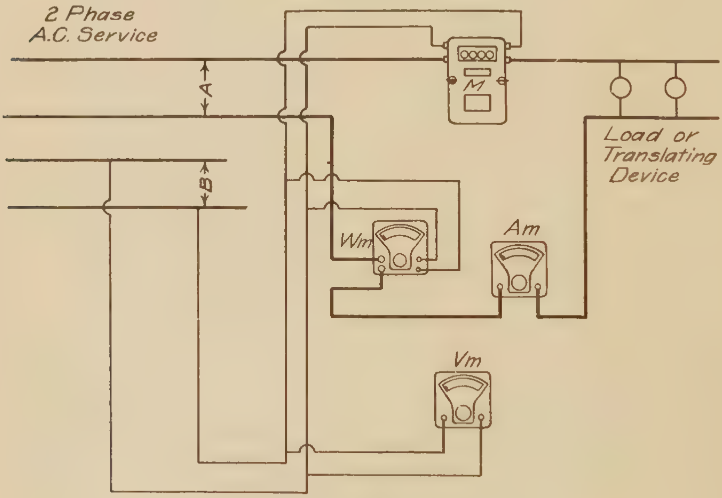


FIG. 261.

determine whether the meter is properly quarter-phased, the absence of torque under proper conditions is the criterion of the test. In making the test, connect the current coil to one phase of a quarter-phase circuit and the voltage coil to the other phase. A suitable method of making these connections is shown in Fig. 261. The exact phase difference is calculated from the readings of the ammeter, voltmeter, and wattmeter, thus

$$\text{Cos } \theta = \frac{watts}{I \times E}.$$

If the quarter-phase relation exists, the wattmeter reading will be zero if corrections are made for resistance, hysteresis, and eddy-current losses. Any inaccuracy in the wattmeter will, however, vitiate the result. Accurate measurements of alternating-current power in circuits of low power-factors require special methods and apparatus.

301. Test of Single-phase Meter on Non-inductive Load.—For non-inductive load test, connect the meter as shown in Fig. 262. It must be remembered that the connections shown are diagrammatic only. For any particular make of meter, the connections will have to conform to the diagrams and directions sent out by the makers.

The voltage during the test should be constant, and the time of a definite number of revolutions of the disk determined by the

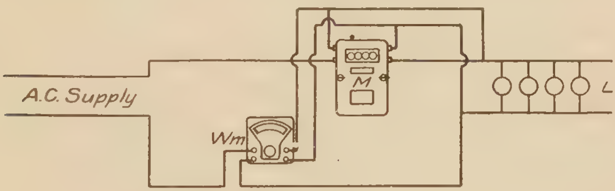


FIG. 262.

aid of an accurate stop watch. It is more accurate to count a definite number of revolutions than to count the revolutions during a definite time, as the fraction of a second is more easily determined than the fraction of a revolution. The per cent accuracy is then determined by

$$\text{Per cent accuracy} = \frac{100 \times KR}{T \times \textit{watts}}$$

where K, R, and T have the significance already explained. The non-inductive load test is to be made at various percentages of load, but with constant voltage. Then repeat the test with both an increase and decrease in voltage of 15 to 20 per cent, but same loads as before. That is, first increase the voltage by 20 per cent above rated voltage and ascertain the accuracy on 10, 50, 100, and 125 per cent of load. Second, decrease the voltage by 20 per cent and ascertain the accuracy at the previously mentioned fractions of load. When the accuracy for the various voltages and loads has been determined, curves should

be plotted from the data, the per cent of load being plotted horizontally and the percentage of accuracy vertically.

EXAMPLE

Test No. 5.—Test of single-phase watt-hour meter.

Apparatus.—Fort Wayne 110-volt, 60-cycle, 5-ampere watt-hour meter.
Weston wattmeter No. 4123.
Lamp bank.
Stop watch No. A.
Temperature 21° C.

TABLE VIII

Per cent of load	No. revolutions	Time Secs.	Meter watts	Correct watts	Percentage accuracy	Remarks
10	2	32.7	55	55.0	100	
25	8	52.4	137.5	137.5	100	Voltage constant at 110 volts
50	12	39.6	273.4	275.0	99.5	
75	16	35.1	408.4	412.5	99.0	
100	20	33.4	540.5	550.0	98.3	
125	25	33.5	673.7	687.5	98.0	Curve Fig. 263a.

302. Test of Single-phase Watt-hour Meter on Inductive Load.—For an inductive load test, any of the methods for varying the power-factor previously discussed may be used. If the station is provided with a three-phase generator, the three-phase ammeter method is perhaps the most convenient. A diagram of connections for a test board as described by Mr. H. B. Taylor in the *Electric Journal* for November, 1906, is shown in Fig. 264. This diagram is for a three-phase circuit and both non-inductive and inductive load tests can readily be made by the aid of a board or table wired as here shown. The three-phase terminals are indicated by A, B, and C. At the left is indicated a potential regulator by means of which gradual changes in secondary voltage from maximum in one direction to maximum in the other direction can be obtained. It may also serve as a phase shifter, depending on whether the primary is connected to the same phase as the meter or to another phase. In the lower left hand corner is shown a small transformer with taps, connected to the potential circuit. The series auto-transformer shown at

the bottom of the diagram is put in simply to economize in energy in testing heavy current meters. This transformer is cut in or out by means of the plug switches and is used only when the current is above 10 amperes. The lamp board is connected between the switches as indicated. When the load is all on one lamp board and the potential circuit is connected to the wires supplying the lamps, the conditions are the same as on

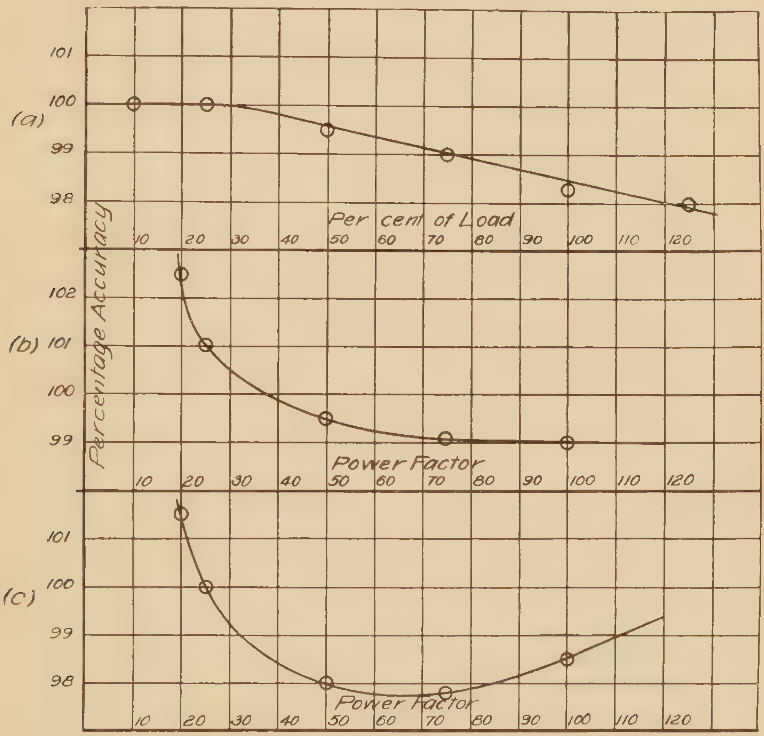


FIG. 263.

any single-phase circuit. Thus, when the lamps are connected to A and B, and the potential plug is inserted at A' and B', the test may be made on practically unity power-factor. Transferring the plug to $B'C'$ gives about zero power-factor. The power-factor will not necessarily be exactly zero, but near enough for commercial practice. For inductive load test, the load is divided between the two lamp boards, and the shunt is connected to outside mains. When the load is evenly divided between the two

lamp boards, the power-factor is 0. Changing the load on either lamp board changes the power-factor, as previously explained.

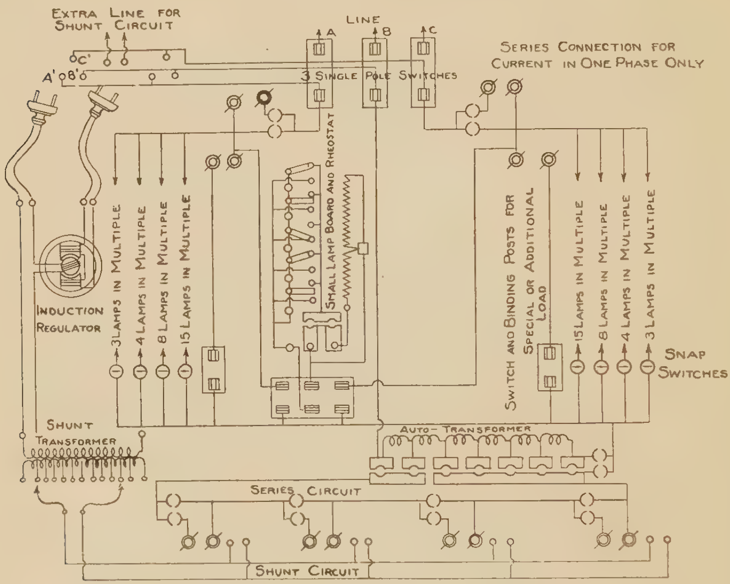


Fig. 264.

EXAMPLE

Test No. 6.—Test of watt-hour meter on inductive load.

Apparatus.—Same as in Test No. 5.

Weston ammeter.

Weston voltmeter.

Temperature.

TABLE IX

PART I

Volts	Amperes	Watts	Power-factor	No. revolutions	Time seconds	Meter watts	Per cent accuracy	Remarks
110	10.20	225	20%	12	46.7	230.6	102.5	Load constant, varying power-factor
110	8.18	225	25%	12	47.5	227.5	101.0	
110	4.10	225	50%	12	48.2	244.0	99.5	
110	2.72	225	75%	12	48.5	223.0	99.1	
110	2.05	225	100%	12	48.7	222.7	99.0	
								Curve Fig. 263b.

PART II

Volts	Amperes	Watts	Power-factor	No. revolutions	Time seconds	Meter watts	Per cent accuracy	Remarks
110	7.5	165.0	20%	6	32.4	167.5	101.5	Current constant.
110	7.5	206 2	25%	12	52.4	206.0	100.0	
110	7.5	412.5	50%	22	49.0	403.7	98.0	
110	7.5	618.6	75%	30	44.6	604.9	97.8	
110	7.5	825.0	100%	44	48.7	813.0	98.5	
								Curve Fig. 263c.

The power-factor had comparatively little effect on the accuracy of this particular meter, but this is not always the case. The curve of Fig. 265 shows what variations in the percentage of accuracy are possible when the meter is not properly adjusted.

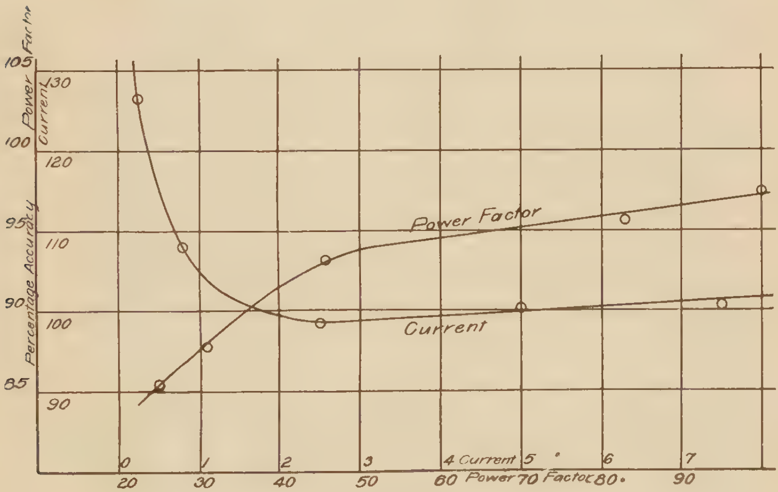


FIG. 265.

303. Testing with Standard Test Meter.—The connections for testing a two-wire meter by means of a standard test meter is shown in Fig. 266. This method of testing is fast superseding the indicating instrument method. This operation of testing with a standard test meter is very simple and consists merely in the determination of the number of rotations of the moving element of the test meter corresponding to a definite number of rotations of the tested meter.

To facilitate the work, some companies equip the test meter with an electrical contact which closes a circuit through a tele-

phone receiver, every rotation giving a click. When this method is used, observations are made by what is called an "eye and ear method." The operator observes the rotations and fractions of a rotation of tested meter corresponding to a whole number of rotations of the disk of the test meter, noting the latter by the aid of the telephone receiver.

In some cases, arrangements are made for starting and stopping the test meter at the instant the spot on the disk of the tested

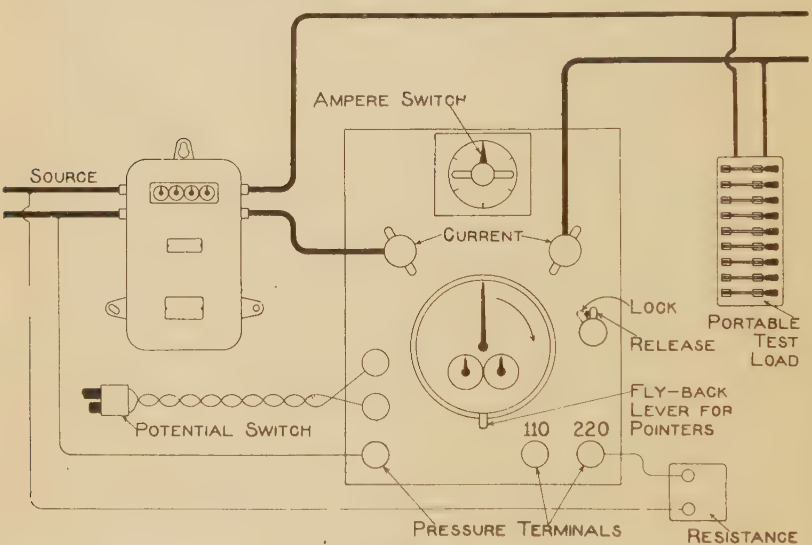


FIG. 266.

meter passes the observation window in the case. This starting or stopping may be accomplished by opening the voltage coil of the test meter, or by short-circuiting its current coil. Instead of stopping the disk, the moving element may be caused to pick up and drop the hand through the agency of a little magnetic clutch.

The chief advantages of the test meter have been pointed out. It evidently eliminates the use of stop watches, which are always troublesome in meter testing; it minimizes the effect of voltage fluctuations; and reduces the work of calculation to a minimum.

When a test meter is used whose constant is not the same as that of the service meter tested, the two meters will not make the same number of rotations. To facilitate the conversion of the number of rotations of the test meter to the equivalent of

TABLE X

WATT-HOUR CONSTANT OF SERVICE METER

WATT-HOUR CONSTANT OF TEST METER

	.125	.2	.25	.3	⅓	.4	5	.6	⅔	¾	1	1¼	1⅓	1½	2	2½	2⅔	3	3⅓	3¾	4	5	6	6⅔	7½
⅛	10	16	20	24	26.6	32.0	40.0	48	53.3	60.0	80	100													
.2	6.25	10	12.5	15	16.6	20.0	25.0	30	33.3	37.5	50	62.5	66.6												
.25	5.0	8.0	10	12	13.3	16.0	20	24	26.6	30.0	40.0	50	53.3	60											
.3	4.16	6.67	8.33	10	11.1	13.3	16.6	20	22.2	25	33.3	41.6	44.4	50	66.6										
⅓	3.75	6.0	7.5	9.0	10	12	15	18	20	22.5	30	37.5	40	45	60	75									
.4	3.12	5.0	6.25	7.5	8.33	10	12.5	15	16.6	18.7	25	31.2	33.3	37.5	50	62.5	66.6								
.5	2.5	4.0	5.0	6.0	6.66	8.0	10.0	12.0	13.3	15.0	20	25	26.7	30	40	50	53.3	60							
.6	2.08	3.33	4.17	5.0	5.55	6.66	8.33	10	11.1	12.5	16.7	20.8	22.2	25	33.3	41.6	44.4	50	55.5						
⅔	1.88	3.0	3.75	4.5	5.0	6.0	7.5	9.0	10	11.2	15.0	18.8	20	22.5	30	37.5	40	45	50	56.3					
¾	1.67	2.67	3.33	4.0	4.44	5.33	6.66	8.0	8.89	10	13.3	16.7	17.7	20.0	26.6	33.3	35.5	40	44.4	50	53.3				
1	1.25	2.0	2.5	3.0	3.33	4.0	5.0	6.0	6.66	7.5	10	12.5	13.3	15.0	20	25	26.6	30	33.3	37.5	40	50			
1¼	1.00	1.6	2.0	2.4	2.66	3.20	4.0	4.8	5.34	6.00	8.0	10	10.6	12	16	20	21.3	24	26.6	30	32	40	48		
1⅓		1.5	1.87	2.25	2.50	3.00	3.75	4.5	5.00	5.63	7.5	9.38	10	11.25	15	18.7	20	22.5	25	28.1	30	37.5	45	50	
1½			1.67	2.0	2.22	2.66	3.33	4.0	4.44	5.00	6.66	8.32	8.9	10	13.3	16.7	17.8	20	22.2	25	26.6	33.3	40	44.4	50
2				1.5	1.67	2.00	2.5	3.0	3.33	3.75	5.0	6.25	6.66	7.5	10	12.5	13.3	15	16.6	18.8	20	25	30	33.3	37.5
2½					1.33	1.6	2.0	2.4	2.66	3.00	4.0	5.0	5.33	6.0	8.0	10.0	10.66	12	13.3	15	16	20	24	26.66	30
2⅔						1.5	1.87	2.25	2.5	2.81	3.75	4.7	5.0	5.62	7.5	9.38	10	11.2	12.5	14.06	15	18.75	22.5	25.0	28.12
3							1.67	2.0	2.22	2.5	33.3	4.16	4.44	5.0	6.66	8.33	8.88	10	11.11	12.5	13.33	16.67	20	22.22	25
3⅓								1.8	2.0	2.25	3.0	3.75	4.0	4.5	6.0	7.5	8.0	9.0	10	11.25	12	15	18	20	22.5
3.75									1.78	2.0	2.67	3.33	3.55	4.0	5.33	6.66	7.11	8.0	8.9	10	10.7	13.3	16	17.8	20
4										1.87	2.5	3.12	33.3	3.75	5.0	6.25	6.66	7.5	8.33	9.37	10	12.5	15	16.66	18.75
5											2.0	2.50	2.66	3.0	4.0	5.0	5.33	6.0	6.66	7.5	8.0	10	12	13.3	15
6												2.08	2.22	2.5	3.33	4.16	4.44	5.0	5.55	6.25	6.66	8.33	10	11.11	12.5
6⅔													2.0	2.25	3.0	3.75	4.0	4.5	5.0	5.62	6.0	7.5	9.0	10	11.25
7½														2.0	2.66	3.33	3.55	4.0	4.44	5.0	5.3	6.66	8.0	8.88	10
8															2.5	3.125	3.33	3.75	4.17	4.68	5.0	6.25	7.6	8.33	9.37
10																2.5	2.66	3.0	3.33	3.75	4.0	5.0	6.0	6.66	7.5
10⅔																	2.5	2.81	3.12	3.51	3.75	4.69	5.62	6.25	7.03
12.5																		2.4	2.66	3.0	3.2	4.0	4.8	5.33	6.0
13⅓																			2.5	2.81	3.0	3.75	4.5	5.0	5.62
15																				2.5	2.66	3.33	4.0	4.44	5.0
16																					2.5	3.12	3.75	4.16	4.69
20																						2.5	3.0	3.33	3.75
25																							2.4	2.66	3.0
26⅔																								2.5	2.81
30																									2.5
33⅓																									
40																									

8	10	10⅔	12½	13⅓	15	16	20	25	26⅔	30	33⅓	40
	40											
	37.5	40										
7	33.3	35.5	41.66									
	30	32	37.5	40								
3	26.6	28.4	33.3	35.5	40							
	25	26.6	31.2	33.3	37.5	40						
	20	21.3	25	26.6	30	32	40					
[illegible]	16.6	17.77	20.8	22.2	25	26.6	3.33	41.6				
[illegible]	15.0	16.0	18.8	20.0	22.5	24	30	37.5	40			
6	13.33	14.22	16.66	17.77	20	21.33	26.66	33.33	35.55	40		
	12.5	13.33	15.62	16.66	18.75	20	25	31.25	33.33	37.5	41.66	
	10	10.66	12.5	13.33	15	16	20	25	26.66	30	33.33	40
	9.38	10	11.73	12.5	14.06	15.0	18.75	23.44	25	28.12	31.25	37.5
	8.0	8.53	10	10.66	12	12.8	16.0	20	21.33	24	26.66	32
	7.5	8.0	9.37	10	11.25	12.0	15.0	18.75	20	22.5	25	30
3	6.66	7.11	8.33	8.88	10	10.66	13.33	16.66	17.77	20	22.22	26.66
	6.25	6.66	7.81	8.33	9.37	10	12.5	15.62	16.66	18.75	20.83	25.0
	5.0	5.33	6.25	6.66	7.5	8.0	10	12.5	13.33	15	16.66	20
	4.0	4.26	5.0	5.33	6.0	6.4	8.0	10	10.66	12	13.33	16
	3.75	4.0	4.68	5.0	5.62	6.0	7.5	9.37	10	11.25	12.5	15
	3.33	3.55	4.16	4.44	5.0	5.33	6.66	8.33	8.88	10	11.11	13.33
	3.0	3.2	3.75	4.0	4.5	4.8	6.0	7.5	8.0	9.0	10	12.0
	2.5	2.66	3.125	3.33	3.75	4.0	5.0	6.25	6.66	7.5	8.33	10

(To face page 336.)

those of the service meter, Table X has been prepared. The table contains the constants of the meters most commonly used.

The table is used as follows:

Suppose a Westinghouse 5-ampere test meter is used to check a Fort Wayne service meter of the same capacity. If both meters are for 110-volt circuits, the constants are 1/3 watt-hour and 1/4 watt-hour respectively. Opposite the test meter constant 1/3 and under the service meter .25 we find 7.5, which means that for every ten rotations of service meter the test meter should make 7.5 rotations.

If a meter is used whose constant is not found in the table, the conversion can be made as follows:

Let K_1 = test meter constant

K_2 = service meter constant

Then $$R=\frac{K_2}{K_1}\times 10.$$

Where R = number of rotations of standard corresponding to ten rotations of service meter. The percentage of accuracy is then calculated as follows:

Count a definite number of revolutions of service meter, and from Table X find number of rotations the standard should have made in the same time. The ratio of the actual number of rotations of test meter to the number it should have made will give the percentage of accuracy.

EXAMPLE

A 50-ampere 110-volt type C, G. E. meter was tested with a 5- to 100-amperes Duncan test meter. The service meter made 20 rotations while the test meter made 15, what is the percentage of accuracy of the service meter? The constant of Duncan test meter is 2.5 and that of service meter 2, hence, opposite 2.5 and under 2 is found the figure 8. The test meter should have made 16 rotations, but as it made only 15 the percentage of accuracy is 15/16 = 93.7.

304. Testing of Polyphase Meters.—Since polyphase meters are simply a combination of two single-phase meters, they may be tested as single-phase meters, each phase being tested separately. Both potential coils must, however, be connected to the circuit during the test. A diagram of connections is shown in Fig. 267. It will be noticed that the current is passed through only one series coil at a time. This is accomplished by changing the connections of the 3-point switch. When one circuit of the

watt-hour meter is fully loaded, the rotating element makes one-half the normal number of revolutions. Pass through the circuit a given number of watts which must be kept constant while readings are being taken. Time the number of revolutions of the disk with a stop watch, and compute the percentage of accuracy in the same manner as in testing single-phase meters. The proper constants to be used will depend upon the make of meter as previously pointed out. When the percentage of accuracy of one circuit has been determined, load the other circuit and repeat the test.

Another method of testing polyphase meters consists in connecting the current coils in series, and the potential coils in parallel. An objection to this method is the inability to deter-

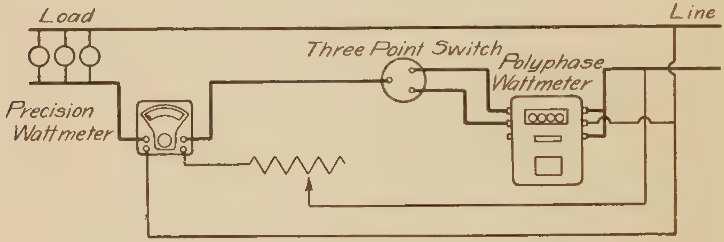


FIG. 267.

mine any unbalancing of one of the circuits. Should the meter be found to be inaccurate, and be adjusted when unbalanced, the meter will give inaccurate results when connected to polyphase circuits.

While conducting tests in accordance with either method, care should be exercised not to reverse either of the potential circuits when connected in parallel. If the series connection of current coils is used, the connections of one of the circuits should be reversed in order that the action of both coils may be in the same direction.

On account of the inconvenience mentioned and the liability of inaccurate results, it is preferable to test each circuit separately, the potential circuits remaining connected in parallel, or, if the test is made on polyphase circuit, the potentials may be left connected to the same phase as in service.

305. Test for Interference of the Two Metering Elements.—By means of a long series of tests the Electrical Laboratories found that various makes of meters differed considerably with respect

to the electromagnetic interaction of the two elements of polyphase, watt-hour meters. In some makes the interference was so small that careful tests failed to detect it. In other makes the interference was so large that serious errors might, under certain conditions, be introduced by it. As a result of these investigations the following specifications for the test of independence of elements has been incorporated in the Meter Code of the National Electric Light Association.

Element *A* of the meter under test is connected to phase I of a two-phase circuit and a certain current is sent through its current coil. The voltage coil of element *B* is also connected to phase I and accuracy readings are taken with the current in the voltage coil of *B* direct and reversed. Next the voltage coil of *B* is connected to phase II and similar readings are taken. Finally, with the voltage coil of element *B* disconnected, a current equal in value to the current passing through current coil of element *A*, is sent through the current coil of element *B*. The current through element *B* is first taken from phase I and then from phase II, and in each case both direct and reverse current readings are taken and compared. Tests are to be made with both light and full-load currents. Under any given set of conditions the difference between the direct and reversed readings must not exceed 1 per cent. In case a meter shows a greater correction than this it must be tested on a polyphase circuit with the direction of rotation on each phase given, so that in installing the meter the same direction may be preserved. If the variation in direct and reversed readings is not over 1 per cent, the meter may be tested on a single-phase circuit, the current coils being connected in series and the potential coils in parallel.

306. Test to Determine Torque.—As previously pointed out, other things being equal, the watt-hour meter whose torque-weight ratio of moving element is the largest, is the best meter; hence, a knowledge of the torque is essential. Fig. 268 shows a so-called torque balance, an instrument for determining this quantity. As shown in the diagram, the instrument consists of two arms at right angles to each other, balanced on a knife edge *C*. With the instrument is provided a clamp to which is attached a light extension rod *EN*. In operation, the arm *EN* is securely clamped to the shaft of the meter, and connected by the link to the vertical arm of balance. Normally, the weight *G* overbalances the arm *H*, and the pointer swings to the

right. When the meter is loaded, the torque exerted by the disk pulls the pointer back to 0. The load necessary to secure balance is measured on an indicating wattmeter, and the torque in gram-centimeters is calculated from the weight G and the levers GC, CV, and EN thus:

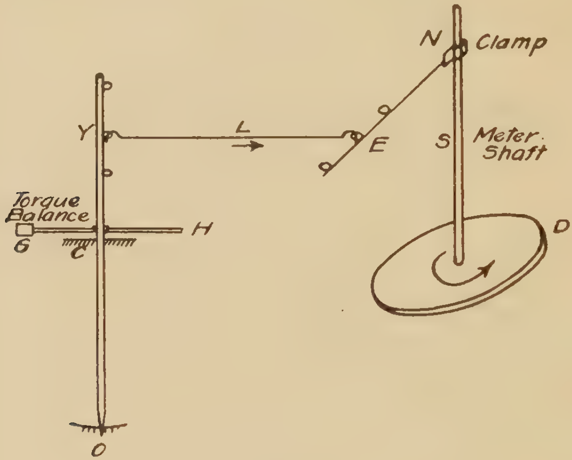


FIG. 268.

Let w = weight of G in grams

f = the pull along link L

and let T = torque of meter

Then $T = f \times EN$

and $f \times YC = w \times CG$

Eliminating f, we get

$$\frac{T}{w \times CG} = \frac{EN}{YC}$$

$$T = \frac{EN \times w \times CG}{YC}.$$

When w is in grams and the other quantities are in centimeters, the above expression gives the torque in gram-centimeters direct. The rods Y and EN are provided with several loops, and two weights are also supplied to permit the use of the balance for measuring a wide range of torques. By accurately weighing the moving elements, the torque-weight ratio is obtained by dividing the torque by the weight. Also the torque per watt of load may readily be obtained from the calculated torque and the reading of the indicating wattmeter.

Another device for measuring the torque of electrical instruments has been devised by Dr. Agnew of the Bureau of Standards and is illustrated in Fig. 269. As the figure clearly shows, it operates on the pendulum principle, the characteristic feature being scale S on a concave spherical surface of 1-meter radius turned from a brass casting. The bob D is supported from an adjustable arm so arranged that the point of support P is at the center of the sphere of which the scale S is the surface. The silk fiber supporting the bob is wound on a friction pin A and passes through a V-shaped notch in the end of the adjustable brass strip B. The whole instrument is mounted on an ordinary clamp stand, the tripod of which is fitted with leveling screws.

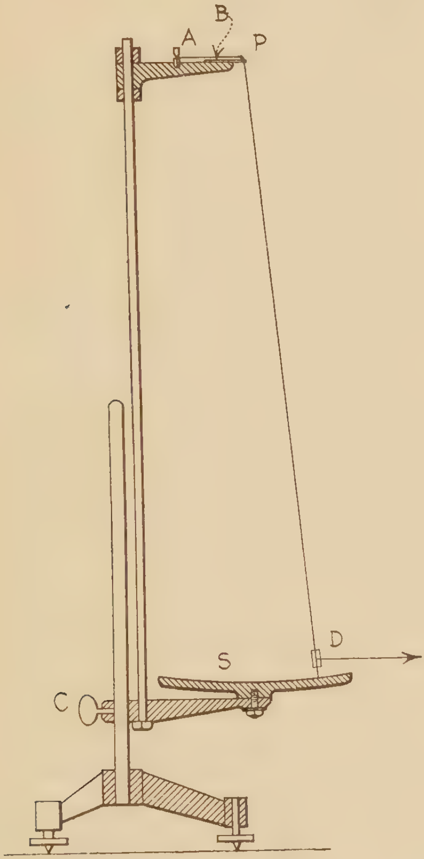


Fig. 269.

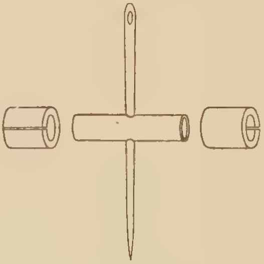


Fig. 270.

The graduations of the scale S consist of 153 concentric circles, the distance between successive circles being so spaced as to give the tangents of the angles of deflection directly. In millimeters the distance between circles is 1000 × tangent of angle of deflection. The bob consists of a small hollow brass cylinder with a fine sewing needle passed through it perpendicular to the axis, as shown in Fig. 270.

The silk fiber, by which the horizontal force to be measured is transmitted to the bob, is attached to the needle and passes out along the axis of the hollow cylinder. The point of attachment is made at the center of mass of the bob, which is adjusted to 0.5 gram. For changing the range of the instrument, concentric cylinders, each cut in halves, are made to fit snugly over the inner cylinder as indicated in Fig. 270.

In measuring the torque of a deflection instrument a horizontal thread, one end of which is connected at D, Fig. 269 is fastened to the pointer of the meter at a convenient distance from the pivot, the torque balance adjusted to the proper height, and the deflection instrument moved horizontally until the desired deflection is obtained. The torque is then given by

$$\text{Torque } T = l \times mg \times .001 \times d$$

where l = distance from pivot to point of attachment of silk fiber on pointer, mg is weight of bob, and d is the number of divisions on scale S.

In measuring the torque of a watt-hour meter it is necessary only to attach the thread to the edge of the disk, apply the current and voltage to the meter, and allow the meter to deflect as far as it will. The calculations for torque may then be made as above. The horizontal thread must be kept tangent to the disk, if this is done the distance l equals the radius of disk.

Tests made show that the torques of A. C. watt-hour meters range from 3.06 to 7.74 gram-centimeters.

307. Test for Influence of Friction.—In connection with the foregoing test, the influence of friction upon the torque may be advantageously determined. First adjust the friction compensation so that the meter is just balanced at no-load. To secure this, the compensation should be just sufficient to cause the meter to creep at no-load when slightly jarred. When the compensation has been properly adjusted, full-load is applied and speed determined. Then the compensating coil is disconnected and the speed is again determined at the same load. The so-called "friction torque ratio" is the ratio of the decrease in speed with compensating coil disconnected to the speed with coil in circuit. Thus a decrease of 5 per cent in speed would mean a ratio of 1 : 20. The smaller this ratio, the less the influence of friction, and from this view point, the better the meter.

308. Test to Determine Influence of Stray Field.—For this test,

mount the meter in such a way that current carrying conductors may be conveniently brought near.

First, test the meter under conditions making impossible the existence of an external magnetic field. Having determined the accuracy under these conditions, place a conductor carrying a current in various positions. Direct-current meters should be tested under the influence of a direct-current field, and alternating-current meters under the influence of alternating-current fields of the same frequency and in phase with the current in the meter. Run the conductor in a horizontal position back of the meter at a distance of 15 in. from the axis of the moving element. Pass a current equal to twice the capacity of the meter through the conductor, andd etermine the accuracy of the meter at 100 per cent load. Change the position of conductor, and repeat the test.

For determining the influence of stray field on the accuracy of alternating-current meters, the meter committee of the National Light Association recommends the following:

"The meter to be tested shall be subjected to an alternating stray field of the same frequency as that of the testing current, and produced by a straight conductor six (6) feet long, with return leads arranged to form a rectangle six (6) feet square, lying in a plane parallel to the switchboard. A current of fifty (50) amperes in phase with the voltage applied to the meter shall be passed through this conductor."

Separate tests are to be made when the stray field conductor is placed successively in the following positions:

1. "Behind the meter in a horizontal position at the level of the moving element and at a distance of fifteen (15) inches from the axis of the moving element.

2. "Directly behind the center line of the meter, in a vertical position and at a distance of fifteen (15) inches from the axis of the moving element.

3. "Vertically, at the same distance in front of the switchboard as the axis of the element and at a distance of fifteen inches to the right or left of the meter center line, the return leads being so arranged that the loop which they form does not surround or include the meter."

In connection with the foregoing, tests may be made to determine the minimum distance which should be maintained between meters when in service. To do this, first determine the accuracy of two meters on 5 and 10 per cent loads. Test each

meter separately when there is no load on the other meter. Maintain a constant load of 100 per cent on one meter and determine the accuracy of the other meter at various distances apart. In varying the distances, always move the meters having the constant load. From the data thus obtained, the distance at which the accuracy of the meter is within permissible limits can readily be determined.

309. Test to Determine Loss in Potential Coil.—To determine this loss, two methods are available; it can be measured directly or computed from the resistance and voltage. The method of measurement will perhaps be the most convenient for alternating-current meters, while for direct-current meters, the method of calculation will give more accurate results.

To measure these losses, connect the potential coils of several meters of the same rated voltage in parallel and, if the meters are for direct-current circuits, measure the total current and voltage applied. The energy lost will be equal to the product of current and applied voltage, which divided by the number of meters will give the average loss per meter.

On account of the low power-factor of the voltage circuit of alternating-current meters it is preferable to measure the resistance of the coil and calculate the loss by

$$\text{Watt loss} = I^2R$$

where I is the voltage coil current and R the coil resistance. If a low reading wattmeter is available, this may be used, providing proper corrections are made for its inaccuracy on low power-factors. If the resistance of the voltage coil is not known, it can be measured in a variety of ways. When direct current is available, it can be measured by the drop of potential method which will necessitate a milliammeter and a high resistance voltmeter.

The loss in current coil can be calculated more accurately than measured. First, the resistance of the coil must be accurately measured, and then the loss at any current will be given by

$$\text{Watts loss} = I^2R \text{ as above.}$$

Since the resistance of the current coil is small special precautions must be taken in its measurement. The drop of potential method may also be used, but in this case the voltage drop will be small, and, hence, a comparatively high resistance millivoltmeter will be necessary. The current can be measured by an accurate ammeter of proper range.

CHAPTER XXII

INSTRUMENT ERRORS

310. Sources of Error.—So far, very little has been said in a systematic way about errors to which measuring instruments are liable, although some of the most important sources of error have been pointed out. The following discussion is adapted from a bulletin on "Testing of Electrical Measuring Instruments," issued by the Bureau of Standards, and is the result of many investigations and tests.

An important matter in connection with the use of electrical instruments, is the question of sources of error and the best means of securing a required degree of accuracy from a given set of instruments. It may be said in the beginning that in very many cases the user underestimates the errors and overestimates the accuracy of the result. This is partly due, in many cases, to the lack of means for checking the results obtained; as there is no check, inaccurate results are passed without suspicion of their inaccuracy, and the maker's representations as to accuracy, which are sometimes much exaggerated, are accepted as correct for an unlimited time after the maker's test and for all conditions of use. Portable instruments are often used in places subject to strong magnetic stray fields or extreme temperatures; subsequent comparison with other instruments in the testing room may show that the working instruments have small errors, while their performance under unfavorable conditions may have been five per cent or more in error.

In all physical measurements, to attain a relatively high degree of accuracy, care must be exercised to distinguish between at least three possible sources of error. These are:

1. Inherent errors of the instrument.
2. Errors due to the method of measurement.
3. Errors of observation.

311. Inherent Errors.—Inherent errors are those due to imperfections of materials, limitations of accuracy in construction, physical conditions determined by quantities measured, etc.

In short, the properties of materials used and the inevitable inaccuracies of construction make it impossible to construct a perfect measuring instrument. Inherent errors cannot be entirely eliminated although by improved and refined methods of construction, and by a knowledge of their presence, their influence may be reduced.

312. Inherent Temperature Errors.—Among inherent errors may first be mentioned those due to the effect of change in the temperature of the various parts of the instrument. Taking, for example, a voltmeter of the permanent-magnet moving-coil type; if it reads correctly at a given point at a certain temperature, it will, in general, show a small error at any other temperature. An increase in temperature of the working parts of an instrument has three effects: a decrease in the strength of the magnet, which tends to reduce the reading at any given voltage; an increase in the resistance of the moving coil which also tends to reduce the reading; and a third effect is the weakening of the controlling spring, which, to some extent, compensates the other two. The temperature coefficient of the spring is about 0.04 per cent per degree centigrade; that of the magnet is not so definite. The resultant temperature coefficient of the instrument is quite small, as a rule. The question of making such an instrument practically free from temperature error is then relatively simple, as it is only necessary to have a low value of the resistance temperature coefficient of the circuit. This is accomplished by making the moving coil of low resistance, usually of copper, and mounting in series with it an external resistance of manganin whose resistance temperature coefficient is negligible within the working temperature range of the instrument. Thus, if the value of the external series resistance is ten times the resistance of coil, the resistance temperature coefficient of both will be reduced to one-tenth. The higher the series manganin resistance, the lower the resulting temperature coefficient. It is thus seen that where a voltmeter has low and high ranges, a greater inaccuracy due to temperature changes is to be expected on the lower ranges. If a low range voltmeter is to have a low temperature coefficient, the moving coil must be constructed with few turns and have a very low resistance.

So far, it has been assumed that the temperature within the instrument is uniform. This would be the case if no source of heat existed within the instrument. Most instruments, however,

contain sources of heat. Unequal heating is, therefore, possible, and some error will result from this cause. When, as in alternating-current voltmeters and wattmeters, a large portion of the resistance is so-called dead resistance in series with the working element, this heat-producing resistance should be partitioned off from the working system and properly ventilated. Unless this is done, the instrument cannot be left in circuit for any length of time without error.

An important instance of large errors through unequal heating within the instrument is found in connection with the permanent-magnet moving-coil ammeter with internal copper shunt. In this instrument the moving coil of copper is connected to the terminals of a copper shunt within the instrument case. The temperature coefficient of the moving coil used alone as an ammeter is quite small, and, as changes of room temperature will not alter the relative values of current in moving coil and shunt, such an instrument would seem at first sight to be almost an ideal one. The performance of the low range instruments is quite good. The performance of the higher range instruments is not very satisfactory and they are suitable for only rough work. In general, it may be said that up to about 25 amperes, well-made instruments of this type will give fairly good service; for currents above that they should not be used for accurate work. The use of manganin for precision shunts is now recognized as the best practice. For large currents, the shunt should be separate from the instrument. In practice, it is desirable to keep down the weight of ammeter shunts as well as the waste of power in them. To fulfill these conditions, millivoltmeters are made to give full-scale reading for very low voltages across the terminals. A very common value of this voltage is 50 millivolts, where the instruments are intended for switchboard use. As the shunts are usually made of a material of a low temperature coefficient, while the millivoltmeter circuit consists largely of copper, the error due to varying room temperature may be considerable. When the instrument is intended for commercial switchboard use, this effect of room temperature is of no great moment; for precision work in the laboratory, or in the testing of direct-current watt-hour meters, the temperature errors above referred to become quite objectionable. To remedy this, most makers manufacture a line of millivoltmeters which have added to copper coil a manganin resistance which has from four to

nine times the resistance of the copper coil. This cuts down the temperature coefficient of the instrument, but requires a higher drop across the shunt, namely, from 150 to 200 millivolts at full-load.

Aside from the errors due to heating, changes of several per cent in the resistance are caused by the method of bolting the copper bar to the shunt. To overcome this difficulty, the terminal blocks should be made longer, so as to make the lines of current flow more nearly parallel at the junction of terminal and resistance metal, near which junction the potential terminals should be located. The same result may be attained by constricting the section of the terminal block considerably between the current and potential terminals.

For precision ammeter shunts, the most satisfactory material is manganin, and the best makers are adopting it in spite of some additional trouble involved in the manufacture of the shunts.

The influence of temperature upon the electromagnetic (soft iron) ammeter, with spring control, is to lower the permeability of the iron, and also to reduce the elasticity of the spring. Since the effect on the spring just about neutralizes the effect on the soft iron core, the ammeter is very nearly independent of ordinary temperature changes.

In the electrodynamometer type of instrument, a temperature change affects mainly the spring. Such instruments will read too low at temperatures below that at which they are calibrated, the temperature correction being about 0.04 per cent per degree centigrade. This assumes that the potential or shunt circuits contain so small a percentage of copper that their change in resistance with temperature does not sensibly affect the result. For ordinary ranges of voltage, this is the case.

In the soft iron voltmeter the temperature coefficient depends mainly upon the ratio of the resistance of the copper coil to the total resistance of the instrument. This ratio is a question of design, depending upon the range of the instrument and the amount of power required for its operation. The temperature coefficient of well-made voltmeters of this type, for the usual commercial voltages, is quite small, and for practical work need not be taken into account, except in extreme cases.

The accuracy of watt-hour meters is also affected by temperature changes. Although enough experimental data are not

available to state definitely the relation between temperature change and resulting error, in general the effect is as follows:

An increase in the temperature of the voltage and compensating coils increases their resistances. This increase in resistance decreases the voltage coil current and consequently decreases the driving torque.

An increase in the temperature of the retarding disk has a like effect upon its resistance, and consequently the retarding torque is decreased both by the influence of temperature upon disk and retarding magnets, whose permeability is decreased by increase in temperature.

Temperature rise thus decreases both driving and retarding torques and, theoretically, the two effects may neutralize each other; actually, however, the error often is 1 per cent, or even 4 per cent per 10° C.

The effect of temperature changes on induction meters is less than upon the electrodynamometer type. This is mainly due to the fact that both the driving and retarding forces operate upon the same disk. The error in induction meters is about 1 per cent per 10° C.

313. Inherent Errors Due to Time and Use.—Another source of errors, to which most instruments are liable, is due to changes in the properties of the materials of which the instruments are made, with time and use. In spite of the labor which has been expended on making permanent magnets and the investigations concerning their properties, individual magnets of the best makes will occasionally show changes with time. When the instrument is new, it may, for a time increase in strength; later, it is more likely to decrease. Controlling springs also show slight changes with time. If the effect of the weakening of the magnet is offset by the weakening of the springs in a direct-current instrument, the accuracy is unchanged.

When direct-current instruments are used in the neighborhood of dynamos or motors, or in other locations subject to strong stray fields, as, for example, near conductors carrying heavy currents, their indications will be considerably affected at the time of use, and in addition permanent changes may occur in the permanent magnets. Stray fields are liable to be found in the neighborhood of switchboard instruments and hence these should be shielded from them. The iron case very generally used for such instruments affords considerable protection, but, in addition,

it is advisable to keep heavy currents well away from the instruments, and, as a further precaution, important instruments that are permanently attached to the switchboard, should be checked in position, under working conditions. Care must be taken that the portable instruments used in this checking are in a location not exposed to stray fields; if this is impossible, the mean of two readings should be taken; for the second reading, the instrument is turned 180 degrees from its first position.

314. Inherent Mechanical Errors.—Among the most common sources of error may be mentioned friction, defective preformance of springs, scale marking, and lack of balance of moving coil. The friction of pivots on a good indicating instrument should not be noticeable, unless it is old or has been roughly used. The friction of the pen on recording instruments is the main cause of inaccuracy of these instruments. It is evident that friction cannot be entirely eliminated, and hence, the problem is to have a spring strong enough to cause the coil to take up its proper position irrespective of friction. If the spring is strong, the torque for a full scale deflection will also have to be high. According to one writer, this torque expressed in gramcentimeters should not be less than one-sixth the weight of the coil in grams; this weight includes that of springs, index, etc. Another author gives a minimum value considerably lower, namely, one-twentieth. Both authorities assume a deflection of about 90°, this being nearly the full scale deflection for direct-current indicating instruments. It is desirable to keep the ratio of torque to weight as high as possible in all electrical measuring instruments. It should be noted, however, that an instrument with a very high torque may really be a poor instrument, if the high torque is obtained by using an excessively heavy moving system.

The most important factor affecting the accuracy of integrating meters is friction of brushes, bearings, and registering mechanism. If this friction were constant, any errors introduced by it could be compensated, but since it is an extremely variable factor, under favorable conditions the error due to it may be appreciable. Tightening the brushes on a commutator meter may cause a 10 per cent error on a 10 per cent load. To reduce the effect of friction to a minimum, the meter should operate at a comparatively low speed and the ratio of driving torque to weight of moving element should be high. High-speed and heavy-moving elements increase friction.

315. Defective Performance of Springs.—An indicating instrument whose pointer stands exactly at zero with no current flowing, will not always indicate zero after use on full-load. If the full-load is on for only a moment, the pointer will usually return to zero within the limit of reading. If full scale deflection be maintained for an hour or so, it will probably be found that the pointer does not return exactly to zero when the circuit is broken; if the full scale deflection lasts several hours, the discrepancy will be still greater. This zero shift is only temporary, and gradually disappears. The amount of this zero shift varies in different classes of instruments, and in different instruments of the same class. In first-class voltmeters it should be just noticeable; in millivoltmeters, as a rule, it is considerably greater, although occasionally a millivoltmeter will show very good performance in this respect. The inaccuracy due to zero shift is most marked if the instrument is used for a small deflection soon after it has sustained a large deflection for a considerable length of time. The reason for the poorer performance of millivoltmeters lies in the necessity of using springs whose electrical properties approach those of copper. For voltmeters no such limitations exist, and the springs may be made of bronze whose mechanical properties are best suited for the purpose irrespective of electrical resistance. The design of a spring determines its performance, as well as the material of which it is made; the shape, length, and thickness determine its elastic limit when made of a given material.

In discussing controlling springs, it was stated that the torque is proportional to the angle or distance through which it has been distorted. This proportionality is not exact, and any measurements based upon the exactness of the assumption may lead to errors of 1 per cent or more. This fact is brought out more clearly in Fig. 271, which shows some calibration curves of several Siemen's dynamometers and a precision instrument. These curves are due to Bradshaw. The error in deflection is plotted vertically and the actual deflection horizontally. Thus, when the deflection on the precision instrument is 150, the instrument whose curve is marked 1 reads two divisions too high. At 50 the reading is 1.5 divisions too low, which shows that the torque of the spring does not follow exactly the assumed law. In the ordinary direct-reading instruments this variation does not appear if the scale has been properly graduated. However, if by accident the spring should be distorted or bent out of its original

shape, the scale will no longer be correct, even though by shifting the spring holder the pointer be brought back to zero.

From the foregoing, it is evident that a first-class instrument should have a scale graduated for that particular instrument. It is not necessary to determine every division by exact test, especially on instruments for commercial use. It is usually considered sufficient to determine, say ten or fifteen points, and fill in the intermediate points, preferably by some mechanical method.

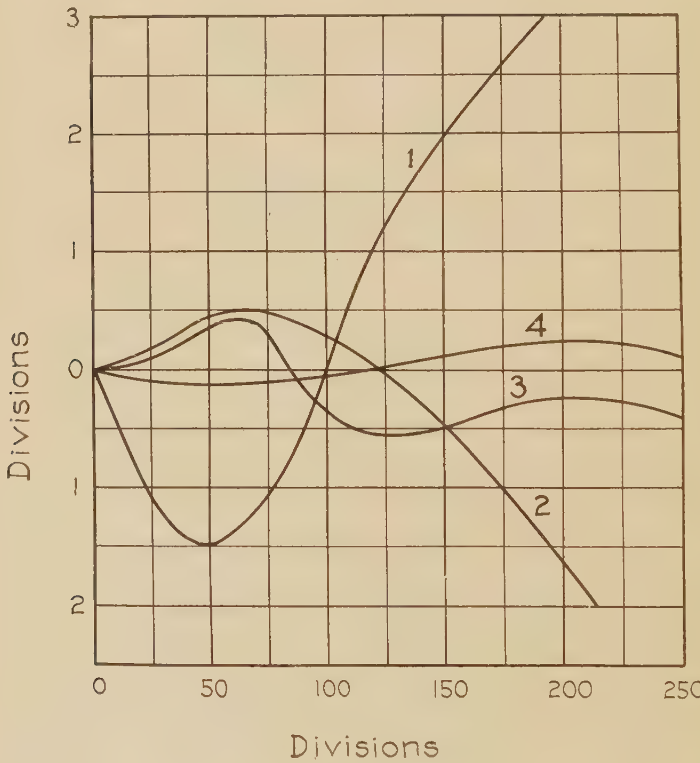


Fig. 271.

Any change in the relative position of the working parts of an instrument will affect its calibration. Thus, the simple removal and replacement of the pole pieces of a direct current instrument—in fact, even the tightening of the screws that hold the pole pieces—will affect the distribution of the magnetic flux so that a scale, which fitted before the operation, will now show appreci-

able errors. Any accident, and mechanical change or adjustment of an instrument should be followed by a test. Some makers claim for their portable direct-current instruments a possible accuracy of 0.1 scale division. No such accuracy need be expected in the average instrument.

316. Errors Due to Balancing.—When a portable instrument is held in different positions when not connected to a circuit, it will be observed that the pointer will not remain at the zero position. This deviation is due to imperfect balancing of the moving parts of the instrument. A portable direct-current voltmeter examined in this way will show a deviation of not more than a few tenths of a scale division if in good balance; millivoltmeters, wattmeters, and alternating-current instruments, all of which usually have a smaller ratio of torque to weight of moving parts than the direct-current voltmeter, may show deviations as great as one division. Most portable instruments are intended to be used on a level support in a horizontal position, and to avoid errors on account of imperfect balancing they should be used and tested in that position. All other instruments should be tested in the position in which they are to be used.

317. Errors of Use.—In addition to the foregoing inherent errors of construction, there are what may be called inherent errors of use. The instrument may be used under circumstances such that errors in the result are inevitable. One of the most common sources of error of this kind is due to stray magnetic fields either from other instruments, or conductors carrying heavy currents. Even the proximity of unmagnetized masses of iron may influence the readings. The effect of stray field depends upon the nature of the field, and the design of the instrument. The influence of a direct-current field is constant so long as the current is constant, and varies with the current. Its effect upon a direct-current moving coil instrument may be obtained by reading the instrument in a given position, quickly turning it through an angle of 180 degrees and reading it again. One-half the sum of the readings will be the true reading. So long as the stray field remains constant, the error may be determined as above and may be allowed for by a percentage correction for readings on any part of the scale, the instrument remaining in a fixed position.

The effect of the stray field is to change the strength of the field of the magnets of the instrument; the distribution of the

field is not perceptibly changed. The resulting deflection of the instrument may then be considered as proportional to the product of current in moving coil and composite field.

Thus let H = original magnet field

let H' = component of stray field parallel to magnet field

and I = current in moving coil of instrument.

Then the reading of the instrument in one position may be written

$$R = K(IH + IH').$$

With the instrument turned through an angle of 180° the effect of the stray field will be opposite to that in the former case, hence,

$$R' = K(IH - IH').$$

Adding the two deflections or readings we get

$$(R + R') = 2KIH$$

whence

$$\frac{(R+R')}{2} = KIH, \text{ the true reading.}$$

Subtracting the two readings the difference is

$$R - R' = 2KIH'$$

or

$$\frac{R - R'}{2} = KIH',$$

the effect or change in reading due to stray field. The percentage error is then

$$\frac{\frac{R-R'}{2}}{\frac{R+R'}{2}} = \frac{KIH'}{KIH}$$

or

$$\frac{R-R'}{R+R'} = \frac{H'}{H}.$$

This shows that so long as H' remains constant, the error is a constant percentage of the true reading.

Since, in most cases it is necessary to calculate the true reading from the indication of the instrument, it is better to give the

error as a per cent of the instrument indication. This can readily be done as follows:

$$\frac{R-R'}{2}=KIH'$$

or change in true reading due to influence of stray field. If R is the indication of the instrument in the original position, the percentage error of R is

$$\frac{\frac{R-R'}{2}}{R}=\frac{KIH'}{KI(H+H')}=\frac{H'}{H+H'}$$

and is also constant. When this percentage error has once been determined, it may be applied for obtaining the true reading at other indications of the instrument. The true reading will be equal to

$$R\left(1\pm\frac{R-R'}{2R}\right)$$

the plus sign being used when the actual indication of the instrument is less than the true reading, and the minus sign when the conditions are the reverse. With instruments of the electro-dynamometer type the case is different. Since the deflection in this type of instrument is due to the reaction of the fields in two coils, a position of the instrument can be found such that the stray field produces no effect for a given reading of the instrument; that is, at a given position of the moving coil the stray field exerts no torque upon the moving coil. At any other position of the moving coil the stray field will have some effect, and this effect will vary with the deflection. Even weak fields, such as that of the earth, have appreciable effects, and the usual method of avoiding error in the test of such instruments, consists in measuring with standard instruments the current, voltage, or power required to bring the pointer of the instrument under test to a given point on the scale; the direction of current is then reversed, and a second measurement made with the same reading of the instrument under test. The arithmetical mean of the two readings of the standard instruments will give the correct value of the quantity measured, and what should be indicated by the instrument under test were no external field present.

When such instruments are used on alternating currents, the constant stray fields will have no effect. Here the trouble is more

likely to come from heavy alternating currents of the same, or nearly the same, frequency as those of the quantity being measured. Errors due to this cause may best be avoided by twisting the leads together in such a way that the inductive effect of the current is eliminated or at least very much weakened. If other sources of stray field are supposed to exist, the instrument may be turned through an angle of 180° and the effect determined, as explained above.

A strong magnetic field, due to an alternating current, is liable to partially demagnetize the permanent magnet and cause the instrument to read low permanently unless repaired. Unless the field is strong enough to cause this partial demagnetization it will have no effect whatever upon the reading. Instruments of the electrodynamometer type are sometimes made astatic to avoid the errors due to stray field. This is accomplished by constructing the instrument with two moving coils which are so connected that a stray field produces equal and opposing torques on the two coils. If the stray field is the same at the two coils, no error is produced. It is not safe, however, to assume that such instruments may be used without error in close proximity to heavy currents, as both theory and experiment show that appreciable errors may result. The same precautions should be taken with astatic instruments as with those of ordinary form.

The hot-wire and electrostatic instruments are not affected to any appreciable extent by stray fields, as their action is not based on magnetic reaction. Induction instruments are also free from serious error from the influence of stray fields, since their air-gap is small and their fields quite strong.

318. Electrostatic Effect.—The electrostatic attraction or repulsion between the moving parts of an instrument and some stationary part may cause appreciable errors. Rubbing the cover-glass with a handkerchief, cloth, or even the hand will often cause the pointer to have an initial deflection. The remedy for this consists in breathing upon the glass, the moisture of the breath causing the charge to disappear.

In calibrating indicating wattmeters by means of two separate sources of electromotive force, a similar effect is likely to be experienced. When the potential applied to the fixed coil is much different from that applied to the moving coil, an electrostatic force is exerted between the two, and an appreciable error in the reading may result.

319. Contact Errors.—Still another source of error is the lack of good contact. This may be the fault of the individual connecting the instrument to the circuit, or it may be due to faulty construction. Millivoltmeter readings are especially liable to be erroneous due to this cause. A millivoltmeter is usually connected to the shunt by two leads, and in most instruments now in use this involves four contacts in the instrument circuit, two at the shunt and two at the binding posts. As the resistance of the instrument is only a few ohms, a corroded or dirty terminal or binding-post surface may introduce errors which may amount to several per cent. Binding posts and lead terminals of all precision instruments should be nickel plated to avoid corrosion. The use of any substance which is liable to corrode the contacts should not be permitted in either the construction or use of the instrument. Soft-rubber tubing contains sulphur which will corrode copper, and for that reason should be avoided in instrument construction.

320. Errors Due to Thermo-electromotive Forces.—In ordinary "station shunts" some errors are due to thermo-electric effects. That is, the heating of the junction of the resistance metal to the terminal block sets up thermo-electric currents which may be quite appreciable.

Errors due to thermo-electric effect may be observed by allowing the current to flow until the shunt has assumed working temperature; on breaking the circuit, the millivoltmeter will show a small current flowing under the action of thermal-electromotive forces; this may be distinguished from zero shift by opening the millivoltmeter circuit.

A bad contact at one end of an ammeter shunt, or even the use of too small a current cable at one end of the shunt, will cause that end to be heated more than the other. With many shunts now in use, this will cause considerable error from thermal-electromotive forces. For accurate work the shunts should be placed in oil and the temperature equalized by constant stirring. Some provision for cooling and constant stirring of the oil must be made when large currents are used unless it is arranged to short circuit the shunt terminals for most of the time.

321. Errors Due to Combination of Instruments.—The amount of energy consumed by the ordinary electrical measuring instruments is small, and may be neglected when the instruments are used in commercial practice. When, however, the instruments

themselves are being tested, and for accurate measurements, it is necessary to know what errors may be introduced by neglecting these losses. Some arrangements of instruments introduce larger errors than others, and also the percentage error with the same arrangement may be higher in one case than in another,

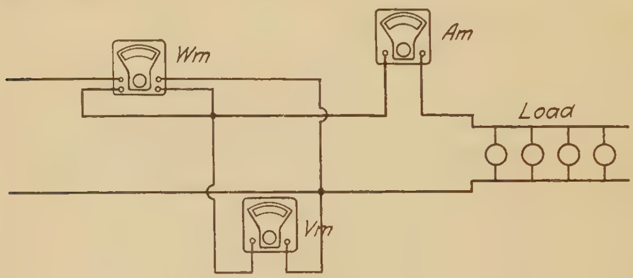


FIG. 272.

depending upon the characteristics and ranges of instruments used. When a wattmeter, ammeter, and a voltmeter are used, the instruments may be connected in several ways, two of which are indicated in Fig. 272 and 273. In Fig. 272 the ammeter and voltmeter are both connected between the wattmeter and load. When such a connection is used, the current indicated by the

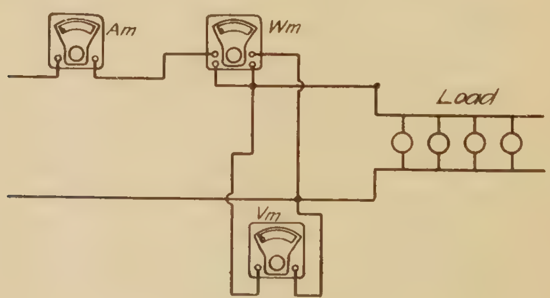


FIG. 273.

ammeter is less than the current passing through the current coil of wattmeter. If the wattmeter is being tested, the product of current and pressure, $I \times E$, is too low; and if power consumption of load is being determined, the wattmeter indication is too high. The excess in current is plainly that flowing through voltmeter and pressure coil of wattmeter. If the wattmeter is properly

compensated, no correction need be made for its pressure coil current. The voltmeter correction will be $\frac{E^2}{R}$, where E is the load pressure and R the resistance of voltmeter coil.

When the connections of Fig. 273 are used, the wattmeter indication should equal IE, but this is greater than the power consumption of load.

In the combined use of the three instruments the best method of connection will depend somewhat upon the conditions of test. The instruments should be so connected that if corrections are necessary they can easily be made.

322. Errors Due to Voltage and Current Transformers.—It is beyond the scope of this text to discuss fully the characteristics of voltage and current transformers and methods of their testing. A comprehensive discussion of this subject may be found in a paper by L. T. Robinson on "Electric Measurements on Circuits Requiring Current and Potential Transformers," page 1005, Vol. XXVII of the Transactions of the A. I. E. E., and also in a paper by P. G. Agnew and T. T. Fitch on "The Determination of the Constants of Instrument Transformers," Bulletin of the Bureau of Standards, Vol. 6, No. 2.

In order that excessive errors may not be introduced in the measurement of electrical quantities when instrument transformers are used, it is necessary to know the ratio of transformation and phase relation between primary and secondary currents. In current transformers, the ratio of transformation must be known in connection with the ammeter and leads with which it is to be used. Slight phase shifting due to the fact that primary and secondary quantities are not exactly in opposition, introduces no errors in current and voltage measurements. In connection with the measurement of power, it is necessary to consider the phase displacement, and make the necessary corrections.

323. Errors Due to Frequency and Wave Form.—Alternating-current instruments that are correct on one frequency may, and quite often will, give erroneous readings on other frequencies. No appreciable errors will be introduced into the readings of the electrodynamometer type instruments by a few per cent variation in the ordinary commercial frequencies, unless the inductance of the coils is excessive.

All instruments that contain iron in their construction, unless

accurately compensated, will give different readings on different frequencies. Many modern instruments can now be obtained in which the compensation makes them practically free from this error. It is a good plan, nevertheless, to test a meter on the frequency at which it is to be used.

Induction watt-hour meters are considerably affected by frequency and wave form, a 5 per cent variation in frequency may cause from 1 to 2 per cent error on one-half load. A certain meter registered correctly on a current from one generator but showed a 15 per cent error on current of same frequency but different wave form supplied by another generator.

324. Errors of Observation.—The errors due to reading are very variable and depend upon two factors, one upon the construction of the instrument, and the other upon the skill and care of the observer. For accurate readings, the pointer should be constructed with a flattened end, the plane of the pointer being perpendicular to the scale, under which should be mounted a mirror. The reading is then made by closing one eye and with the other noting that the pointer is exactly over its reflection from the mirror. In this way errors due to parallax are easily avoided.

In any series of measurements of the same physical quantity, we find that the results differ slightly one from another owing to imperfections in the instruments, or errors in making observations. Errors of observation are likely to be positive as often as negative, and, if a sufficient number of readings are taken, may in the long run, be considered as having little influence upon the result. The greater the number of individual observations, the less likely will the mean of the individual observations deviate very far from the correct value. The conditions, and degree of accuracy desired, will in each case determine the number of readings to be taken.

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